Cambridge AS

Physics

(Code: 9702)

Chapter 2
Accelerated motion

The meaning of acceleration

The rate of change of velocity per unit of time is known as acceleration.

Calculating acceleration

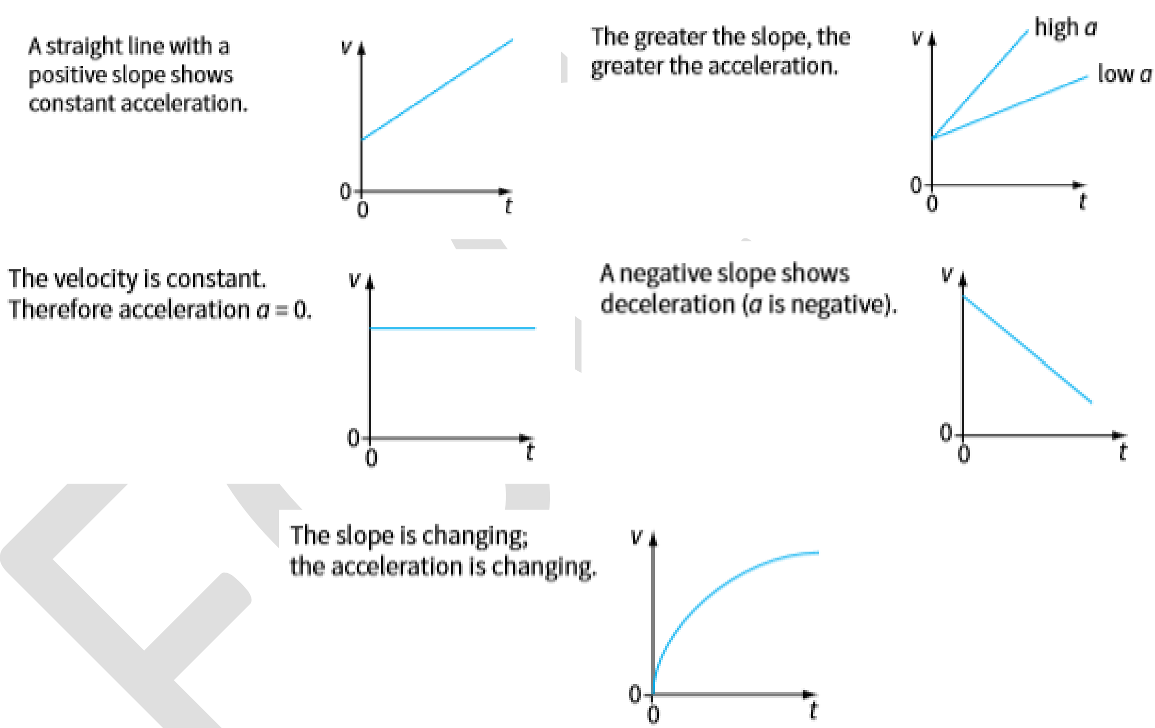
$$\boxed{a = \frac{\Delta v}{\Delta t}} \qquad \boxed{a = \frac{v - u}{t}}$$

Units of acceleration

The SI unit of acceleration is m s^{-2} (metres per second squared).

Deducing acceleration

✓ Acceleration = gradient of velocity–time graph



Deducing displacement

✓ displacement = area under velocity–time graph

The equations of motion

equation 1:	$v = u + at$
equation 2:	$s = \frac{(u+v)}{2} \times t$
equation 3:	$s = ut + \frac{1}{2}at^2$
equation 4:	$v^2 = u^2 + 2as$

s	displacement	a	acceleration
u	initial velocity	t	time taken
v	final velocity		

The equations can only be used:

- for motion in a straight line
- for an object with constant acceleration

Deriving the equations of motion

Equation 1

The acceleration is defined as:

$$a = \frac{v - u}{t}$$

By rearranging:

$$\boxed{v = u + at}$$

Equation 2

Displacement = average velocity × time taken

Hence:

$$\boxed{s = \frac{(u + v)}{2} \times t}$$

Equation 3

From equations 1 and 2, we can derive equation 3. Substituting v from equation 1 gives:

$$s = \frac{u + u + at}{2} \times t$$

$$s = \frac{2ut}{2} + \frac{at^2}{2}$$

So,

$$\boxed{s = ut + \frac{1}{2}at^2}$$

Equation 4

Equation 4 is also derived from equations 1 and 2. Substituting for time t from equation 1 gives:

$$s = \frac{u + v}{2} + \frac{v + u}{a}$$

Rearranging this gives

$$2as = (u + v)(v - u) = v^2 - u^2$$

Simply:

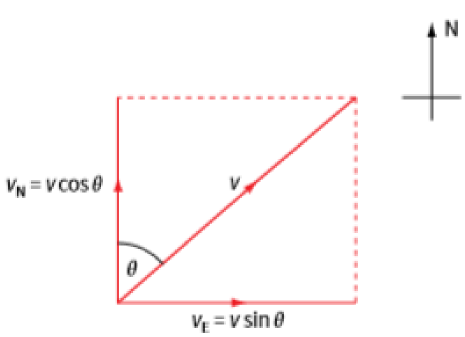
$$\boxed{v^2 = u^2 + 2as}$$

Acceleration caused by gravity

Acceleration of free fall, g = 9.81 m s^{-2}

Motion in two dimensions – projectiles

Components of a vector



Understanding projectiles

1 A stone is thrown horizontally with a velocity of $12\,m\,s^{-1}$ from the top of a vertical cliff.

Calculate how long the stone takes to reach the ground 40 m below and how far the stone lands from the base of the cliff.

Step 1 Consider the ball's vertical motion. It has zero initial speed vertically and travels 40 m with acceleration $9.81\,m\,s^{-2}$ in the same direction.

$s = ut + \frac{1}{2}at^2$

$40 = 0 + \frac{1}{2} \times 9.81 \times t^2$

Thus $t = 2.86\,s$.

Step 2 Consider the ball's horizontal motion. The ball travels with a constant horizontal velocity, $12\,m\,s^{-1}$, as long as there is no air resistance.

distance travelled $= u \times t = 12 \times 2.86 = 34.3\,m$

Hint: *You may find it easier to summarise the information like this:*

vertically	$s = 40$	$u = 0$	$a = 9.81$	$t = ?$	$v = ?$
horizontally	$u = 12$	$v = 12$	$a = 0$	$t = ?$	$s = ?$

2 A ball is thrown with an initial velocity of $20\,m\,s^{-1}$ at an angle of 30° to the horizontal (Figure 2.32). Calculate the horizontal distance travelled by the ball (its **range**).

Step 1 Split the ball's initial velocity into horizontal and vertical components:

initial velocity $= u = 20\,m\,s^{-1}$

horizontal component of initial velocity

$= u\cos\theta = 20 \times \cos 30° = 17.3\,m\,s^{-1}$

vertical component of initial velocity

$= u\sin\theta = 20 \times \sin 30° = 10\,m\,s^{-1}$

Step 2 Consider the ball's vertical motion. How long will it take to return to the ground? In other words, when will its displacement return to zero?

$u = 10\,m\,s^{-1}$ $a = -9.81\,m\,s^{-2}$ $s = 0$ $t = ?$

Using $s = ut + \frac{1}{2}at^2$, we have:

$0 = 10t - 4.905t^2$

This gives $t = 0\,s$ or $t = 2.04\,s$. So the ball is in the air for 2.04 s.

Step 3 Consider the ball's horizontal motion. How far will it travel horizontally in the 2.04 s before it lands? This is simple to calculate, since it moves with a constant horizontal velocity of $17.3\,m\,s^{-1}$.

horizontal displacement $s = 17.3 \times 2.04$

$= 35.3\,m$

Hence the horizontal distance travelled by the ball (its range) is about 35 m.

EXCERSISE

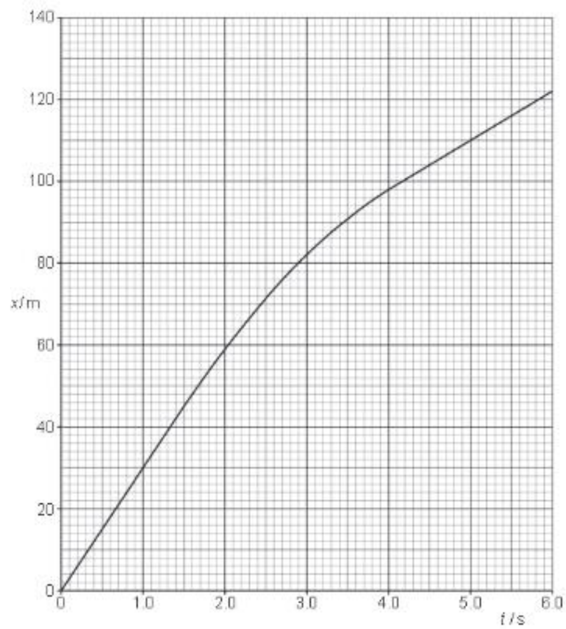
1.

a. Define

i. Velocity

ii. acceleration

b. A car of mass 1500 kg travels along a straight hon zontal road. The variation with time t of the displacement x of the car is shown in Fig. 3.1.



i. Use Fig. 3.1 to describe qualitatively the velocity of the car during the first six seconds of the motion shown. Give reasons for your answers.

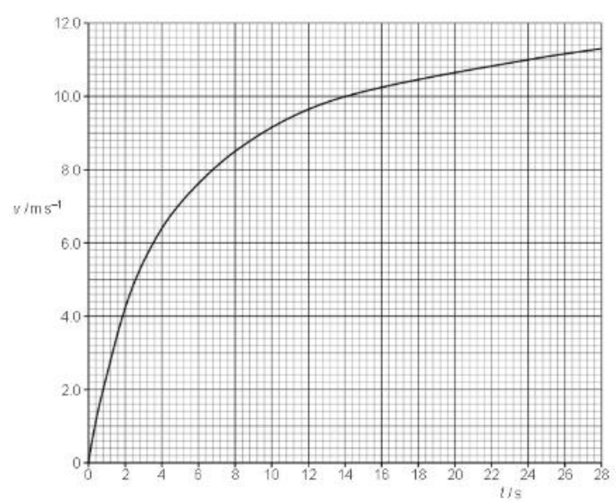
ii. Calculate the average velocity during the time interval t = 0 to t = 1.5 s.

iii. Show that the average acceleration between t = 1.5s and t = 4.0 s is $-7.2ms^{-2}$.

iv. Calculate the average force acting on the car between t = 1.5s and t = 4.0s.

2.

a. A cyclist travels along a horizontal road. The variation with time f of speed v is shown. The cyclist maintains a constant power and after some time reaches a constant speed of $12ms^{-1}$.



i. Describe and explain the motion of the cyclist.

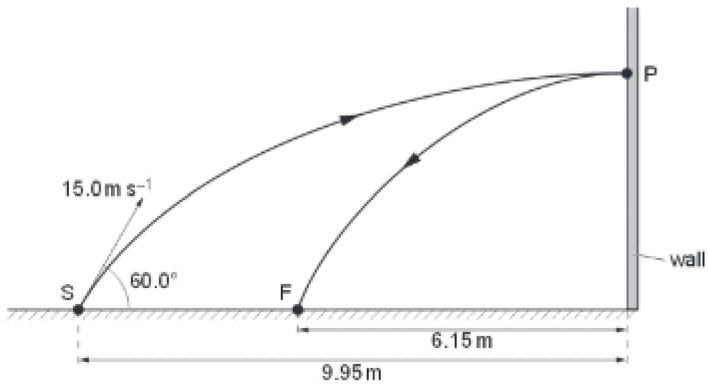
ii. When the cyclist is moving at a constant speed of 12ms the resistive force is 48N. Show that the power of the cyclist is about 600W. Explain your working.

iii. show that the acceleration of the cyclist when his speed is $8.0ms^{-1}$ is about $0.5ms^{-2}$

iv. The total mass of the cyclist and bicycle is 80kg. Calculate the resistive force R acting on the cyclist when his speed is $8.0ms^{-1}$. Use the value for the acceleration given in (iii).

v. Use the information given in (ii) and your answer to (iv) to show that, in this situation, the resistive force R is proportional to the speed v of the cyclist.

3. A ball is thrown against a vertical wall. The path of the ball is shown



The ball is thrown from S with an initial velocity of 15.0ms-1 at 60.0° to the horizontal. Assume that air resistance is negligible.

a. For the ball at S, calculate

i. its horizontal component of velocity.

ii. its vertical component of velocity.

b. The horizontal distance from S to the wall is 9.95 m. The ball hits the wall at P with a velocity that is at right angles to the wall. The ball rebounds to a point F that is 6.15m from the wall. Using your answers in (a).

i. calculate the vertical height gained by the ball when it travels from S to P.

ii. show that the time taken for the ball to travel from S to P is 1.33s,

iii. show that the velocity of the ball immediately after rebounding from the wall is about $4.6ms^{-1}$

c. The mass of the ball is 60 x 10-3kg.

i. Calculate the change in momentum of the ball as it rebounds from the wall.

ii. State and explain whether the collision is elastic or inelastic.

4. The variation with time t of velocity v of a car is shown in Fig. 2.1.

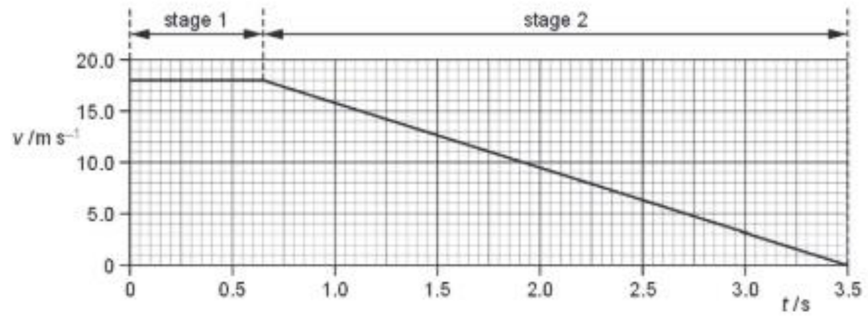


Fig. 2.1

At time t = 0, the driver sees an obstacle in the road. A short time later, the driver applies the brakes. The car travels in two stages, as shown in Fig. 2.1.

a. Use Fig. 2.1 to describe the velocity of the car in
    1. stage 1,
    2. stage 2.

b.
    i. Calculate the distance travelled by the car from t = 0 to t = 3.5s.
    ii. The car has a total mass of 1250kg. Determine the total resistive force acting on the car in stage 2.

c. For safety reasons drivers are asked to travel at lower speeds. For each stage, describe and explain the effect on the distance travelled for the same car and driver travelling at half the initial speed shown in Fig. 2.1.
    1. stage 1:
    2. stage 2:

5.

a.
    i. Define displacement.
    ii. Use your definition to explain how it is possible for a car to travel a certain distance and yet have zero displacement.

b. A car starts from rest and travels upwards along a straight road inclined at an angle of 5.0° to the horizontal, as illustrated in Fig.2.1.

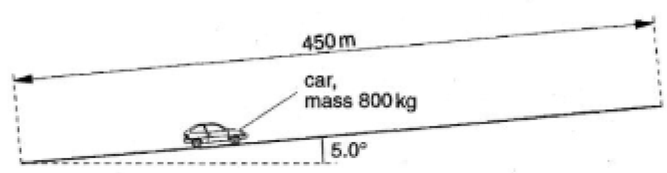


Fig. 2.1

The length of the road is 450m and the car has mass 800 kg. The speed of the car increases at a constant rate and is 28 m s-1 at the top of the slope.

i. Determine, for this car travelling up the slope,
   1. its acceleration,
   2. the time taken to travel the length of the slope,

6. A student has been asked to determine the linear acceleration of a toy car as it moves down a slope. He sets up the apparatus as shown in Fig. 3.1.

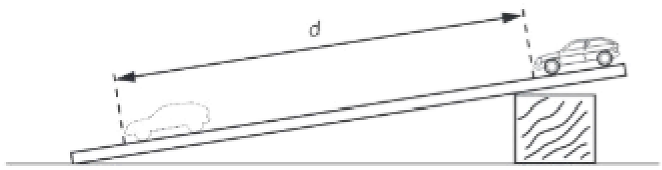
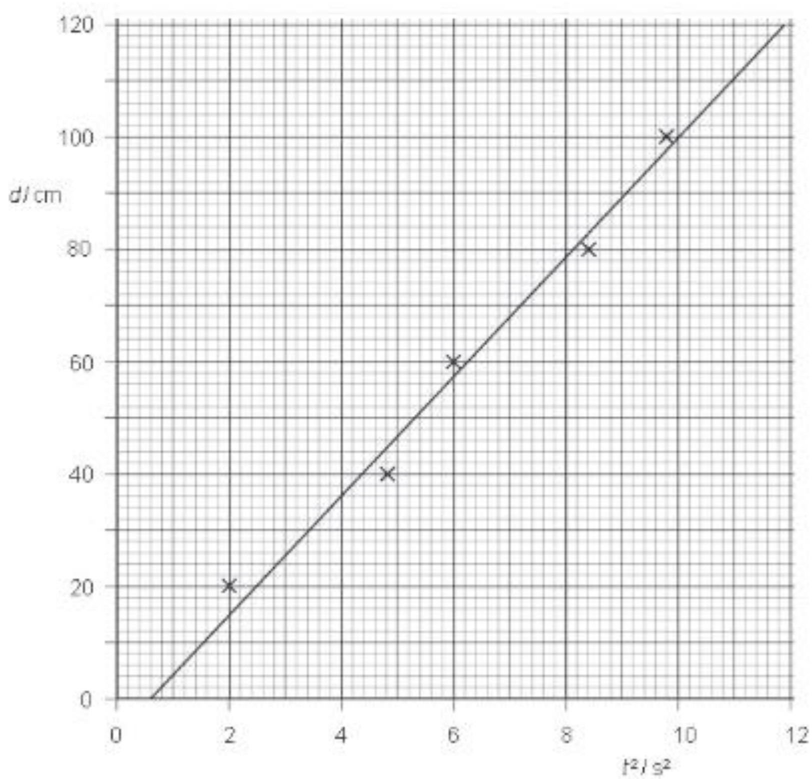


Fig. 3.1

The time tto move from rest through a distance d is found for different values of d. A graph of d (y-axis) is plotted against t2 (x-axis) as shown in Fig. 3.2.



a. Theory suggests that the graph is a straight line through the origin. Name the feature on Fig. 3.2 that indicates the presence of
   i. random error,
   ii. systematic error.

b.
   i. Determine the gradient of the line of the graph in Fig. 3.2.
   ii. Use your answer to (i) to calculate the acceleration of the toy down the slope. Explain your working.

7.

a. During a fireworks display, a firework is launched into the air and explodes once it reaches a maximum height of 250 m. The vertical component of the velocity at launch depends on both the initial velocity of the firework and the angle between the initial velocity and the horizontal. The maximum initial velocity a firework can be launched at is 110 ms^{-1}.
   i. Calculate the initial launch velocity for a firework that is fired directly upwards.
   ii. Show that the minimum launch angle between the initial velocity and the horizontal to reach 250 m is about 40°.
b. On Fig. 1.1, sketch the variation of angle with the initial velocity required for the firework to reach the maximum height of 250 m. Include values from (a) on the axes.
c. The length of the fuse within the firework is chosen to ensure it explodes just as it reaches its highest point above the ground.
d. The firework is launched into the air at an angle of 75° to the horizontal, as shown in Fig. 1.2.

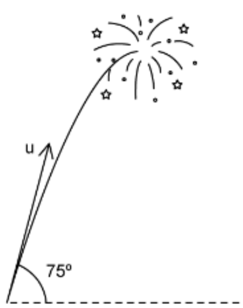


Fig. 1.2

The firework contains a fuse with a burn rate of 1.5 cm s^{-1}.
Determine the length of the fuse used in the firework.

8. During the display, one of the fireworks is lit but falls over before launching. When it launches, it leaves the ground with an initial velocity of 75 m s1 at an angle of 30° to the horizontal, as shown in Fig. 1.3.

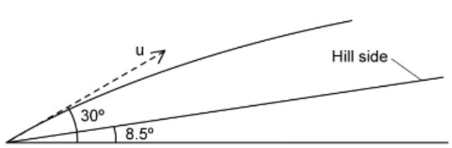


Fig. 1.3

The firework moves in the direction of a hill. The side of the hill is inclined at 8.5° to the horizontal, as shown.
Deduce whether the firework ignites before it lands on the hillside.

9.

a. Fig. 1.1 shows a stunt motorcyclist descending down a ramp.

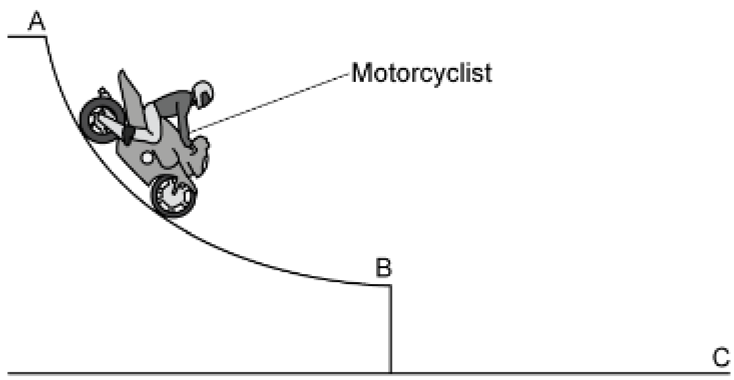


Fig. 1.1

The motorcyclist starts from rest at the top of the ramp at A and leaves the ramp at B horizontally.
After leaving the ramp at B, the motorcyclist lands on the ground at C, which is 6.4 m away.
At C, the motorcyclist lands with a resultant velocity of 13.4 m s1at an angle of 29.5° with the horizontal.
Calculate the initial velocity of the motorcyclist at B.
In this question, assume that air resistance is negligible.

b. The motorcyclist practices a stunt on a different ramp inclined at 25° and 3 m high as shown in Fig. 1.2.

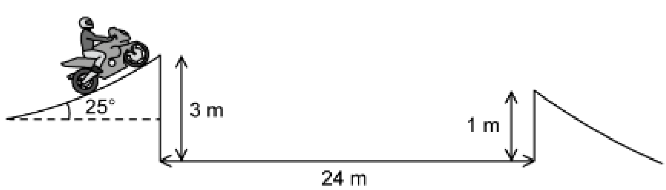


Fig. 1.2

The other ramp is 24 m away and has a height of 1 m, as shown. Calculate the minimum initial velocity the motorcyclist requires in order to reach the other ramp.

c. The stunt motorcyclist's ultimate goal is to jump across a river and land on the other side. Fig. 1.3 shows the motorcyclist driving off a ramp at the edge of a river.

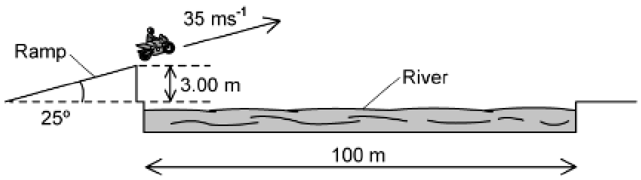


Fig. 1.3

The ramp is at an angle of 25° to the horizontal and the height at the end of the ramp is 3.0 m. The width of the river is 100 m. The initial velocity of the motorcyclist is 35 ms-1. Deduce whether the motorcyclist lands on the other side of the river.

d. Explain how air resistance would affect the jump.

10.

a. An object is released near the surface of the Moon at time t = 0. Fig. 1.1 shows the variation of displacement s with time t of the object from the point of release.

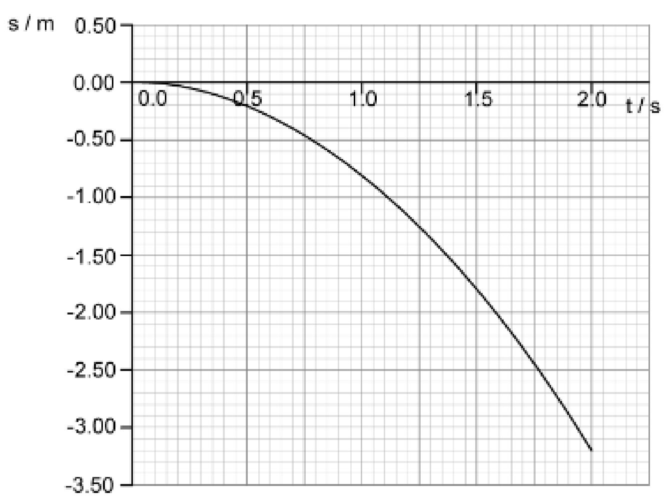


Fig. 1.1

i. State the significance of the negative values of s.
ii. State two assumptions about the motion of the object.

b. Use the graph in Fig. 1.1 to determine a value for the acceleration of free fall close to the surface of the Moon.

c. Use Fig. 1.1 to estimate the instantaneous velocity of the object when t = 1.5 s.

d.

i. On Fig. 1.1, sketch the variation of displacement s with time t if the same object was released close to the surface of the Earth instead.
ii. Describe and explain the features of your sketch.