Cambridge A2

Physics

(Code: 9702)

Chapter 19

Oscillations

Free and forced oscillations

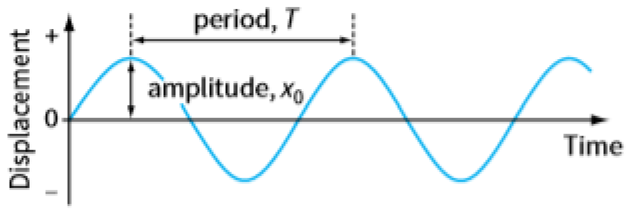
An object oscillates when it moves back and forth repeatedly, on either side of some equilibrium position. If we stop the object from oscillating, it returns to the equilibrium position.
Every oscillator has a *natural frequency* of vibration, the frequency with which it vibrates freely after an initial disturbance.

Observing oscillations

Our eyes cannot respond rapidly enough if the frequency of oscillation is more than about 5 Hz (five oscillations per second); anything faster than this appears as a blur.

Describing oscillations

Amplitude, period and frequency



The shape of this graph is a sine curve, and the motion is described as sinusoidal. the displacement changes between positive and negative values.

<u>Amplitude (x_0)</u>
The maximum displacement from the equilibrium position.

<u>Period (T)</u>
The time for one complete oscillation

<u>Frequency (f)</u>
The number of oscillations per unit time

$$\boxed{f = \frac{1}{T}}$$

Phase

The term phase describes the point that an oscillating mass has reached within the complete cycle of an oscillation.

Simple harmonic motion

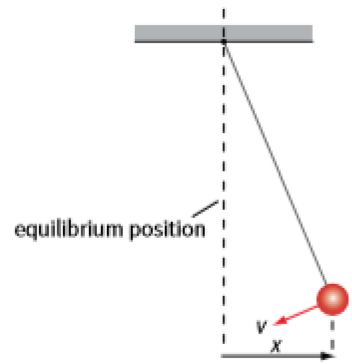
A body executes simple harmonic motion if its acceleration is directly proportional to its displacement from its equilibrium position, and in the opposite direction to its displacement.

Situations where simple harmonic motion can be found

- When a pure (single tone) sound wave travels through air, the molecules of the air vibrate with s.h.m.
- When an alternating current flows in a wire, the electrons in the wire vibrate with s.h.m.
- There is a small alternating electric current in a radio or television aerial when it is tuned to a signal, in the form of electrons moving with s.h.m.
- The atoms that make up a molecule vibrate with s.h.m.

The requirements for s.h.m.

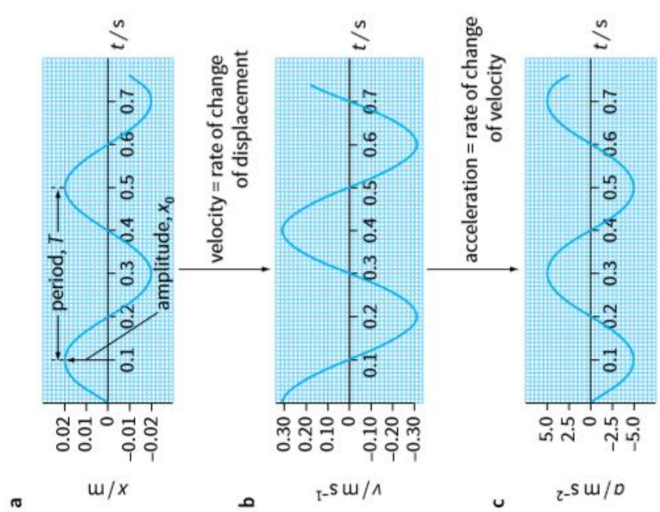
1. A mass that oscillates
2. A position where the mass is in equilibrium (conventionally, displacement x to the right of this position is taken as positive; to the left it is negative)
3. A restoring force that acts to return the mass to its equilibrium position; the restoring force F is directly proportional to the displacement x of the mass from its equilibrium position and is directed towards that point.



The changes of velocity in s.h.m.

- As it swings from right to left its velocity is negative. It has positive velocity as it swings back from left to right.
- It accelerates towards the equilibrium position and then decelerates as it approaches the other end of the oscillation.
- It has maximum speed as it travels through the equilibrium position and decelerates as it swings up to its starting position.

Representing s.h.m. graphically

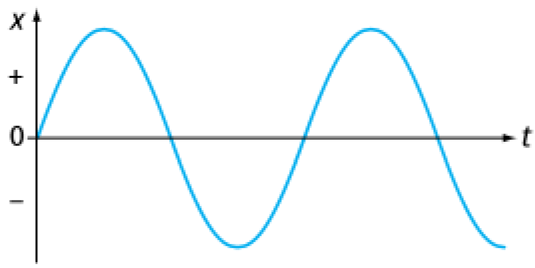


Frequency and angular frequency

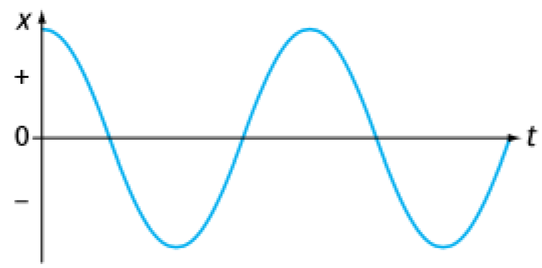
$$\boxed{\omega = 2\pi f}$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

Equations of s.h.m.



$x = x_0 \sin\omega t$

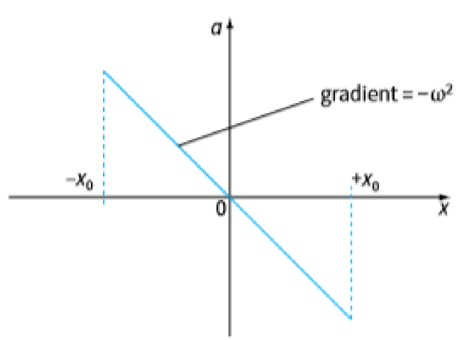


$x = x_0 \cos\omega t$

Acceleration and displacement

$$\boxed{a = -\omega^2 x}$$

The acceleration a is directly proportional to displacement x; and the minus sign shows that it is in the opposite direction.



- The graph is a straight line through the origin (a ∝ x).
- It has a negative slope (the minus sign in the equation ($a = -\omega^2 x$). This means that the acceleration is always directed towards the equilibrium position.
- The magnitude of the gradient of the graph is ω^2.
- The gradient is independent of the amplitude of the motion. This means that the frequency f or the period T of the oscillator is independent of the amplitude and so a simple harmonic oscillator keeps steady time

Equations for velocity

$$\boxed{v = v_0 \cos \omega t}$$

$$\boxed{v = \pm\omega \sqrt{x_0^2 - x^2}}$$

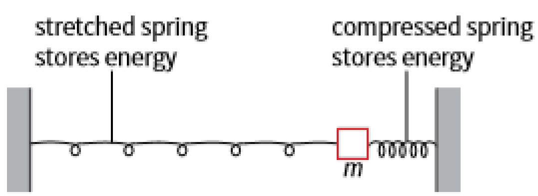
Maximum speed of an oscillator

Maximum speed when it passes through its equilibrium position. This is when its displacement x is zero.

$\boxed{v_{max} = \omega x_0}$ $\boxed{v_{max} = (2\pi f)x_0}$

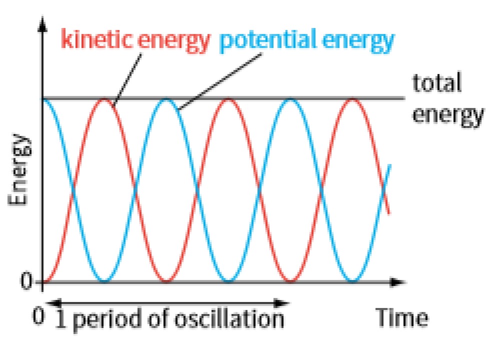
Energy changes in s.h.m.

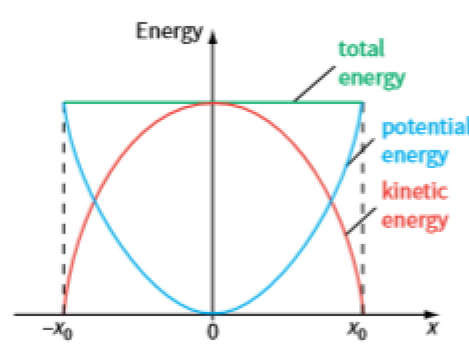
During simple harmonic motion, there is a constant interchange of energy between two forms: potential and kinetic



Energy graphs

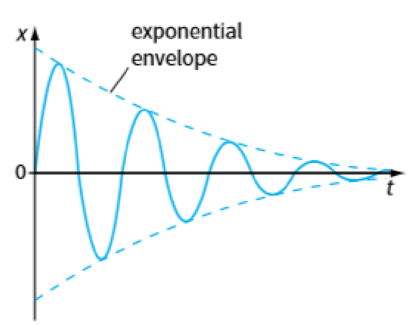
- Kinetic energy is maximum when displacement $x = 0$
- Potential energy is maximum when $x = \pm x_0$
- At any point on this graph, the total energy (k.e. + p.e.) has the same value.





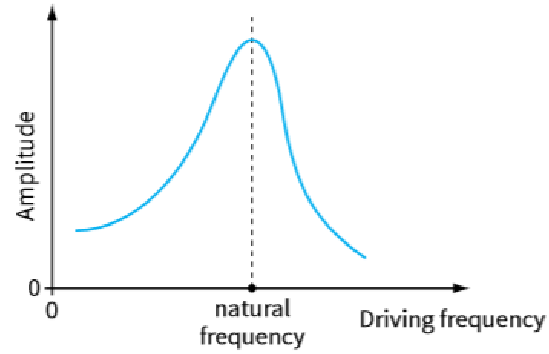
Damped oscillations

They die out, either rapidly or gradually. We describe these oscillations as damped. Their amplitude decreases according to a particular pattern. The amplitude of damped oscillations does not decrease linearly. Notice that the frequency of the oscillations does not change as the amplitude decreases. This is a characteristic of simple harmonic motion.Damping is achieved by introducing the force of friction into a mechanical system By introducing friction, damping has the effect of removing energy from the oscillating system, and the amplitude and maximum speed of the oscillation decrease



Resonance

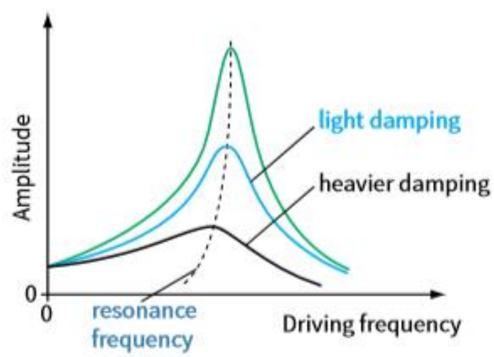
Resonance, in physics, relatively large selective response of an object or a system that vibrates in step or phase, with an externally applied oscillatory force.

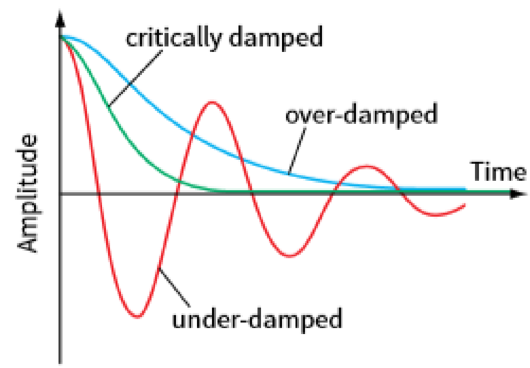


The following statements apply to any system in resonance:

- Its natural frequency is equal to the frequency of the driver.
- Its amplitude is maximum.
- It absorbs the greatest possible energy from the driver.

Resonance and damping



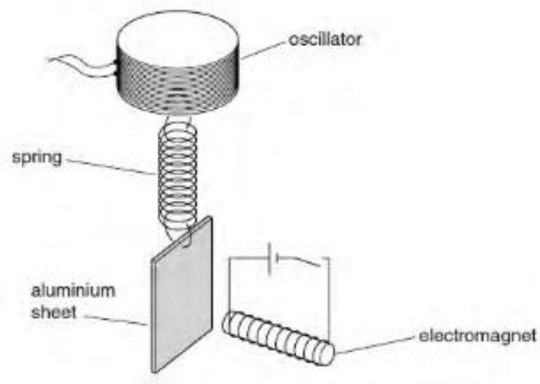


Using resonance

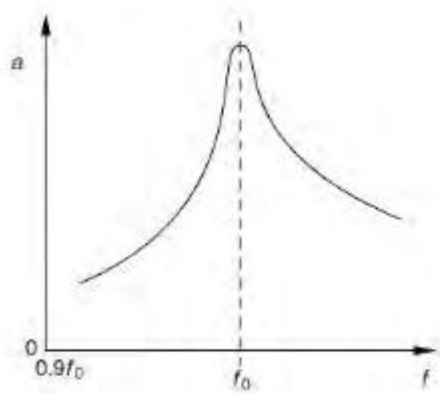
- Magnetic resonance imaging (MRI) is increasingly used in medicine to produce images
- A radio or television also depends on resonance for its tuning circuitry
- microwave cooking

EXCERSISE

1. An aluminium sheet is suspended from an oscillator by means of a spring, as illustrated spring



An electromagnet is placed a short distance from the centre of the aluminium sheet.
The electromagnet is switched off and the frequency f of oscillation of the oscillator is gradually increased from a low value. The variation with frequency f of the amplitude a of vibration of the sheet is shown in the graph below.



A peak on the graph appears at frequency f_0

a. Explain why there is a peak at frequency f_0
b. The electromagnet is now switched on and the frequency of the oscillator is again gradually increased from a low value. Draw a line to show the variation with frequency f of the amplitude a of vibration of the sheet.
c. The frequency of the oscillator is now maintained at a constant value. The amplitude of vibration is found to decrease when the current in the electromagnet is switched on. Use the laws of electromagnetic induction to explain this observation.

2. A piston moves vertically up and down in a cylinder, as illustrated in Fig. 4.1.

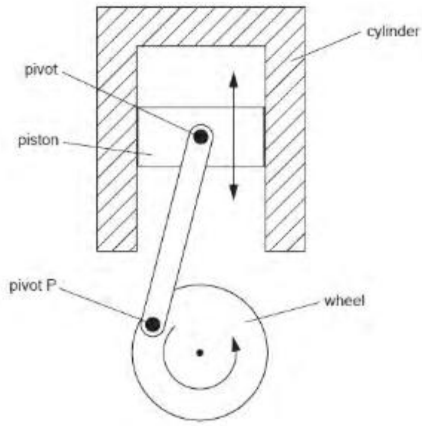


Fig. 4.1

The piston is connected to a wheel by means of a rod that is pivoted at the piston and at the wheel. As the piston moves up and down, the wheel is made to rotate.

a.
   i. State the number of oscillations made by the piston during one complete rotation of the wheel.
   ii. The wheel makes 2400 revolutions per minute. Determine the frequency of oscillation of the piston.

b. The amplitude of the oscillations of the piston is 42 mm.
   Assuming that these oscillations are simple harmonic, calculate the maximum values for the piston of
   i. the linear speed
   ii. the acceleration.

c. On Fig. 4.1, mark a position of the pivot P for the piston to have
   i. maximum speed (mark this position S)
   ii. maximum acceleration (mark this position A).

3.

a. State what is meant by
   i. oscillations,
   ii. free oscillations,
   iii. simple harmonic motion.

b. Two inclined planes RA and LA each have the same constant gradient. They meet at their lower edges, as shown in Fig. 3.1.

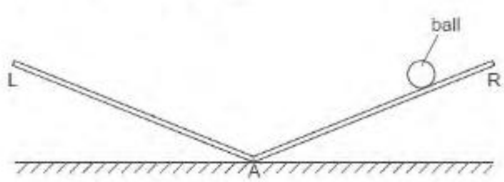


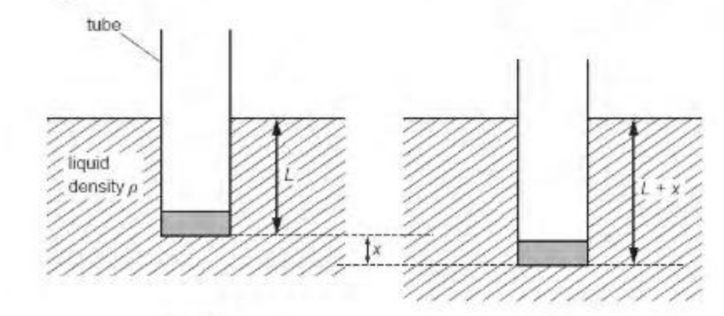
Fig. 3.1

A small ball moves from rest down plane RA and then rises up plane LA. It then moves down plane LA and rises up plane RA to its original height. The motion repeats itself. State and explain whether the motion of the ball is simple harmonic.

4.

a. Define simple harmonic motion.

b. A tube, sealed at one end, has a total mass m and a uniform area of cross-section A. The tube floats upright in a liquid of density p with length L submerged, as shown in Fig. 3.1a.



The tube is displaced vertically and then released. The tube oscillates vertically in the liquid. At one time, the displacement is x, as shown in Fig. 3.1b.

Theory shows that the acceleration a of the tube is given by the expression

$$a = -\frac{a\rho g}{m}x$$

i. Explain how it can be deduced from the expression that the tube is moving with simple harmonic motion.

ii. The tube, of area of cross-section $4.5 cm^2$, is floating in water of density $1.0 \times 10^3\ kgm^{-3}$

Calculate the mass of the tube that would give rise to oscillations of frequency 1.5 Hz.