Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

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ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2019**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

DC (PQ) 180655

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $y = \dfrac{\sin x}{\ln x^2}$, find an expression for $\dfrac{dy}{dx}$. [4]

2 Find the values of k for which the equation $(k-1)x^2 + kx - k = 0$ has real and distinct roots. [4]

3 **(i)** Given that $x-2$ is a factor of ax^3-12x^2+5x+6, use the factor theorem to show that $a=4$. [2]

(ii) Showing all your working, factorise $4x^3-12x^2+5x+6$ and hence solve $4x^3-12x^2+5x+6=0$. [4]

4 A circle has diameter x which is increasing at a constant rate of $0.01\,\text{cm}\,\text{s}^{-1}$. Find the exact rate of change of the area of the circle when $x = 6\,\text{cm}$. [5]

5 **(i)** Express $5x^2 - 15x + 1$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

(ii) Hence state the least value of $x^2 - 3x + 0.2$ and the value of x at which this occurs. [2]

6 **(a)** State the order of the matrix $\begin{pmatrix} 0 & 1 & 4 & 8 \\ 5 & 8 & 1 & 6 \end{pmatrix}$. [1]

(b) $\mathbf{A} = \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix}$

(i) Find $\mathbf{A}^{-1}$. [2]

(ii) Hence, given that $\mathbf{ABA} = \mathbf{I}$, find the matrix $\mathbf{B}$. [3]

7 **(a)** Solve $\lg(x^2 - 3) = 0$. [2]

(b) **(i)** Show that, for $a > 0$, $\dfrac{\ln a^{\sin(2x+5)} + \ln\left(\dfrac{1}{a}\right)}{\ln a}$ may be written as $\sin(2x+5) + k$, where k is an integer. [3]

(ii) Hence find $\displaystyle\int \frac{\ln a^{\sin(2x+5)} + \ln\left(\frac{1}{a}\right)}{\ln a}\,\mathrm{d}x$. [3]

8 **(a)** In the binomial expansion of $\left(a-\frac{x}{2}\right)^6$, the coefficient of x^3 is 120 times the coefficient of x^5. Find the possible values of the constant a. [4]

(b) **(i)** Expand $(1+2x)^{20}$ in ascending powers of x, as far as the term in x^3. Simplify each term. [2]

(ii) Use your expansion to show that the value of 0.98^{20} is 0.67 to 2 decimal places. [2]

9 **(a)** Solve $6\sin^2 x - 13\cos x = 1$ for $0° \leqslant x \leqslant 360°$. [4]

(b) **(i)** Show that, for $-\frac{\pi}{2} < y < \frac{\pi}{2}$, $\frac{4\tan y}{\sqrt{1+\tan^2 y}}$ can be written in the form $a\sin y$, where a is an integer. [3]

(ii) Hence solve $\frac{4\tan y}{\sqrt{1+\tan^2 y}} + 3 = 0$ for $-\frac{\pi}{2} < y < \frac{\pi}{2}$ radians. [1]

10 **(a)** Find the unit vector in the direction of $5\mathbf{i} - 15\mathbf{j}$. [2]

(b) The position vectors of points A and B relative to an origin O are $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ 7 \end{pmatrix}$ respectively. The point C lies on AB such that $AC : CB$ is $2 : 1$.

(i) Find the position vector of C relative to O. [3]

The point D lies on OB such that $OD : OB$ is $1 : \lambda$ and $\overrightarrow{DC} = \begin{pmatrix} 6 \\ 1.25 \end{pmatrix}$.

(ii) Find the value of λ. [3]

11 The velocity, $v\,\mathrm{m\,s^{-1}}$, of a particle travelling in a straight line, t seconds after passing through a fixed point O, is given by $v = \dfrac{4}{(t+1)^3}$.

(i) Explain why the direction of motion of the particle never changes. [1]

(ii) Showing all your working, find the acceleration of the particle when $t = 5$. [3]

(iii) Find an expression for the displacement of the particle from O after t seconds. [3]

(iv) Find the distance travelled by the particle in the fourth second. [2]

12 **(a)** The functions f and g are defined by

$$f(x) = 5x - 2 \quad \text{for } x > 1,$$
$$g(x) = 4x^2 - 9 \quad \text{for } x > 0.$$

(i) State the range of g. [1]

(ii) Find the domain of gf. [1]

(iii) Showing all your working, find the exact solutions of $gf(x) = 4$. [3]

Question 12(b) is printed on the next page.

(b) The function h is defined by $h(x) = \sqrt{x^2 - 1}$ for $x \leqslant -1$.

(i) State the geometrical relationship between the graphs of $y = h(x)$ and $y = h^{-1}(x)$. [1]

(ii) Find an expression for $h^{-1}(x)$. [3]