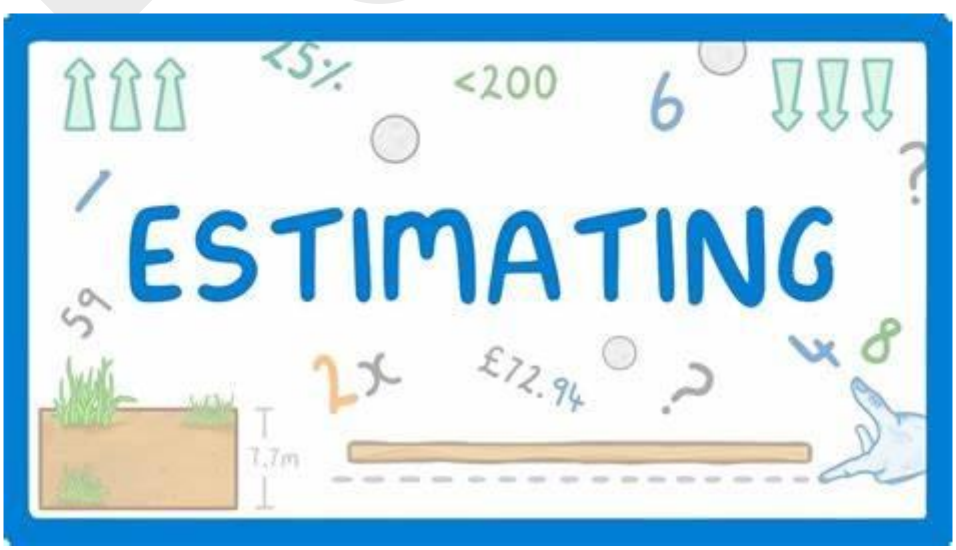
Cambridge OL

Mathematics

CODE: (4024)

Chapter 09 and Chapter 10

Estimation and Limits of accuracy



Estimating lengths

To estimate lengths, compare with measures you know.

Example 9.1

Question

Estimate the height of the lamp post.

Solution

7.5 m

Any answer from 6.5 to 8.5 m would be acceptable.

The lamp post is about four times the height of the man.

Note

The height to an adult person's waist is about 1 m (100 cm) and their full height is about 1.7 m (170 cm).

Note

When estimating, do not try to be too accurate. What is wanted is a rough estimate and anything about right is acceptable. Decide if it is bigger or smaller than a measure you know and work from there.

Note

Make sure that you include units with your answer and that they are sensible. For example, the length of a motorway should be in kilometres, not metres.

Rounding to a given number of decimal places

Find the digit in the decimal place to be rounded. Look at the next digit.

- If that is less than 5, leave the decimal as it is.
- If it is 5 or more, round the decimal up by 1.

You ignore any digits further to the right

Example 9.2

Question

Write each of these numbers correct to 2 decimal places (2 d.p.).

a 9.368 **b** 0.0438 **c** 84.655

Solution

a 9.37 The digits in the first two decimal places are 36.
The digit in the third decimal place is 8, which is more than 5, so you add 1 to the 6.

b 0.04 The digits in the first two decimal places are 04.
The digit in the third decimal place is 3, which is less than 5, so you leave the 4 as it is.

c 84.66 The digit in the third decimal place is 5, so you add 1 to the digit in the second decimal place.

Note

The mean of a set of numbers is the sum of the numbers divided by how many numbers there are.

Rounding to a given number of significant figures

Significant figures are counted from left to right, starting from the first non-zero digit.

5 346 000

1st significant figure 2nd significant figure 3rd significant figure

You use a similar technique to the one you use for rounding to a given number of decimal places.

You look at the first digit that is not required. If this is less than 5, you round down. If it is 5 or more, you round up.

Rounding 5 346 000 to

1 significant figure gives 5 000 000

2 significant figures gives 5 300 000

3 significant figures gives 5 350 000

Note

Zeros are used to maintain the place value of the digits. In this case, 5 (millions), 3 (hundred thousands) and 5 (ten thousands).

0.005 346 23

1st significant figure 2nd significant figure 3rd significant figure

Rounding 0.005 346 23 to

1 significant figure gives 0.005

2 significant figures gives 0.0053

3 significant figures gives 0.005 35

Note

Again, zeros are used to maintain the place value of the digits. In this case, 5 (thousandths), 3 (ten thousandths).

Note

Zeros to the right of the first non-zero digit *are* significant. For example, 0.070 56 = 0.0706 to 3 significant figures.

Example 9.3

Question

a Round 52 617 to 2 significant figures.
b Round 0.072 618 to 3 significant figures.
c Round 17 082 to 3 significant figures.

Solution

a 52 617 = 53 000 to 2 s.f.

To round to 2 significant figures, look at the third significant figure. It is 6, so the second significant figure changes from 2 to 3. Remember to add zeros for placeholders.

b 0.072 618 = 0.0726 to 3 s.f.

The first significant figure is 7. To round to 3 significant figures, look at the fourth significant figure. It is 1, so the third significant figure does not change.

c 17 082 = 17 100 to 3 s.f.

The 0 in the middle here is a significant figure. To round to 3 significant figures, look at the fourth figure. It is 8, so the third figure changes from 0 to 1. Remember to add zeros for placeholders.

Note
Always state the accuracy of your answers when you have rounded them.

Example 9.4

Question

Write each of these numbers to 1 significant figure.

a 5126 **b** 5.817 **c** 0.047 15 **d** 146 500 **e** 1968

Solution

a 5000 The second significant figure is 1, so the 5 stays as it is. You use three zeros to show the size.
b 6 The second significant figure is 8, so you change the 5 to 6. No zeros are needed.
c 0.05 The second significant figure is 7, so the 4 becomes 5. You keep the zero before the 5 to show the size.
d 100 000 The second significant figure is 4 so the 1 stays as it is. You use zeros to show the size.
e 2000 The second figure is 9, so the 1 changes to 2.

Note
Zeros are used to show size. A common error is to use the wrong number of zeros.

Estimating answers to problems

To estimate answers to problems, you can round each number to 1 significant figure and then carry out the calculation.

Example 9.5

Question

Jim buys 24 golf balls at \$3.75 each.
What is the approximate cost?

Solution

$20 \times 4 = \$80$ Round to 1 significant figure.

Example 9.6

Question

Estimate the result of each of these calculations by rounding to 1 significant figure.

a 3.7×2.1 **b** 47×63 **c** 4.32×8.76 **d** 2.95×0.321 **e** $0.815 \div 3.41$

Solution

a $4 \times 2 = 8$ **b** $50 \times 60 = 3000$ **c** $4 \times 9 = 36$ **d** $3 \times 0.3 = 0.9$

e $0.8 \div 3 = 0.266 = 0.27$ (2 s.f.) or 0.3 (1 s.f.)

Note

Do not try to be too accurate with your answer. Give the answer to 1 or 2 significant figures.

Example 9.7

Question

Estimate the result of each of these calculations.

In each case, say whether the estimated result is bigger or smaller than the actual result and explain your answer.

a 4.7×3.9 **b** 516×2.34 **c** $79.6 \div 23.7$

Solution

a $5 \times 4 = 20$

The estimated result is bigger than the actual result, because both the rounded values used in the calculation are bigger than the actual values.

b $500 \times 2 = 1000$

The estimated result is smaller than the actual result, because both the rounded values used in the calculation are smaller than the actual values.

c $80 \div 20 = 4$

The estimated result is bigger than the actual result, because the rounded value of the number being divided is bigger than the actual value, and the rounded value of the number it is divided by is smaller than the actual value.

Estimating answers to problems involving division and square roots

It is sometimes more convenient to use numbers other than the given values rounded to 1 significant figure.

When division is involved, you can round the numbers so that there are common factors.

When rounding numbers to estimate a square root, round to the nearest perfect square.

Example 9.8

Question

Find approximate answers to these.

a $83.7 \div 2.87$ **b** $0.73 \times \sqrt{18.2}$

Solution

a Round to $90 \div 3 = 30$

or $81 \div 3 = 27 = 30$ to 1 s.f.

(Rounding to $80 \div 3$ is more difficult, but would still give 30 to 1 s.f.)

b Round to $0.7 \times \sqrt{16} = 0.7 \times 4 = 2.8 = 3$ to 1 s.f.

Working to a sensible degree of accuracy

Measurements and calculations should always be expressed to a suitable degree of accuracy.

As a general rule, the answer you give after a calculation should not be given to a greater degree of accuracy than any of the values used in the calculation.

Example 9.9

Question

Bilal measured the length and width of a table as 1.84 m and 1.32 m.

He calculated the area as $1.84 \times 1.32 = 2.4288\,m^2$.

How should he have given the answer?

Solution

Bilal's answer has four places of decimals (4 d.p.) so it is more accurate than the measurements he took.

His answer should be $2.43\,m^2$.

Key points

- Know how to round a number to the nearest 100, 1000, and so on.
- When rounding to a given number of decimal places, count the number of places from the decimal point. This is where the rounded answer will end. If the next digit along is less than five, leave the rounded answer as it is. If the next digit along is greater than or equal to five, add one to the final digit of the rounded answer.
- When rounding to a given number of significant figures, count the number of figures from the first non-zero digit starting at the left of the number. This is where the digits will end in the answer. As with decimal places, check the next digit along to see if the final digit needs altering.
- After rounding to a number of significant figures, make sure the answer has the same order of magnitude as the original number.
- A zero within a series of digits should be counted as one of the decimal places or one of the significant figures.
- To find an estimate to the answer to a problem, round each of the numbers in the problem to one significant figure and then perform the calculation.
- Know that an answer should not be given to a greater degree of accuracy than any of the values used in the calculation.

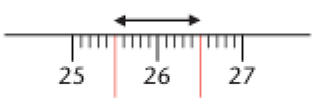
Chapter 10 – Limits of accuracy

Bounds of measurement

Ali measures the length of a sheet of paper. She says the length is 26 cm to the nearest centimetre. What does this mean?

It means the length is nearer to 26 cm than the closest measurements on each side, 25 cm and 27 cm. Any measurement that is nearer to 26 cm than to 25 cm or 27 cm will be counted as 26 cm.

This is the marked interval on the number line.



The boundaries of this interval are 25.5 cm and 26.5 cm. These values are exactly halfway between one measurement and the next. Usually when rounding to a given number of decimal places or significant figures, you would round 25.5 up to 26 and 26.5 up to 27.

- The interval for 26 cm to the nearest centimetre is m cm where $25.5 \leqslant m < 26.5$.
- 25.5 cm is called the **lower bound** of the interval.
- 26.5 cm is called the **upper bound** of the interval but it is not actually included in the interval.

Example 10.1

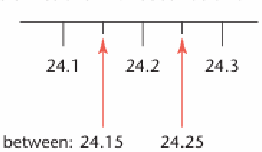
Question

Por won the 200 m race in a time of 24.2 seconds to the nearest tenth of a second. Complete the sentence below.

Por's time was between … seconds and … seconds.

Solution

As the measurement is stated to the nearest tenth of a second, the next possible times are 24.1 seconds and 24.3 seconds.



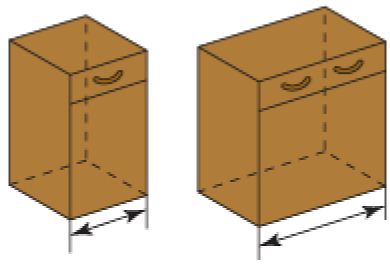
Por's time was between 24.15 seconds and 24.25 seconds.

Note

Many people are confused about the upper bound. The convention is that the lower bound is contained in the interval and the upper bound is in the next, higher interval.

Sums and differences of measurements

Two kitchen cupboards have widths of 300 mm and 500 mm correct to the nearest millimetre.



	300 mm	500 mm
Lower bound of width:	299.5 mm	499.5 mm
Upper bound of width:	300.5 mm	500.5 mm

If the cupboards are put next to each other then

- the lower bound of w, their joint width = 299.5 + 499.5 = 799 mm
- the upper bound of w, their joint width = 300.5 + 500.5 = 801 mm
- so 799 mm $\leqslant w \leqslant$ 801 mm.

To find the lower bound of a sum, add the lower bounds.

To find the upper bound of a sum, add the upper bounds.

However, the difference between the widths of the kitchen cupboards is largest when the smallest possible width of the smaller cupboard is subtracted from the largest possible width of the larger cupboard. The difference is least when the largest possible width of the smaller cupboard is subtracted from the smallest possible width of the larger cupboard. So

- The upper bound of the difference in their widths = 500.5 – 299.5 = 201 mm
- The lower bound of the difference in their widths = 499.5 – 300.5 = 199 mm

Example 10.2

Question

A piece of red string is 35.2 cm long to the nearest millimetre.

A piece of blue string is 12.6 cm long to the nearest millimetre.

a What is the minimum length of the two pieces laid end to end?

b What is the lower bound of the difference in the lengths?

Solution

For the red string:	LB = 35.15 cm	UB = 35.25 cm
For the blue string:	LB = 12.55 cm	UB = 12.65 cm

a Minimum total length = sum of lower bounds

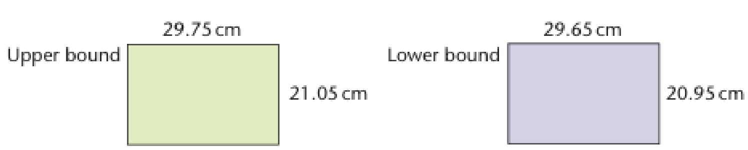
= 35.15 + 12.55 = 47.7 cm

b LB of difference in lengths = LB of longer piece – UB of shorter piece

= 35.15 – 12.65 = 22.5 cm

Multiplying and dividing measurements

The measurements of a piece of A4 paper are given as 21.0 cm and 29.7 cm to the nearest millimetre. What are the upper and lower bounds of the area of the piece of paper?



Upper bound of area
$= 29.75 \times 21.05$
$= 626.2375\ \text{cm}^2$

Lower bound of area
$= 29.65 \times 20.95$
$= 621.1675\ \text{cm}^2$

When multiplying

- to find the upper bound, multiply the upper bounds
- to find the lower bound, multiply the lower bounds.

When dividing, however, the situation is different.
Dividing by a larger number makes the answer smaller

When dividing

- to find the upper bound, divide the upper bound by the lower bound
- to find the lower bound, divide the lower bound by the upper bound.

Note

You will learn more about density and population density in Chapter 12.

Example 10.3

Question

Gan cycles 14.2 km (to 3 significant figures) in a time of 46 minutes (to the nearest minute).
What are the upper and lower bounds of his average speed in kilometres per hour?
Give your answer to 3 significant figures.

Solution

Gan cycles 14.2 km.
This distance lies between 14.15 km and 14.25 km.
Gan takes 46 minutes.
This time lies between 45.5 minutes and 46.5 minutes.

$$\text{Upper bound of speed} = \frac{\text{upper bound of distance}}{\text{lower bound of time}} = \frac{14.25}{45.5}\text{ km/minute} = \frac{14.25}{45.5 \div 60}\text{ km/h} = 18.8\text{ km/h to 3 significant figures}$$

$$\text{Lower bound of speed} = \frac{\text{lower bound of distance}}{\text{upper bound of time}} = \frac{14.15}{46.5 \div 60}\text{ km/h} = 18.3\text{ km/h to 3 significant figures}$$

Note

To find the upper bound of any combined measurement, work out which of the upper and lower bounds of the given measurements you need to use to give you the greatest result.
If you aren't sure, experiment!

Key points

- A value to a given unit of accuracy has an upper bound that is the value plus half the given unit and a lower bound that is the value minus half the given unit.
- When calculating with two values, A and B, each given to a unit of accuracy, the upper bound (U) and lower bound (L) of the answer can be found using the following.

$U(A + B) = U(A) + U(B)$ $L(A + B) = L(A) + L(B)$

$U(A - B) = U(A) - L(B)$ $L(A - B) = L(A) - U(B)$

$U(A \times B) = U(A) \times U(B)$ $L(A \times B) = L(A) \times L(B)$

$U\left(\frac{A}{B}\right) = \frac{U(A)}{L(B)}$ $L\left(\frac{A}{B}\right) = \frac{L(A)}{U(B)}$

Revision questions

Q1 Work out an estimate for $\dfrac{62 \times 9.67}{16.1}$

Q2 Lionel has enough ingredients to make 31 batches of marmalade.
Each batch will have a total mass of 10.9kg.
He is going to put the marmalade into jars which hold 0.454kg each.

(a) Work out an estimate for the number of jars he will need.

(b) Is your answer to part (a) an underestimate or an overestimate. Given a reason for your answer.

3) Round the following numbers to 2 decimal places.

i) 345.256

ii) 0.295 631

iii) 4.998

4)

Calculate an estimate for $\frac{17.3 \times 3.81}{11.5}$. State, with a reason, whether the estimate is an overestimate or an underestimate.

5)

The length of a road, l, is given as $l = 3.6$ km, correct to 1 decimal place.

Find the lower and upper bounds for l.

6)

(a) A room measures 4 m by 7 m, where each measurement is made to the nearest metre

Find the upper and lower bounds for the area of the room

(b) David is trying to work out how many slabs he needs to buy in order to lay a garden path.

Slabs are 50 cm long, measured to the nearest 10 cm.

The length of the path is 6 m, measured to the nearest 10 cm.

Find the maximum number of slabs David will need to buy.

7) a) A number is 2.94, correct to 2 decimal places.
Using inequalities, write down the error interval for the number.

(b) John does a calculation and truncates his answer by cutting off the decimal places.
He writes down his answer as 8.
Using inequalities, write down the error interval for his answer.

8)

(a) A number is 3.7, correct to 2 significant figures.
Write down the upper bound for the number.

(b) The distance between two houses is 35 miles, correct to the nearest 5 miles.
Write down the lower bound for the distance.

9) Round the following numbers to 3 significant figures.

i) 345 256

ii) 0.002 956 314

iii) 3.997