

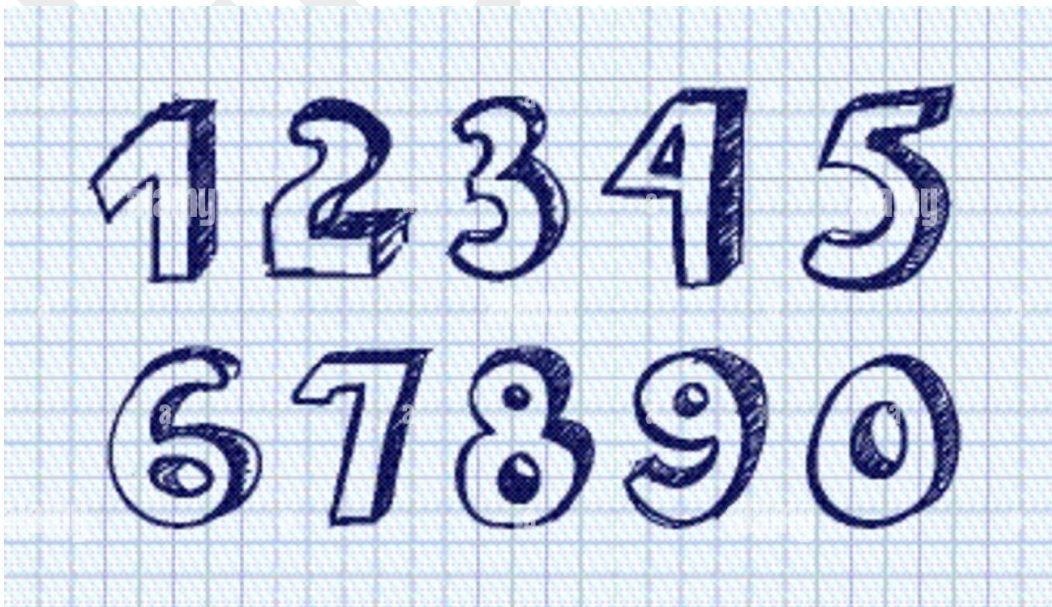
Cambridge OL

Mathematics

CODE: (4024)

Chapter 01

Number



Types of number

Integers

Integers are positive and negative whole numbers: ..., -2, -1, 0, 1, 2, ...

Natural numbers

Natural numbers are integers that can be used for counting: 1, 2, 3, 4, 5, ...

Rational numbers

A rational number is a number that can be written as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Rational numbers include:

- all integers
- all terminating decimals, for example, 5.81 can be written as $\frac{581}{100}$
- all recurring decimals, for example, $0.\dot{6} = 0.666\ 666\dots$ can be written as $\frac{2}{3}$.

Irrational numbers

An irrational number is a number that cannot be written as a fraction, such as $\sqrt{2}$ or π . They give decimals that do not terminate or recur.

Reciprocals

The reciprocal of a number is $\frac{1}{\text{the number}}$. So the reciprocal of 3 is $\frac{1}{3}$, of $\frac{1}{3}$ is 3 and of $\frac{2}{3}$ is $\frac{3}{2}$.

Prime factors

The **factors** of a number are all the numbers that divide exactly into that number.

A **prime number** is a number with only two factors. The factors of 7 are 1 and 7, so 7 is a prime number.

The factors of 12 are 1, 2, 3, 4, 6 and 12, so 12 is not a prime number.

The only factor of 1 is 1, so 1 is not a prime number.

Any number that is not prime can be written as the product of its prime factors.

The prime factors of a number can be found either by using a factor tree or by dividing repeatedly by prime numbers.

Example 1.1

Question

Sort the numbers in the list below into rational and irrational numbers. Show how you decide.

0.8652 $\sqrt{12}$ 4π $\frac{67}{19}$ $\sqrt{64}$

Solution

Rational numbers can be written as fractions, so 0.8652, $\frac{67}{19}$ and $\sqrt{64}$ are rational.

$0.8652 = \frac{8652}{10000}$ $\sqrt{64} = 8$

Irrational numbers cannot be written as fractions, so $\sqrt{12}$ and 4π are irrational.

$\sqrt{12} = 3.464\ 101\dots$

$4\pi = 12.566\ 370\dots$

Example 1.2

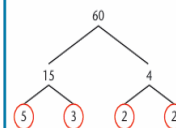
Question

Write 60 as the product of its prime factors.

Solution

Factor tree method

It doesn't matter how you start the factor tree, the ends of the branches will be the same.



The prime factors of 60 are 2, 3 and 5.

Written as a product of its prime factors, $60 = 2 \times 2 \times 3 \times 5$.

Using index notation, $60 = 2^2 \times 3 \times 5$.

Division method

2	60
2	30
3	15
5	5
	1

Divide by the prime numbers, starting with 2, until the result is 1.

Common factors and common multiples

A **common factor** of two numbers is a number that is a factor of both of them.

2 is a common factor of 8 and 12 because $8 \div 2 = 4$ and $12 \div 2 = 6$.

The **highest common factor (HCF)** of two numbers is the highest number that is a factor of both numbers.

The highest common factor of 8 and 12 is 4.

A **multiple** of a number is the product of the number and any integer.

A **common multiple** of two numbers is a number that is a multiple of both of them.

20 is a common multiple of 2 and 5 because $2 \times 10 = 20$ and $5 \times 4 = 20$.

The **lowest common multiple (LCM)** of two numbers is the lowest number that is a multiple of both numbers.

The lowest common multiple of 2 and 5 is 10.

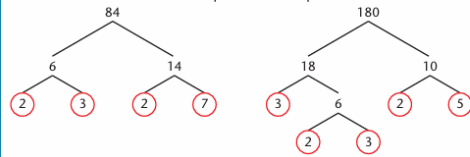
Example 1.3

Question

- Find the highest common factor (HCF) of 84 and 180.
- Find the lowest common multiple (LCM) of 84 and 180.

Solution

First write each number as the product of its prime factors.



$$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$$

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$$

- To find the highest common factor, find all numbers that appear in *both* lists.

$$84 = 2 \times 2 \times 3 \times 7$$

$$180 = 2 \times 2 \times 3 \times 3 \times 5$$

The highest common factor will be no higher than the smaller of the two numbers.

The highest common factor of 84 and 180 is $2 \times 2 \times 3 = 12$.

This means that 12 is the highest number that is a factor of both 84 and 180.

Key points

- Numbers can be classified as natural numbers, integers, rational and irrational numbers.
- The reciprocal of a number is $\frac{1}{\text{the number}}$.
- Prime numbers have only 1 and themselves as factors.
- The highest common factor (HCF) and the lowest common multiple (LCM) can be found when the numbers are split into prime factors.

- To find the lowest common multiple, find all of the prime numbers that appear in each list and use the higher power of each.

$$84 = 2^2 \times 3 \times 7$$

$$180 = 2^2 \times 3^2 \times 5$$

The lowest common multiple will be no lower than the larger of the two numbers.

The lowest common multiple of 84 and 180 is $2^2 \times 3^2 \times 5 \times 7 = 1260$.

This means that 1260 is the lowest number that is a multiple of both 84 and 180.

It is the lowest number that has both 84 and 180 as factors.

Revision questions

1) 4024/11/M/J/23/Q1

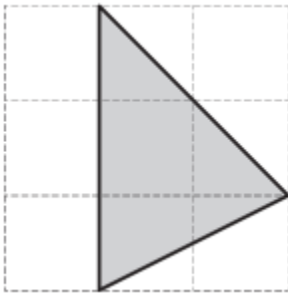
Work out.

(a) $1234.4 \div 8$

(b) $\frac{3}{7}$ of 56

2) 4024/11/M/J/23/Q2

(a) Write down the fraction of this 3 3 # square that is shaded.



(b) Evaluate. 05^2 .

3) 4024/11/M/J/23/Q5

(a) Insert one set of brackets to make the calculation correct.

$$3 + 5 \times 2 - 7 = 9$$

(b) Insert +, - and $\times$ to make the calculation correct.

$$3 \quad 5 \quad 2 \quad 7 \quad = \quad 20$$

4) 4024/11/M/J/23/Q10

(a) Work out $1\frac{1}{3} \times \frac{8}{9}$.

Give your answer as a mixed number in its simplest form.

(b) Kate has a bunch of grapes.

She ate $\frac{1}{4}$ of the grapes in the morning.

She ate $\frac{2}{3}$ of the grapes in the afternoon.

Work out the fraction of the grapes that she has not eaten.

5) 4024/12/M/J/23/Q1

Work out.

(a) $3.25 - 1.73$ (b) 1.2^2

6) 4024/12/M/J/23/Q8

Work out $1\frac{3}{4} + \frac{5}{6}$.

Give your answer as a mixed number in its simplest form.

7) 4024/12/M/J/23/Q14

(a) Write 325 as a product of its prime factors.

(b) $P = x^n y^2$ and $Q = x^{n-1} y^4$, where x and y are prime.Find the highest common factor (HCF) of P and Q .Give your answer in terms of x , y and n

7) 4024/11/O/N/22/Q11

(a) Write 120 as a product of its prime factors.

(b) $315 = 3^2 \times 5 \times 7$ Use this information to find the smallest integer value of n , such that $315n$ is a square number.

8) 4024/12/M/J/22/Q12

a) Write 0.002 035 61 correct to 3 significant figures.

(b) By writing each number correct to 1 significant figure, estimate the value of

$$\frac{\sqrt{3.93} \times 63.7}{0.425}$$

9) 4024/11/M/J/22/Q6

Write down

(a) a prime number between 10 and 15,

(b) an irrational number between 10 and 15.

10) 4024/11/M/J/22/Q1

Write down the value of the 5 in the number 253 624.

(b) The crowd at a sports event is exactly 35 687.

Write this number correct to the nearest ten.

11) 4024/12/O/N/21/Q13

The mean of five numbers is 17.

The numbers are listed in order of size, starting with the smallest.

The three smallest numbers are equal.

The middle three numbers add to 35.

The largest number is four times the smallest number.

List the five numbers in order of size.

12) 4024/12/O/N/21/Q9

(a) Write 216 as a product of its prime factors.

b) Two positive integers are each greater than 25.

Their lowest common multiple (LCM) is 216.

Their highest common factor (HCF) is 18.

Find the two integers.

13) 4024/11/O/N/21/Q14

(a) Express 60 as the product of its prime factors.

(b) A school buys packs of pens and packs of rulers.

There are 60 pens in each pack of pens.

There are 42 rulers in each pack of rulers.

The school wants to buy exactly the same number of pens and rulers.

Work out the smallest number of each pack the school should buy.

14) 4024/11/M/J/21/Q6

(a) Write 308 as a product of its prime factors.

(b) Find the highest common factor (HCF) of 308 and 66.

15) 4024/11/O/N/20/Q24

(a) Express 99 as the product of prime factors.

(b) Expressed as the product of prime factors,

$$p = 2^{n+2} \times 3^n \times 5 \quad \text{and} \quad q = 2^n \times 3^{n+1} \times 5^2$$

where n is a positive integer.

(i) The lowest common multiple (LCM) of p and q is $2^n \times 3^n \times R$.

Express R as the product of prime factors.

ii) Express $p + q$ as the product of prime factors.

15) 4024/12/M/J/20/Q13

Write 108 as the product of its prime factors.

Find the lowest common multiple (LCM) of 108 and 180.

16) 4024/11/O/N/18/Q17

$$120 = 2^3 \times 3 \times 5$$

(a) Express 1200 as the product of its prime factors.

(b) Find the smallest value of n , such that $120n$ is a square number.

16) 4024/12/M/J/17/Q20

- (a) (i) Write 54 as the product of its prime factors.
(ii) Find the smallest possible integer m such that $54m$ is a cube number.

17) 4024/11/M/J/17/Q4

- (a) Express 36 as the product of its prime factors.
(b) Write down two prime numbers whose sum is 15.

18) 4024/11/M/J/17/Q8

- (a) A car travels at 84 km/h.
Calculate the number of metres that the car travels in one minute.

19) 4024/12/M/J/16/Q6

- (a) Express 96 as a product of its prime factors.
(b) 24 is a common factor of 96 and the integer n .
Given that n is less than 96, find the largest possible value of n .

20) 4024/11/M/J/16/Q21

- (a) Express 500 as the product of its prime factors.
(b)

$$M = 2 \times 3^2 \qquad N = 2^4 \times 3^2$$

Find the values of p and q when,

(i) $M \times N = 2^p \times 3^q$,

(ii) $M \div N = 2^p \times 3^q$,

(iii) $N^2 = 2^p \times 3^q$.