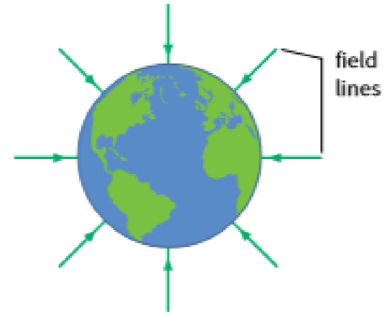
Cambridge A2 Physics

(Code: 9702)

Chapter 18
Gravitational fields

Representing a gravitational field



We can represent the Earth's gravitational field by drawing field lines. The field lines show two things:

- The arrows on the field lines show us the direction of the gravitational force on a mass placed in the field.
- The spacing of the field lines indicates the strength of the gravitational field – the further apart they are, the weaker the field.

We describe the Earth's gravitational field as *radial*, since the field lines diverge (spread out) radially from the centre of the Earth. However, on the scale of a building, the gravitational field is *uniform*, since the field lines are equally spaced.

Newton's law of gravitation

Newton's law of gravitation states that any two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of their separation.

$$F = \frac{GMm}{r^2}$$

The gravitational constant *G* is sometimes referred to as the universal gravitational constant because it is believed to have the same value, 6.67×10^{-11} N m^2 kg^{-2}, throughout the Universe.

Where m_1 and m_2 are the masses of the two bodies.

$$F = \frac{Gm_1m_2}{r^2}$$

Gravitational field strength

The gravitational field strength is the familiar quantity g. Its value is approximately 9.81 ms^{-2}. The weight of a body of mass m is mg.

The gravitational field strength g at any point in a gravitational field is defined as:

> The gravitational field strength at a point is the gravitational force exerted per unit mass on a small object placed at that point.

$$g = \frac{GM}{r^2}$$

Gravitational field strength is a vector. The field strength g is not a constant; it decreases as the distance r increases.

Gravitational field strength g also has units ms^{-2}; it is an acceleration. Another name for g is 'acceleration of free fall'.

Energy in a gravitational field

We need a more general approach to calculating gravitational energy, for two reasons:

- If we use g.p.e. = $mg\Delta h$, we are assuming that an object's g.p.e. is zero on the Earth's surface. This is fine for many practical purposes but not, for example, if we are considering objects moving through space, far from Earth. For these, there is nothing special about the Earth's surface.
- If we lift an object to a great height, g decreases and we would need to take this into account when calculating g.p.e.

Gravitational potential

Gravitational potential is defined as: The gravitational potential at a point is the work done per unit mass in bringing a mass from infinity to the point.
The symbol used for potential is Ø (Greek letter phi), and unit mass means one kilogram.

$$\emptyset = -\frac{GM}{r}$$

Gravitational potential is always negative because, as a mass is brought towards another mass, its g.p.e. decreases. Since g.p.e. is zero at infinity, it follows that, anywhere else, g.p.e. and potential are less than zero.

Fields – terminology

<u>Field strength</u>
Tells us about the force on unit mass at a point

<u>Potential</u>
Tells us about *potential energy* of unit mass at a point.

Orbiting under gravity

$$v^2 = \frac{GM}{r}$$

The orbital period

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Geostationary orbits

The satellite remains above a fixed point on the Earth's equator. This kind of orbit is called a *geostationary orbit*.

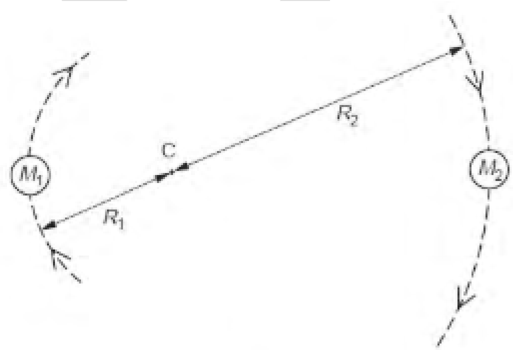
For a satellite to stay above a fixed point on the equator, it must take exactly 24 hours to complete one orbit. For a satellite to occupy a geostationary orbit, it must be at a distance of 42 300 km from the centre of the Earth and at a point directly above the equator. The orbital radius is 6.6 Earth radii from the centre of the Earth

EXCERSISE

1.
    a. Define gravitational potential.
    b. Explain why values of gravitational potential near to an isolated mass are all negative
    c. The Earth may be assumed to be an isolated sphere of radius 6.4×10^3 km with its mass of 6.0×10^{24} kg concentrated at its centre. An object is projected vertically from the surface of the Earth so that it reaches an altitude of 1.3×10^4 km.
    Calculate, for this object,
        i. The change in gravitational potential,
        ii. The speed of projection from the Earth's surface, assuming air resistance is negligible.
    d. Suggest why the equation
    $$v^2 = u^2 + 2as$$
    is not appropriate for the calculation in (c)(ii).

2. A binary star consists of two stars that orbit about a fixed point C, as shown

3. The star of mass M_1 has a circular orbit of radius R_1 and the star of mass M_2 has a circular orbit of radius R_2. Both stars have the same angular speed ω, about C.



    a. State the formula, in terms of G, M_1, M_2, R_1, R_2 and ω for
        i. the gravitational force between the two stars,
        ii. the centripetal force on the star of mass M_1

b. The stars orbit each other in a time of 1.26×10^8 s (4.0 years). Calculate the angular speed ω for each star.

c.

i. Show that the ratio of the masses of the stars is given by the expression

$$\frac{M_1}{M_2} = \frac{R_1}{R_2}$$

ii. The ratio $\frac{M_1}{M_2}$ is equal to 3.0 and the separation of the stars is 3.2×10^{11} m.

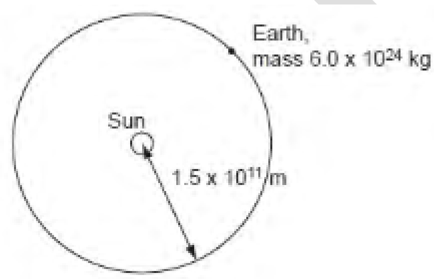
iii. Calculate the radii R_1 and R_2

d.

i. By equating the expressions you have given in (a) and using the data calculated in (b) and (c), determine the mass of one of the stars.

ii. State whether the answer in (i) is for the more massive or for the less massive star

4. 1 The orbit of the Earth, mass 6.0 x 1024 kg, may be assumed to be a circle of radius 1.5x 1011m with the Sun at its centre, as illustrated



The time taken for one orbit is 3.2 x 107s.

a. Calculate

i. the magnitude of the angular velocity of the Earth about the Sun,

ii. the magnitude of the centripetal force acting on the Earth.

b.

i. State the origin of the centripetal force calculated in (a)(ii).

ii. Determine the mass of the Sun.

5. The Earth may be considered to be a uniform sphere with its mass M concentrated at its centre. A satellite of mass m orbits the Earth such that the radius of the circular orbit is r.

a. Show that the linear speed v of the satellite is given by the expression

$$V = \sqrt{\frac{GM}{r}}$$

b. For this satellite, write down expressions, in terms of G, M, m and r_1 for

i. its kinetic energy.

ii. its gravitational potential energy.

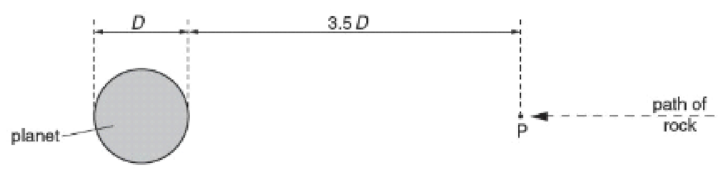
iii. its total energy.

c. The total energy of the satellite gradually decreases
State and explain the effect of this decrease on
i. the radius r of the orbit,
ii. the linear speed of the satellite.

6.
a. Define gravitational field strength.
b. A spherical planet has diameter 12 x 10^4km. The gravitational field strength at the surface of the planet is 8.6Nkg^{-1}
The planet may be assumed to be isolated in space and to have its mass concentrated at its centre.
Calculate the mass of the planet
c. The gravitational potential at a point X above the surface of the planet in (b) is $-5.3 \times 10^7 Jkg^{-1}$
For point Y above the surface of the planet, the gravitational potential is $-6.8 \times 10^7 Jkg^{-1}$
i. State, with a reason, whether point X or point Y is nearer to the planet.
ii. A rock falls radially from rest towards the planet from one point to the other.
Calculate the final speed of the rock

7.
a. Newton's law of gravitation applies to point masses.
i. State Newton's law of gravitation.
ii. Explain why, although the planets and the Sun are not point masses, the law also applies to planets orbiting the Sun.
b. Gravitational fields and electric fields show certain similarities and certain differences. State one aspect of gravitational and electric fields where there is
i. a similarity,
ii. a difference.

8. An isolated spherical planet has a diameter of 6.8x10^6 m. Its mass of 6.4x10^{23} kg may be assumed to be a point mass at the centre of the planet.
a. Show that the gravitational field strength at the surface of the planet is 3.7Nkg^{-1}.
b. A stone of mass 2.4kg is raised from the surface of the planet through a vertical height of 1800m.
Use the value of field strength given in (a) to determine the change in gravitational potential energy of the stone.
Explain your working.
c. A rock, initially at rest at infinity, moves towards the planet. At point P, its height above the surface of the planet is 3.5D, where D is the diameter of the planet, as shown. Calculate the speed of the rock at point P, assuming that the change in gravitational potential energy is all transferred to kinetic energy.



9.

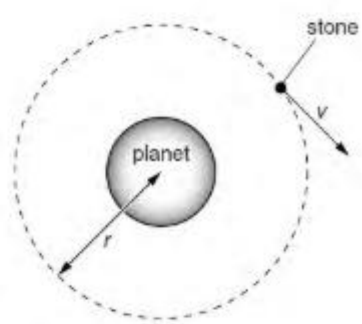
a. Define gravitational potential at a point.

b. A stone of mass m has gravitational potential energy Ep at a point X in a gravitational field. The magnitude of the gravitational potential at X is Ø
State the relation between m, Ep and Ø.

c. An isolated spherical planet of radius R may be assumed to have all its mass concentrated at its centre. The gravitational potential at the surface of the planet is -6.30 x 10^7 Jkg^{-1}.
A stone of mass 1.30 kg is travelling towards the planet such that its distance from the centre of the planet changes from 6R to 5R.
Calculate the change in gravitational potential energy of the stone.

10. The mass Mof a spherical planet may be assumed to be a point mass at the centre of the planet.

a. A stone, travelling at speed v, is in a circular orbit of radius r about the planet, as illustrated

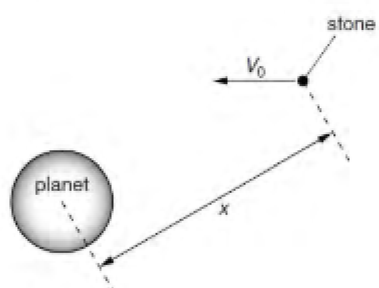


Show that the speed v is given by the expression

$$V = \sqrt{\frac{GM}{r}}$$

where G is the gravitational constant. Explain your working.

b. A second stone, initially at rest at infinity, travels towards the planet, as illustrated



The stone does not hit the surface of the planet.

i. Determine, in terms of the gravitational constant G and the mass M of the planet, the speed V, of the stone at a distance x from the centre of the planet. Explain your working. You may assume that the gravitational attraction on the stone is due only to the planet.

ii. Use your answer in (i) and the expression in (a) to explain whether this stone could enter a circular orbit about the planet.