Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0765613034

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2021**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

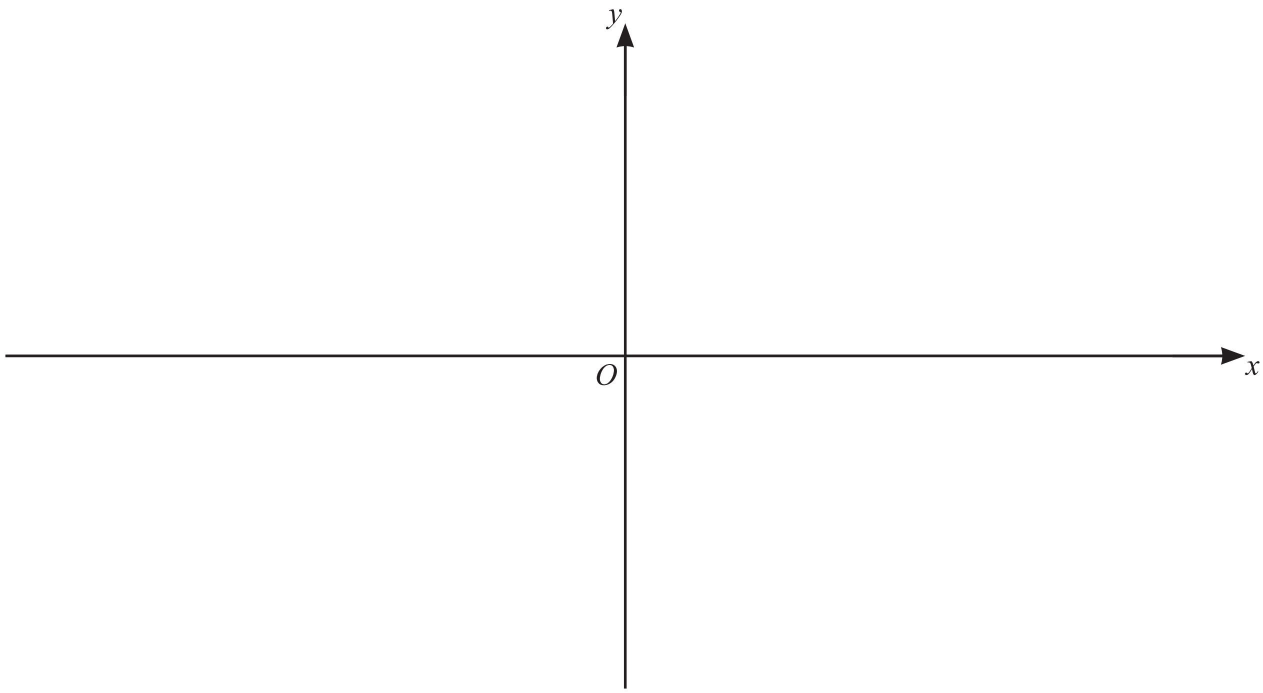
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

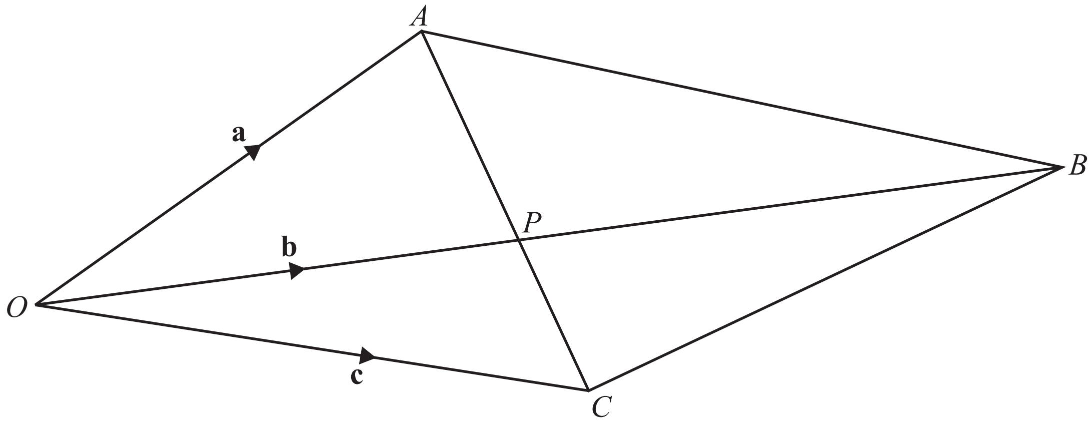
1 Write $\dfrac{(pqr)^{-2}r^{\frac{1}{3}}}{\left(p^{2}r\right)^{-1}q^{3}}$ in the form $p^{a}q^{b}r^{c}$, where a, b and c are constants. [3]

2 **(a)** On the axes, sketch the graph of $y = |4 - 3x|$, stating the intercepts with the coordinate axes. [2]



(b) Solve the inequality $|4 - 3x| \geqslant 7$. [3]

3



The diagram shows the quadrilateral $OABC$ such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The lines OB and AC intersect at the point P, such that $AP : PC = 3 : 2$.

(a) Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [3]

(b) Given also that $OP : PB = 2 : 3$, show that $2\mathbf{b} = 3\mathbf{c} + 2\mathbf{a}$. [2]

4 A curve is such that $\frac{d^2y}{dx^2} = (3x+2)^{-\frac{1}{3}}$. The curve has gradient 4 at the point $(2, 6.2)$. Find the equation of the curve. [6]

5 **(a)** Given that $\log_a p + \log_a 5 - \log_a 4 = \log_a 20$, find the value of p. [2]

(b) Solve the equation $3^{2x+1} + 8(3^x) - 3 = 0$. [3]

(c) Solve the equation $4\log_y 2 + \log_2 y = 4$. [3]

6 DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation $y = \left(3+\sqrt{5}\right)x^2 - 8\sqrt{5}x + 60$.

(a) Find the x-coordinate of the stationary point on the curve, giving your answer in the form $a+b\sqrt{5}$, where a and b are integers. [4]

(b) Hence find the y-coordinate of this stationary point, giving your answer in the form $c\sqrt{5}$, where c is an integer. [3]

7 **(a)** A six-character password is to be made from the following eight characters.

Digits	1	3	5	8	9
Symbols	*	$	#		

No character may be used more than once in a password.

Find the number of different passwords that can be chosen if

(i) there are no restrictions, [1]

(ii) the password starts with a digit and finishes with a digit, [2]

(iii) the password starts with three symbols. [2]

(b) The number of combinations of 5 objects selected from n objects is six times the number of combinations of 4 objects selected from $n-1$ objects. Find the value of n. [3]

8 Variables x and y are such that $y = Ax^b$, where A and b are constants. When $\lg y$ is plotted against $\lg x$, a straight line graph passing through the points (0.61, 0.57) and (5.36, 4.37) is obtained.

(a) Find the value of A and of b. [5]

Using your values of A and b, find

(b) the value of y when $x = 3$, [2]

(c) the value of x when $y = 3$. [2]

9 **(a)** The first three terms of an arithmetic progression are $-4, 8, 20$. Find the smallest number of terms for which the sum of this arithmetic progression is greater than 2000. [4]

(b) The 7th and 9th terms of a geometric progression are 27 and 243 respectively. Given that the geometric progression has a positive common ratio, find

(i) this common ratio, [2]

(ii) the 30th term, giving your answer as a power of 3. [2]

(c) Explain why the geometric progression $1, \sin\theta, \sin^2\theta, \ldots$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, where θ is in radians, has a sum to infinity. [2]

10 **(a)** Solve the equation $\sin\alpha \operatorname{cosec}^2\alpha + \cos\alpha \sec^2\alpha = 0$ for $-\pi < \alpha < \pi$, where α is in radians. [4]

(b) **(i)** Show that $\dfrac{\cos\theta}{1-\sin\theta}+\dfrac{1-\sin\theta}{\cos\theta}=2\sec\theta$. [4]

(ii) Hence solve the equation $\dfrac{\cos 3\phi}{1-\sin 3\phi}+\dfrac{1-\sin 3\phi}{\cos 3\phi}=4$ for $0° \leqslant \phi \leqslant 180°$. [4]

Question 11 is printed on the next page.

11 The normal to the curve $y = \dfrac{\ln(x^2 + 2)}{2x - 3}$ at the point where $x = 2$ meets the y-axis at the point P.

Find the coordinates of P. [7]