



Cambridge O Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



ADDITIONAL MATHEMATICS

4037/21

Paper 2

May/June 2021

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A \end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A \end{aligned}$$

1 (a) Write the expression $x^2 - 6x + 1$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(b) Hence write down the coordinates of the minimum point on the curve $y = x^2 - 6x + 1$. [1]

2 Variables x and y are such that, when $\ln y$ is plotted against $\ln x$, a straight line graph passing through the points $(6, 5)$ and $(8, 9)$ is obtained. Show that $y = e^p x^q$ where p and q are integers. [4]

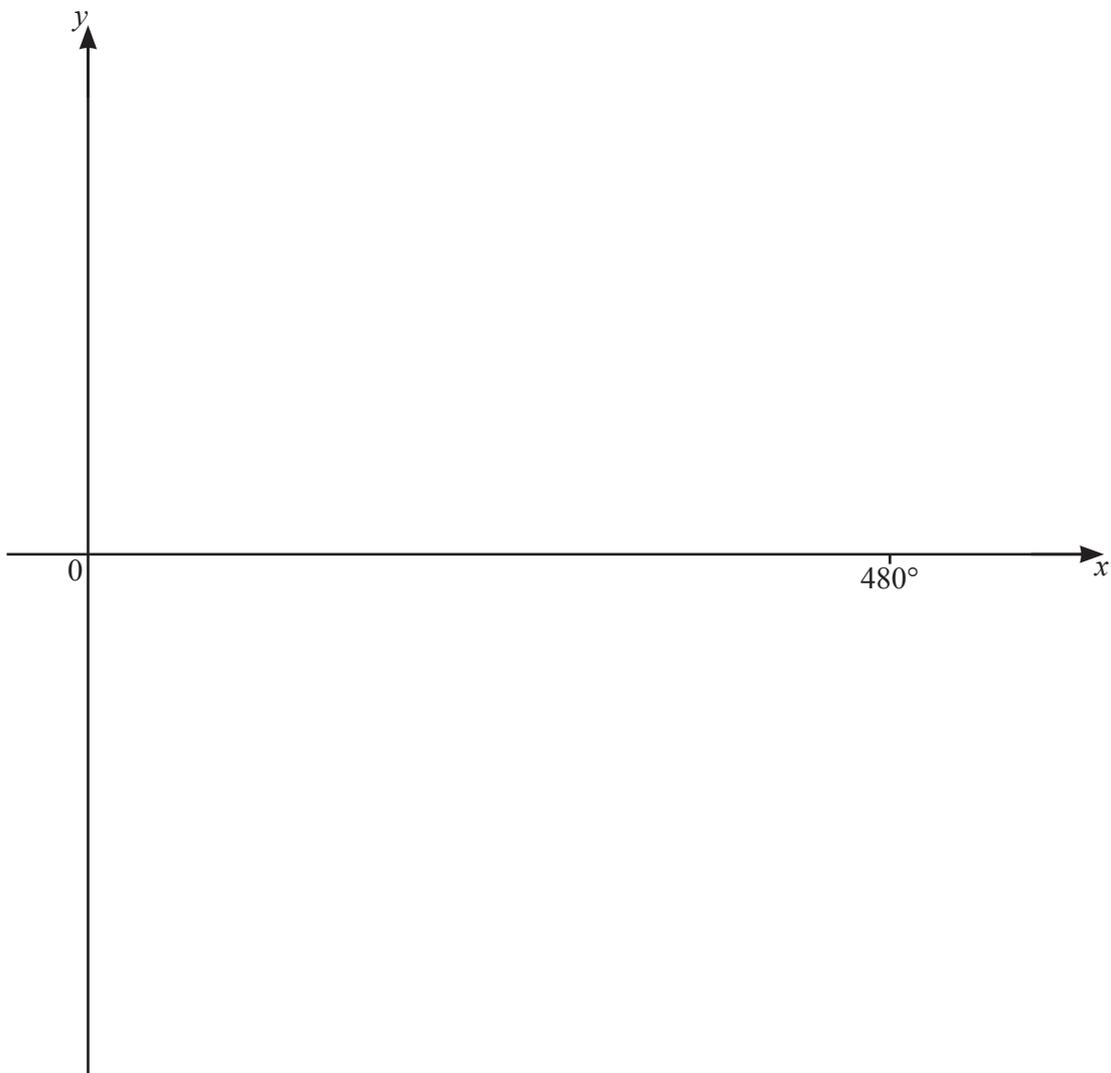
3 (a) Solve the inequality $|4x - 1| > 9$. [3]

(b) Solve the equation $2x - 11\sqrt{x} + 12 = 0$. [3]

- 4 The graph of $y = a + 2 \tan bx$, where a and b are constants, passes through the point $(0, -4)$ and has period 480° .

(a) Find the value of a and of b . [3]

(b) On the axes, sketch the graph of y for values of x between 0° and 480° . [2]



- 5 The curves $y = x^2$ and $y^2 = 27x$ intersect at $O(0, 0)$ and at the point A . Find the equation of the perpendicular bisector of the line OA . [8]

- 6 Variables x and y are such that $y = e^{\frac{x}{2}} + x \cos 2x$, where x is in radians. Use differentiation to find the approximate change in y as x increases from 1 to $1 + h$, where h is small. [6]

- 7 Find the exact values of the constant k for which the line $y = 2x + 1$ is a tangent to the curve $y = 4x^2 + kx + k - 2$.

[6]

8 In this question, a , b , c and d are positive constants.

(a) (i) It is given that $y = \log_a(x+3) + \log_a(2x-1)$. Explain why x must be greater than $\frac{1}{2}$. [1]

(ii) Find the exact solution of the equation $\frac{\log_a 6}{\log_a(y+3)} = 2$. [3]

(b) Write the expression $\log_a 9 + (\log_a b)(\log_{\sqrt{b}} 9a)$ in the form $c + d\log_a 9$, where c and d are integers. [4]

- 9 A curve is such that $\frac{d^2y}{dx^2} = \sin\left(6x - \frac{\pi}{2}\right)$. Given that $\frac{dy}{dx} = \frac{1}{2}$ at the point $\left(\frac{\pi}{4}, \frac{13\pi}{12}\right)$ on the curve, find the equation of the curve. [7]

10 Relative to an origin O , the position vectors of the points A , B , C and D are

$$\vec{OA} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}, \quad \vec{OC} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \vec{OD} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}.$$

(a) Find the unit vector in the direction of \vec{AB} . [3]

(b) The point A is the mid-point of BC . Find the value of x and of y . [2]

(c) The point E lies on OD such that $OE : OD$ is $1 : 1 + \lambda$. Find the value of λ such that \vec{BE} is parallel to the x -axis. [3]

11 The 2nd, 8th and 44th terms of an arithmetic progression form the first three terms of a geometric progression. In the arithmetic progression, the first term is 1 and the common difference is positive.

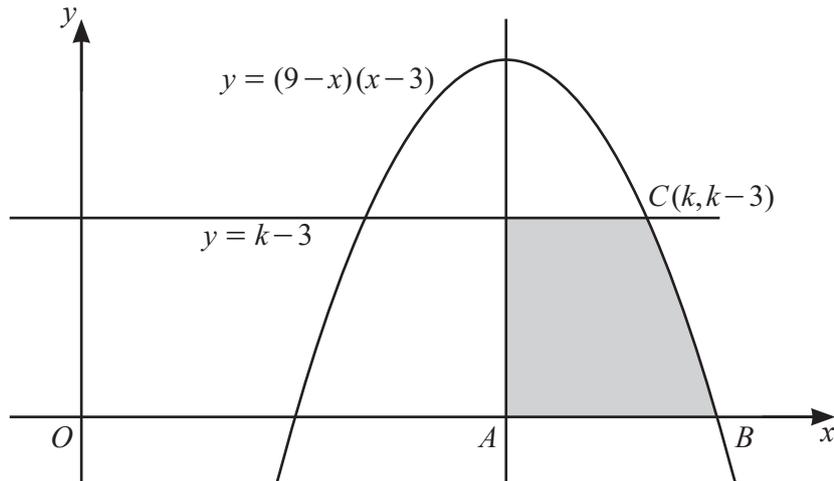
(a) (i) Show that the common difference of the arithmetic progression is 5. [5]

(ii) Find the sum of the first 20 terms of the arithmetic progression. [2]

(b) (i) Find the 5th term of the geometric progression. [2]

(ii) Explain whether or not the sum to infinity of this geometric progression exists. [1]

12



The diagram shows part of the curve $y = (9-x)(x-3)$ and the line $y = k-3$, where $k > 3$. The line through the maximum point of the curve, parallel to the y -axis, meets the x -axis at A . The curve meets the x -axis at B , and the line $y = k-3$ meets the curve at the point $C(k, k-3)$. Find the area of the shaded region. [9]

Continuation of working space for Question 12.

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.