Cambridge

A2 Level

Physics

CODE: (9702)

Chapter 23

Chapter 23: Coulomb's law

Electric fields

In Chapter 8, we presented some fundamental ideas about electric fields:

- An electric field is a field of force and can be represented by field lines.
- The electric field strength at a point is the force per unit positive charge that acts on a stationary charge:

  $\text{field strength} = \frac{\text{force}}{\text{charge}}$ $\qquad E = \frac{F}{Q}$

- There is a uniform field between charged parallel plates:

  $\text{field strength} = \frac{\text{potential difference}}{\text{separation}}$ $\qquad E = \frac{V}{d}$

Coulomb's law

Any electrically charged object produces an electric field in the space around it. It could be something as small as an electron or a proton, or as large as a planet or star.

A statement of Coulomb's law is as follows:

> Any two point charges exert an electrical force on each other that is proportional to the product of their charges and inversely proportional to the square of the distance between them.

We consider two point charges Q1 and Q2 separated by a distance r (Figure 23.2). The force each charge exerts on the other is F. According to Newton's third law of motion, the point charges interact with each other and therefore exert equal but opposite forces on each other.

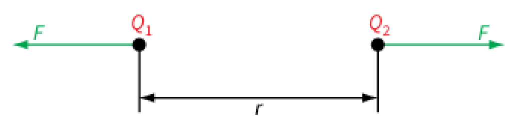


Figure 23.2 The variables involved in Coulomb's law.

The force depends on each of the properties producing it (in this case, the charges), and it is an inverse square law with distance – if the particles are twice as far apart, the electrical force is a quarter of its previous value (Figure 23.3)

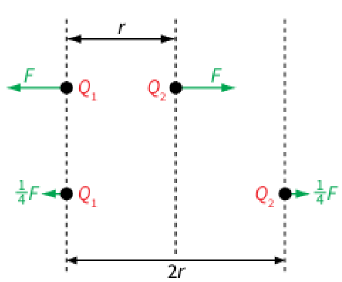


Figure 23.3 Doubling the separation results in one-quarter of the force, a direct consequence of Coulomb's law.

According to Coulomb's law, we have:

force ∝ product of the charges $\qquad F \propto Q_1 Q_2$

$\text{force} \propto \frac{1}{\text{distance}^2}$ $\qquad F \propto \frac{1}{r^2}$

Therefore:

$$F \propto \frac{Q_1 Q_2}{r^2}$$

We can write this in a mathematical form:

$$F = \frac{k Q_1 Q_2}{r^2}$$

The constant of proportionality is:

$$k = \frac{1}{4\pi\varepsilon_0}$$

where ε_0 is known as the **permittivity of free space** (ε is the Greek letter epsilon). The value of ε_0 is approximately $8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$. An equation for Coulomb's law is thus:

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

By substituting for π and ε_0, we can show that the force F can also be given by the equation:

$$F \approx 9.0 \times 10^9 \frac{Q_1 Q_2}{r^2}$$

Electric field strength for a radial field

In Chapter 8, we saw that the electric field strength at a point is defined as the force per unit charge exerted on a positive charge placed at that point, $E = \frac{F}{Q}$.

So, to find the field strength near a point charge Q_1 (or outside a uniformly charged sphere), we have to imagine a small positive test charge Q_2 placed in the field, and determine the force per unit charge on it. We can then use the definition above to determine the electric field strength for a point (or spherical) charge.

The force between the two point charges is given by:

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

The electric field strength E due to the charge Q_1 at a distance of r from its centre is thus:

$$E = \frac{\text{force}}{\text{test charge}} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2 Q_2}$$

or:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Electric potential

When we discussed gravitational potential (page 276), we started from the idea of potential energy. The potential at a point is then the potential energy of unit mass at the point. We will approach the idea of electrical potential in the same way.

Electric potential energy

When an electric charge moves through an electric field, its potential energy changes. Think about this concrete example: if you want to move one positive charge closer to another positive charge, you have to push it (Figure 23.6).

Figure 23.6 Work must be done to push one positive charge towards another.

Energy changes in a uniform field

We can also think about moving a positive charge in a uniform electric field between two charged parallel plates. If we move the charge towards the positive plate, we have to do work. The potential energy of the charge is therefore increasing. If we move it towards the negative plate, its potential energy is decreasing (Figure 23.7a).

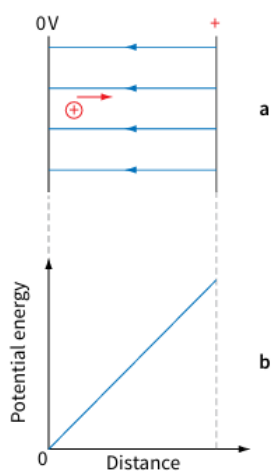


Figure 23.7 Electrostatic potential energy changes in a uniform field.

Potential difference is defined as the energy change per coulomb between two points. Hence, for charge Q, the work done in moving it from the negative plate to the positive plate is:

$$W = QV$$

We can rearrange this equation as:

$$V = \frac{W}{Q}$$

We can extend the idea of electric potential to measurements in electric fields. In Figure 23.9, the power supply provides a potential difference of 10 V. The value of the potential at various points is shown. You can see that the middle resistor has a potential difference across it of (8 − 2) V = 6 V.

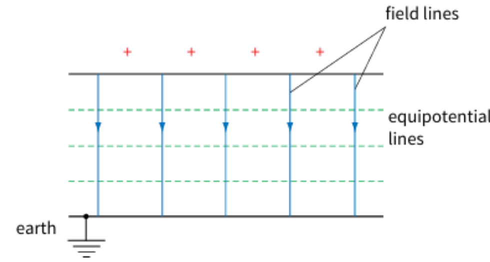


Figure 23.8 Equipotential lines in a uniform electric field.

Energy in a radial fields

The potential energy of the test charge increases as you push it. It increases more and more rapidly the closer you get to the repelling charge. This is shown by the graph in Figure 23.10. We can write an equation for the potential V at a distance r from a charge Q:

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

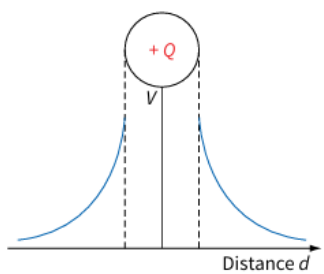


Figure 23.10 The potential changes according to an inverse law near a charged sphere.

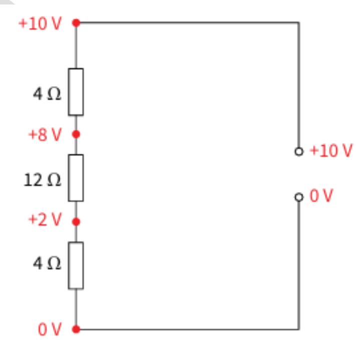


Figure 23.9 Changes in potential (shown in red) around an electric circuit.

The equipotentials get closer together as we get closer to the charge (Figure 23.11)
T his allows us to give a definition of electric potential:

> The **electric potential** at a point is equal to the work done in bringing unit positive charge from infinity to that point.

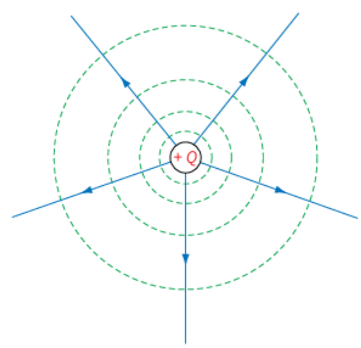


Figure 23.11 The electric field around a positive charge. The dashed equipotential lines are like the contour lines on a map; they are spaced at equal intervals of potential.

Field strength and potential gradient

We can picture electric potential in the same way that we thought about gravitational potential. A negative charge attracts a positive test charge, so we can regard it as a potential 'well'. A positive charge is the opposite – a 'hill' (Figure 23.12). The strength of the field is shown by the slope of the hill or well:

field strength = −potential gradient

This relationship applies to all electric fields. For the special case of a uniform field, the potential gradient E is constant. Its value is given by E = ΔV / Δd

where V is the potential difference between two points separated by a distance d.

Figure 23.12 A 'potential well' near a negative charge, and a 'potential hill' near a positive charge.

Comparing gravitational and electric fields

There are obvious similarities between the ideas we have used in this chapter to describe electric fields and those we used in Chapter 18 for gravitational fields. This can be helpful, or it can be confusing! The summary given in Table 23.1 is intended to help you to sort them out.

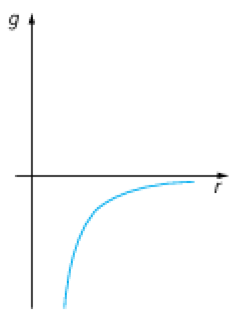
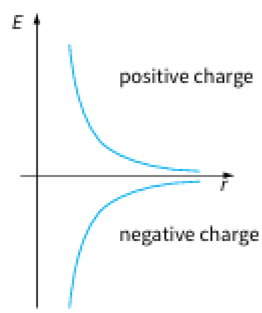
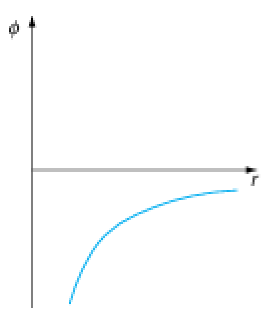
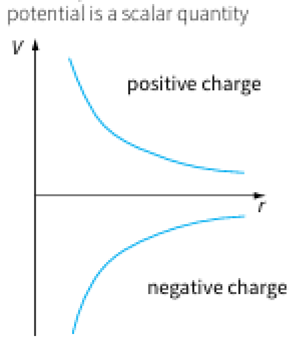
Gravitational fields	Electric fields
Origin arise from masses	**Origin** arise from electric charges
Vector forces only gravitational attraction, no repulsion	**Vector forces** both electrical attraction and repulsion are possible (because of positive and negative charges)
All gravitational fields field strength $g = \frac{F}{m}$ i.e. field strength is force per unit mass	**All electric fields** field strength $E = \frac{F}{Q}$ i.e. field strength is force per unit positive charge
Units F in N, g in $\mathrm{N\,kg^{-1}}$ or $\mathrm{m\,s^{-2}}$	**Units** F in N, E in $\mathrm{N\,C^{-1}}$ or $\mathrm{V\,m^{-1}}$
Uniform gravitational fields parallel gravitational field lines g = constant	**Uniform electric fields** parallel electric field lines $E = \frac{V}{d}$ = constant
Spherical gravitational fields radial field lines force given by Newton's law: $F = \frac{GMm}{r^2}$ field strength is therefore: $g = \frac{GM}{r^2}$ (Gravitational forces are always attractive, so we show g on a graph against r as negative.) force and field strength obey an inverse square law with distance 	**Spherical electric fields** radial field lines force given by Coulomb's law: $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$ field strength is therefore: $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ (A negative charge gives an attractive field, a positive charge gives a repulsive field.) force and field strength obey an inverse square law with distance 
Gravitational potential given by: $\phi = -\frac{GM}{r}$ potential obeys an inverse relationship with distance and is zero at infinity potential is a scalar quantity and is always negative 	**Electric potential** given by: $V = \frac{Q}{4\pi\varepsilon_0 r}$ potential obeys an inverse relationship with distance and is zero at infinity potential is a scalar quantity 

Table 23.1 Gravitational and electric fields compared.