Cambridge AS

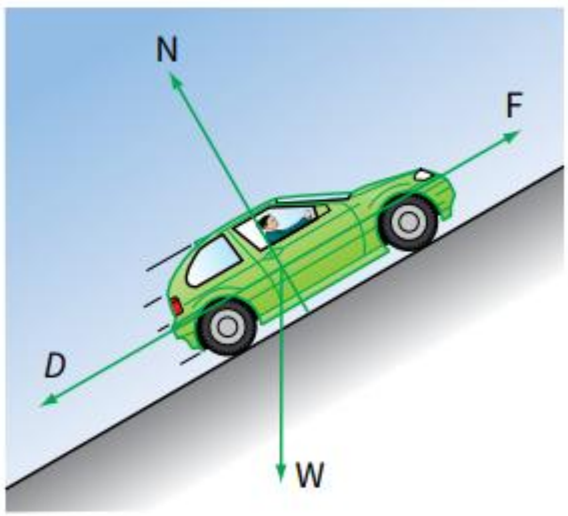
Physics

(Code: 9702)

Chapter 4

Forces – vectors and moments

Combining forces

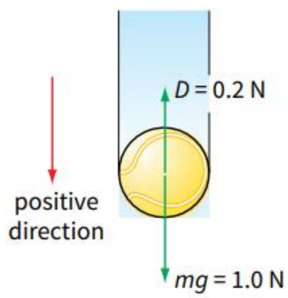


There are several forces acting on the car as it struggles up the steep hill. They are:

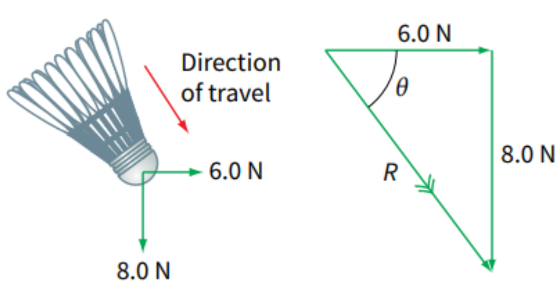
- its weight W (= mg)
- the contact force N of the road (its normal reaction)
- air resistance D
- the forward force F caused by friction between the car tyres and the road.

Two forces in a straight line

we adopt a sign convention to help us decide which is which.



Two forces at right angles

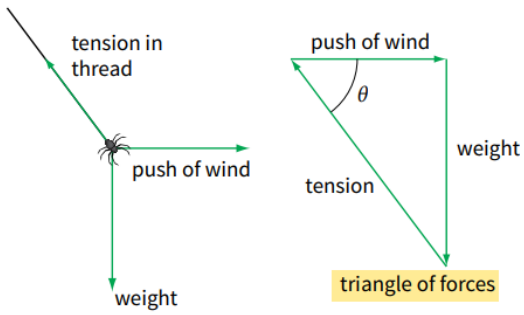


- First, a horizontal arrow is drawn to represent the 6.0N push of the wind.
- Next, starting from the end of this arrow, we draw a second arrow, downwards, representing the weight of 8.0N.
- Now we draw a line from the start of the first arrow to the end of the second arrow. This arrow represents the resultant force R, in both magnitude and direction.

If you draw a scale drawing be careful to:

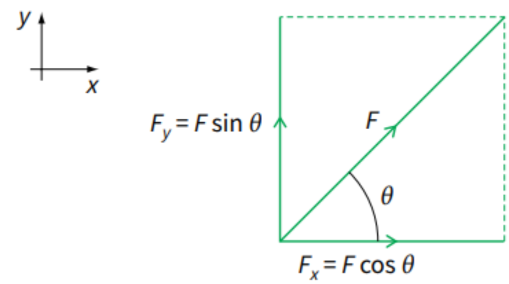
- state the scale used
- draw a large diagram to reduce the uncertainty

Three or more forces



horizontal component of F, $F_x = F\cos\theta$

vertical component of F, $F_y = F\sin\theta$



Solving problems by resolving forces

WORKED EXAMPLE

1 A boy of mass 40 kg is on a waterslide which slopes at 30° to the horizontal. The frictional force up the slope is 120 N. Calculate the boy's acceleration down the slope. Take the acceleration of free fall g to be $9.81\ \text{m s}^{-2}$.

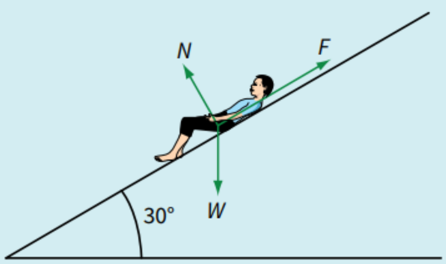


Figure 4.13 For Worked example 1.

Step 1 Draw a labelled diagram showing all the forces acting on the object of interest (Figure 4.13). This is known as a **free-body force diagram**. The forces are:

the boy's weight $W = 40 \times 9.81 = 392\ \text{N}$

the frictional force up the slope $F = 120\ \text{N}$

the contact force N at 90° to the slope.

Step 2 We are trying to find the resultant force on the boy which makes him accelerate down the slope. We resolve the forces down the slope, i.e. we find their components in that direction.

component of W down the slope $= 392 \times \cos 60° = 196\ \text{N}$

component of F down the slope $= -120\ \text{N}$
(negative because F is directed up the slope)

component of N down the slope $= 0$
(because it is at 90° to the slope)

It is convenient that N has no component down the slope, since we do not know the value of N.

Step 3 Calculate the resultant force on the boy:

resultant force $= 196 - 120 = 76\ \text{N}$

Step 4 Calculate his acceleration:

$$\text{acceleration} = \frac{\text{resultant force}}{\text{mass}} = \frac{76}{40} = 1.9\ \text{m s}^{-2}$$

So the boy's acceleration down the slope is $1.9\ \text{m s}^{-2}$. We could have arrived at the same result by resolving vertically and horizontally, but that would have led to two simultaneous equations from which we would have had to eliminate the unknown force N. It often helps to resolve forces at 90° to an unknown force.

Centre of gravity

The centre of gravity of an object is defined as the point where all the weight of the object may be considered to act.

The turning effect of a force

Moment of a force

The quantity which tells us about the turning effect of a force is its moment. The moment of a force depends on two quantities:

- the magnitude of the force (the bigger the force, the greater its moment)
- the perpendicular distance of the force from the pivot (the further the force acts from the pivot, the greater its moment).

$$\boxed{moment = F \times x}$$

Balanced or unbalanced?

For any object that is in equilibrium, the sum of the clockwise moments about any point provided by the forces acting on the object equals the sum of the anticlockwise moments about that same point.

The torque of a couple

A pair of forces like that in Figure 4.25 is known as a couple. A couple has a turning effect, but does not cause an object to accelerate. To form a couple, the two forces must be:

- equal in magnitude
- parallel, but opposite in direction
- separated by a distance d.

The turning effect or moment of a couple is known as its torque.

$$\boxed{torque\ of\ a\ couple\ =\ one\ of\ the\ forces \times perpendicular\ distance\ between\ the\ forces}$$

Pure turning effect

For an object to be in equilibrium, two conditions must be met at the same time:

- The resultant force acting on the object is zero.
- The resultant moment is zero.

EXCERSISE

1.
   a. Define the torque of a couple.
   b. A wheel is supported by a pin P at its centre of gravity, as shown in Fig. 4.1.
      The plane of the wheel is vertical. The wheel has radius 25 cm.
      Two parallel forces each of 35 N act on the edge of the wheel in the vertical directions shown in Fig. 4.1. Friction between the pin and the wheel is negligible.

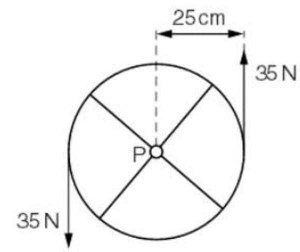


Fig. 4.1

      i. List two other forces that act on the wheel. State the direction of these forces and where they act.
      ii. Calculate the torque of the couple acting on the wheel.
      iii. The resultant force on the wheel is zero. Explain, by reference to the four forces acting on the wheel, how it is possible that the resultant force is zero.
      iv. State and explain whether the wheel is in equilibrium.

2. Two planks of wood AB and BC are inclined at an angle of 15° to the horizontal. The two wooden planks are joined at point B, as shown in Fig. 2.1.

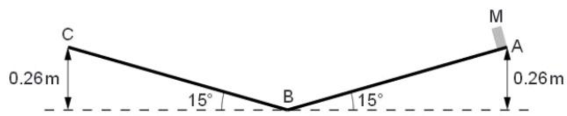


Fig. 2.1

A small block of metal M is released from rest at point A. It slides down the slope to B and up the opposite side to C. Points A and C are 0.26 m above B. Assume frictional forces are negligible.

   a.
      i. Describe and explain the acceleration of M as it travels from A to B and from B to C.
      ii. Calculate the time taken for M to travel from A to B.
      iii. Calculate the speed of M at B.
   b. The plank BC is adjusted so that the angle it makes with the horizontal is 30°. M is released from rest at point A and slides down the slope to B. It then slides a distance along the plank from B towards C.
      Use the law of conservation of energy to calculate this distance. Explain your working.

3.
   a. State Newton's second law.
   b. A ball of mass 65 g hits a wall with a velocity of 5.2ms1 perpendicular to the wall. The ball rebounds perpendicularly from the wall with a speed of 3.7 m s-1. The contact time of the ball with the wall is 7.5 ms.
      Calculate, for the ball hitting the wall,
      i. the change in momentum,
      ii. the magnitude of the average force.

c.

i. For the collision in (b) between the ball and the wall, state how the following apply:
   1. Newton's third law,
   2. the law of conservation of momentum.

ii. State, with a reason, whether the collision is elastic or inelastic.

4.

a. Distinguish between scalars and vectors.

b. Underline all the vector quantities in the list below.
   A. acceleration
   B. kinetic energy
   C. momentum
   D. power
   E. weight

c. A force of 7.5N acts at 40° to the horizontal, as shown in Fig. 1.1.

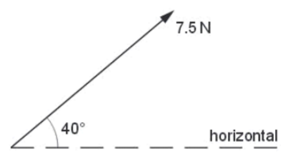


Fig. 1.1

Calculate the component of the force that acts
   i. horizontally,
   ii. vertically

d. Two strings support a load of weight 7.5N, as shown in Fig. 1.2.

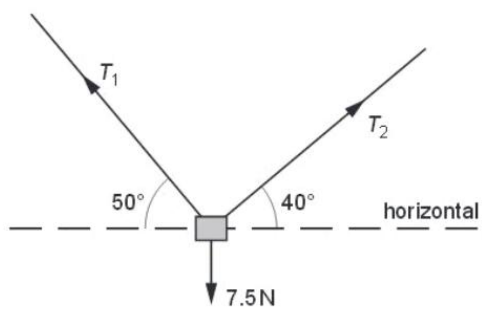


Fig. 1.2

One string has a tension T, and is at an angle 50° to the horizontal. The other string has a tension T, and is at an angle 40° to the horizontal. The object is in equilibrium. Determine the values of T1 and T2 by using a vector triangle or by resolving forces.

5.

a. Define
   - i. force,
   - ii. work done.

b. A force Facts on a mass m along a straight line for a distance s. The acceleration of the mass is a and the speed changes from an initial speed u to a final speed v.
   - i. State the work W done by F.
   - ii. Use your answer in (i) and an equation of motion to show that kinetic energy of a mass can be given by the expression

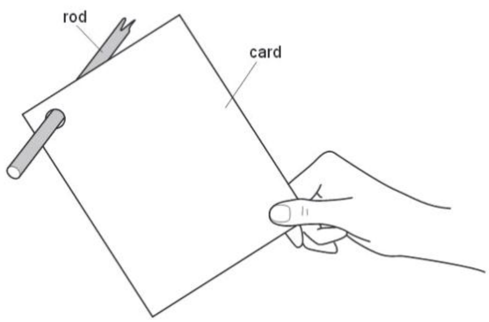
$$kinetic\ energy = \frac{1}{2} \times mass \times speed^2$$

c. A resultant force of 3800N causes a car of mass of 1500 kg to accelerate from an initial speed of 15m s^{-1} to a final speed of 30m s^{-1}.
   - i. Calculate the distance moved by the car during this acceleration.
   - ii. The same force is used to change the speed of the car from 30m s^{-1} to 45ms^{-1}. Explain why the distance moved is not the same as that calculated in (i).

6.

a. State what is meant by the centre of gravity of a body.

b. A uniform rectangular sheet of card of weight W is suspended from a wooden rod. The card is held to one side, as shown in Fig. 3.1.



   - i. mark, and label with the letter C, the position of the centre of gravity of the card,
   - ii. mark with an arrow labelled Wthe weight of the card.

c. The card in (b) is released. The card swings on the rod and eventually comes to rest.
   - i. List the two forces, other than its weight and air resistance, that act on the card during the time that it is swinging. State where the forces act.
   - ii. By reference to the completed diagram of Fig. 3.1, state the position in which the card comes to rest.
     Explain why the card comes to rest in this position.

7. A stationary nucleus of mass 220u undergoes radioactive decay to produce a nucleus D of mass 216u and an a-particle of mass 4u, as illustrated in Fig. 3.1.

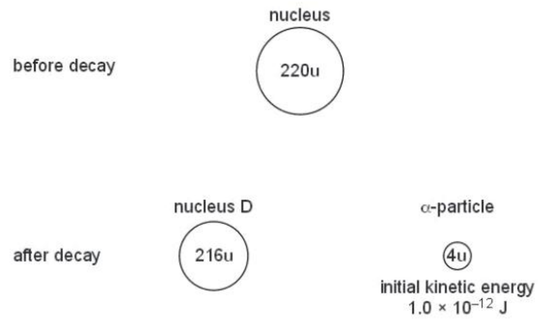


Fig. 3.1

a.
   i. State the law of conservation of linear momentum.
   ii. Explain why the initial velocities of the nucleus D and the a-particle must be in opposite directions.

b.
   i. Show that the initial speed of the a-particle is $1.7 \times 10^{7} ms^{-1}$.
   ii. Calculate the initial speed of nucleus D.

c. Calculate the average deceleration of the a-particle as it is stopped by the air.

8.

a. Distinguish between the moment of a force and the torque of a couple.
   i. moment of a force
   ii. torque of a couple

b. One type of weighing machine, known as a steelyard, is illustrated in Fig. 3.1.

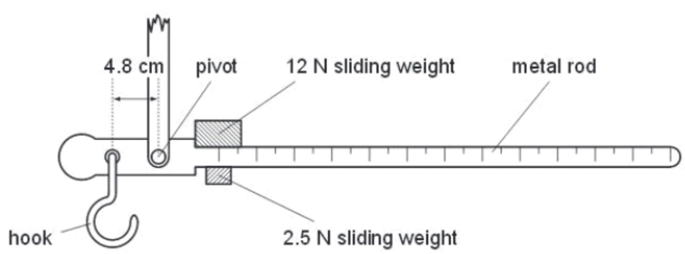


Fig. 3.1

The two sliding weights can be moved independently along the rod.
With no load on the hook and the sliding weights at the zero mark on the metal rod, the metal rod is horizontal. The hook is 4.8 cm from the pivot.
A sack of flour is suspended from the hook. In order to return the metal rod to the horizontal position, the 12N sliding weight is moved 84 cm along the rod and the 2.5N weight is moved 72 cm.

i. Calculate the weight of the sack of flour.

ii. Suggest why this steelyard would be imprecise when weighing objects with a weight of about 25N.

9. Francium-208 is radioactive and emits a-particles with a kinetic energy of 1.07 × 10-12 J to form nuclei of astatine, as illustrated in Fig. 3.1.

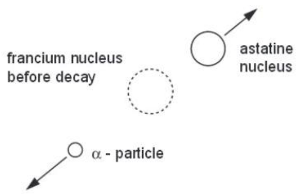


Fig. 3.1

a. State the nature of an α-particle.

b. Show that the initial speed of an a-particle after the decay of a francium nucleus is approximately 1.8×10^7 m s^{-1}.

c.

i. State the principle of conservation of linear momentum.

ii. The Francium-208 nucleus is stationary before the decay. Estimate the speed of the astatine nucleus immediately after the decay.

d. Close examination of the decay of the francium nucleus indicates that the astatine nucleus and the o-particle are not ejected exactly in opposite directions.
suggest an explanation for this observation.

10.

a. Distinguish between mass and weight.

i. mass:

ii. weight

b. An object O of mass 4.9 kg is suspended by a rope A that is fixed at point P. The object is pulled to one side and held in equilibrium by a second rope B, as shown in Fig. 2.1.

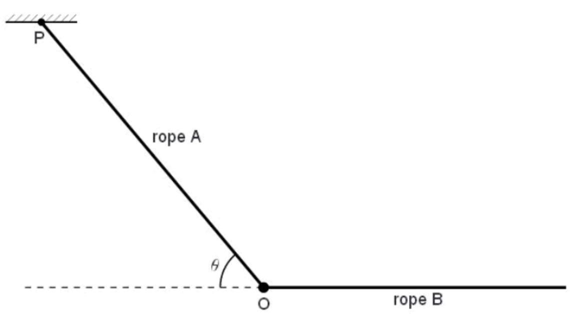


Fig. 2.1

Rope A is at an angle to the horizontal and rope B is horizontal. The tension in rope A is 69 N and the tension in rope B is T.

i. On Fig. 2.1, draw arrows to represent the directions of all the forces acting on object O.

ii. Calculate

1. the angle 0,

2. the tension T.