Cambridge

A2 Level

Physics

CODE: (9702)

Chapter 31

Chapter 31: Nuclear physics

- In α decay, the nucleon number decreases by 4 and the proton number decreases by 2.
- In β^- decay, the nucleon number is unchanged and the proton number increases by 1.
- In β^+ decay, the nucleon number is unchanged and the proton number decreases by 1.
- In γ emission there is no change in nucleon or proton number.

Balanced equations

When an unstable nucleus undergoes radioactive decay, the nucleus before the decay is often referred to as the parent nucleus and the new nucleus after the decay of the α-particle is known as the daughter nucleus.

Mass and energy

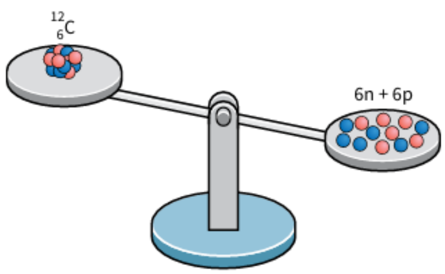

Figure 31.3 The mass of a nucleus is less than the total mass of its component protons and neutrons.

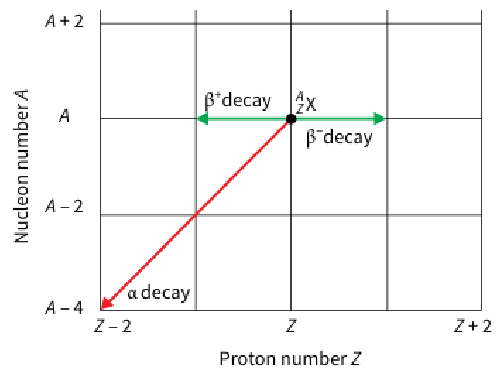

Figure 31.2 Emission of α- and β-particles.

The law of conservation of mass was broken when dismantling a stable nucleus, $^{12}{}_{6}$ C, revealing that separate nucleons have more mass than the nucleus itself. This contradicts the fundamental law of nature, which states that energy has mass. Einstein proposed that energy has mass, but this is not an easy idea. In everyday life, the extra mass is small, but large changes in energy occur in nuclear physics and high-energy physics.

Einstein's mass–energy equation

Albert Einstein produced his famous mass–energy equation, which links energy E and mass m:

$$E = mc^2$$

Generally, we will be concerned with the changes in mass owing to changes in energy, when the equation becomes:

$$\Delta E = \Delta mc^2$$

According to Einstein's equation:

- ■ the mass of a system **increases** when energy is supplied to it
- ■ when energy is released from a system, its mass decreases.

Particle	Rest mass / 10^{-27} kg
^1_1p	1.672623
^1_0n	1.674929
$^{12}_{6}C$ nucleus	19.926483

Table 31.1 Rest masses of some particles. It is worth noting that the mass of the neutron is slightly greater than that of the proton (roughly 0.1% greater).

When six protons and six neutrons combine to form the nucleus of carbon-12, there is a very small loss of mass Δm, known as the mass defect.

The mass defect of a nucleus is equal to the difference between the total mass of the individual, separate nucleons and the mass of the nucleus.

Mass–energy conservation

Einstein pointed out that his equation $\Delta E = \Delta mc^2$ applied to all energy changes, not just nuclear processes. So, for example, it applies to chemical changes, too. If we burn some carbon, we start off with carbon and oxygen. At the end, we have carbon dioxide and energy.

Another unit of mass

When calculating energy values using $E = mc^2$, it is essential to use values of mass in kg, the SI unit of mass. As an alternative, atomic and nuclear masses are often given in a different unit, the **atomic mass unit** (symbol u).

1 u is defined as $\frac{1}{12}$ of the mass of a neutral atom of carbon-12.

For example, the mass of uranium-235 is slightly more than 235. The extra bit is known as the **mass excess**

mass excess = mass (in u) – nucleon number

Nuclide	Symbol	Mass / u
proton	${}^{1}_{1}p$	1.007 825
neutron	${}^{1}_{0}n$	1.008 665
helium-4	${}^{4}_{2}He$	4.002 602
carbon-12	${}^{12}_{6}C$	12.000 000
potassium-40	${}^{40}_{19}K$	39.963 998
uranium-235	${}^{235}_{92}U$	235.043 930

Table 31.3 Masses of some nuclides in atomic mass units. Some have been measured to several more decimal places than are shown here.

Energy released in radioactive decay

Unstable nuclei may emit α- and β-particles with large amounts of kinetic energy. We can use Einstein's mass–energy equation $\Delta E = \Delta mc^2$ to explain the origin of this energy.

mass of ${}^{238}_{92}U$ nucleus $= 3.952\,83 \times 10^{-25}$ kg

total mass of ${}^{234}_{90}Th$ nucleus and α-particle (${}^{4}_{2}He$) $= 3.952\,76 \times 10^{-25}$ kg

change in mass $\Delta m = (3.952\,76 - 3.952\,83) \times 10^{-25}$ kg

$\approx -7.0 \times 10^{-30}$ kg

The minus sign shows a decrease in mass, hence, according to the equation $\Delta E = \Delta mc^2$, energy is released in the decay process:

energy rele[illegible]

$\approx 6.3 \times 10$[illegible] J

The minimum energy needed to pull a nucleus apart into its separate nucleons is known as the **binding energy** of the nucleus.

Binding energy and stability

We can now begin to see why some nuclei are more stable than others. If a nucleus is formed from separate nucleons, energy is released. In order to pull the nucleus apart, energy must be put in; in other words, work must be done against the strong nuclear force which holds the nucleons together. The more energy involved in this, the more stable is the nucleus

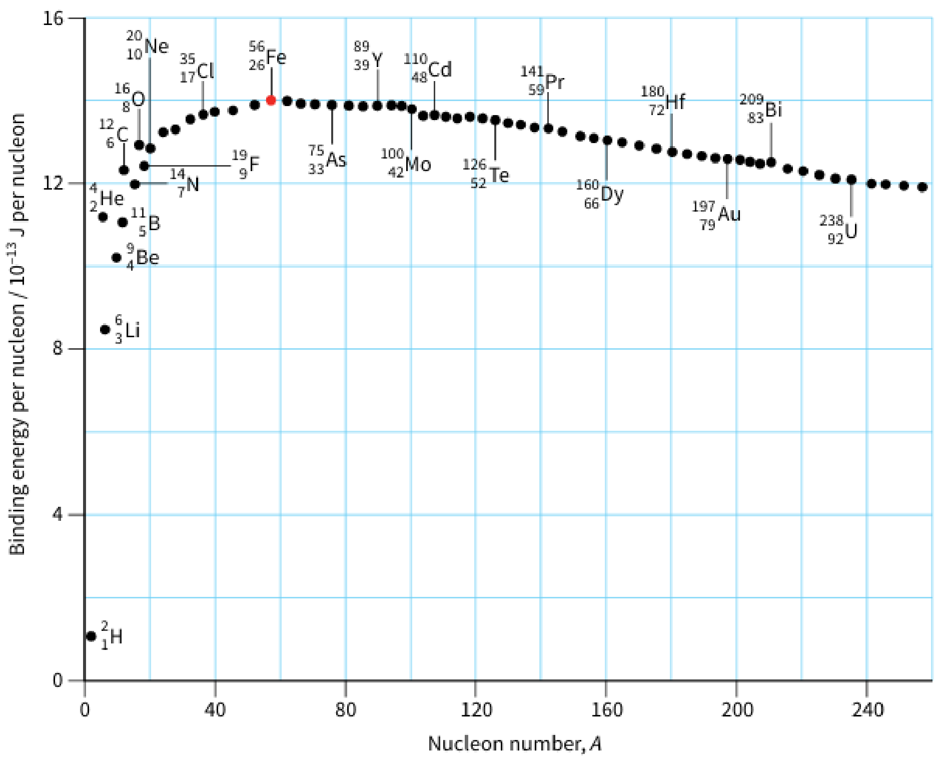

Figure 31.4 This graph shows the binding energy per nucleon for a number of nuclei. The nucleus becomes more stable as binding energy per nucleon increases.

For nuclides with A > 20 approximately, there is not much variation in binding energy per nucleon. The greatest value of binding energy per nucleon is found for $^{56}_{26}Fe$.

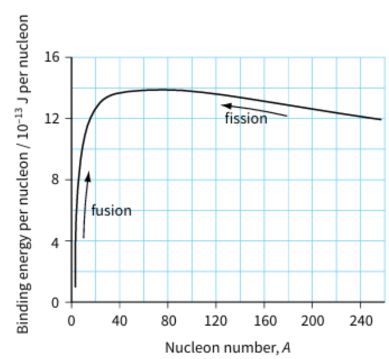

Figure 31.6 Both fusion and fission are processes that tend to increase the binding energy per nucleon of the particles involved.

Binding energy, fission and fusion

We can use the binding energy graph to help us decide which nuclear processes – fission, fusion, radioactive decay – are likely to occur (Figure 31.6).

Fission

Fission is the process in which a massive nucleus splits to form two smaller fragments (rather than simply emitting α- or β-radiation).

Fusion

Fusion is the process by which two very light nuclei join together to form a heavier nucleus. (This is the process by which energy is released in the Sun, when hydrogen nuclei fuse to form helium nuclei.)

Randomness and decay

Listen to a counter connected to a Geiger–Müller (GM) tube that is detecting the radiation from a weak source, so that the count rate is about one count per second. Each count represents the detection of a single α-particle or a β-particle or a γ-ray photon.

Figure 31.8 shows a graph of count rate against time, with a smoothing of a few seconds. The count rate decreases with time as the number of radioactive nuclei that are left decreases. The fluctuations either side are caused by the randomness of the decay.

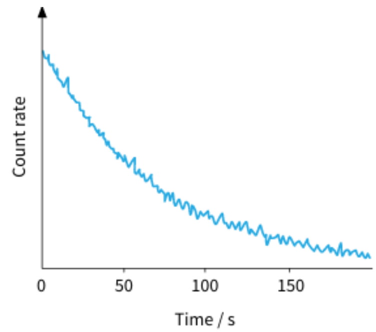

Figure 31.8 Count rate showing randomness of decay.


Figure 31.7 The time constant of this ratemeter can be adjusted to smooth out rapid fluctuations in the count rate.

Spontaneous decay

Radioactive decay occurs within the unstable nucleus of an atom. A nucleus emits radiation and becomes the nucleus of an atom of a different element. This is a spontaneous process, which means that we cannot predict, for a particular nucleus, when it will happen.

To summarise, nuclear decay is spontaneous because:

■ ■ the decay of a particular nucleus is not affected by the presence of other nuclei

■ ■ the decay of nuclei cannot be affected by chemical reactions or external factors such as temperature and pressure.

Nuclear decay is random because:

■ ■ it is impossible to predict when a particular nucleus in a sample is going to decay

■ ■ each nucleus in a sample has the same chance of decaying per unit time.

The mathematics of radioactive decay

We have seen that radioactive decay is a random, spontaneous process. Because we cannot say when an individual nucleus will decay, we have to start thinking about very large numbers of nuclei.

The probability that an individual nucleus will decay per unit time interval is called the **decay constant**, λ.

The **activity** *A* of a radioactive sample is the rate at which nuclei decay or disintegrate.

Count rate

Although we are often interested in finding the activity of a sample of radioactive material, we cannot usually measure this directly. This is because we cannot easily detect all of the radiation emitted. Some will escape past our detectors, and some may be absorbed within the sample itself.

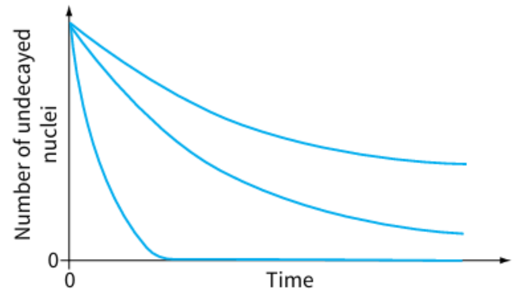

Figure 31.9 Some radioactive materials decay faster than others.

Decay graphs and equations

The activity of a radioactive substance gradually diminishes as time goes by. The atomic nuclei emit radiation and become different substances. The pattern of radioactive decay is an example of a very important pattern found in many different situations, a pattern called exponential decay.

> The half-life $t_{1/2}$ of a radioisotope is the mean time taken for half of the active nuclei in a sample to decay.

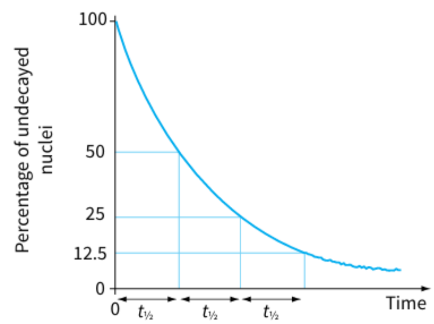

Figure 31.10 All radioactive decay graphs have the same characteristic shape.

Mathematical decay

We can write an equation to represent the graph shown in Figure 31.10. If we start with N0 undecayed nuclei, then the number N that remain undecayed after time t is given by:

$$N = N_0 e^{(-\lambda t)}$$

Decay constant and half-life

A radioactive isotope that decays rapidly has a short half-life $t_{1/2}$. Its decay constant must be large, since the probability per unit time of an individual nucleus decaying must be high. What is the connection between the decay constant and the half-life?

In a time equal to one half-life $t_{1/2}$, the number of undecayed nuclei is halved. Hence the equation:

$$N = N_0 e^{(-\lambda t)}$$

becomes:

$$\frac{N}{N_0} = e^{(-\lambda t_{1/2})} = \frac{1}{2}$$

Therefore:

$$e^{(\lambda t_{1/2})} = 2$$

$$\lambda t_{1/2} = \ln 2 \approx 0.693$$

(remember if $e^x = y$, then $x = \ln y$).

The half-life of an isotope and the decay constant are inversely proportional to each other. That is:

$$\lambda = \frac{0.693}{t_{1/2}}$$