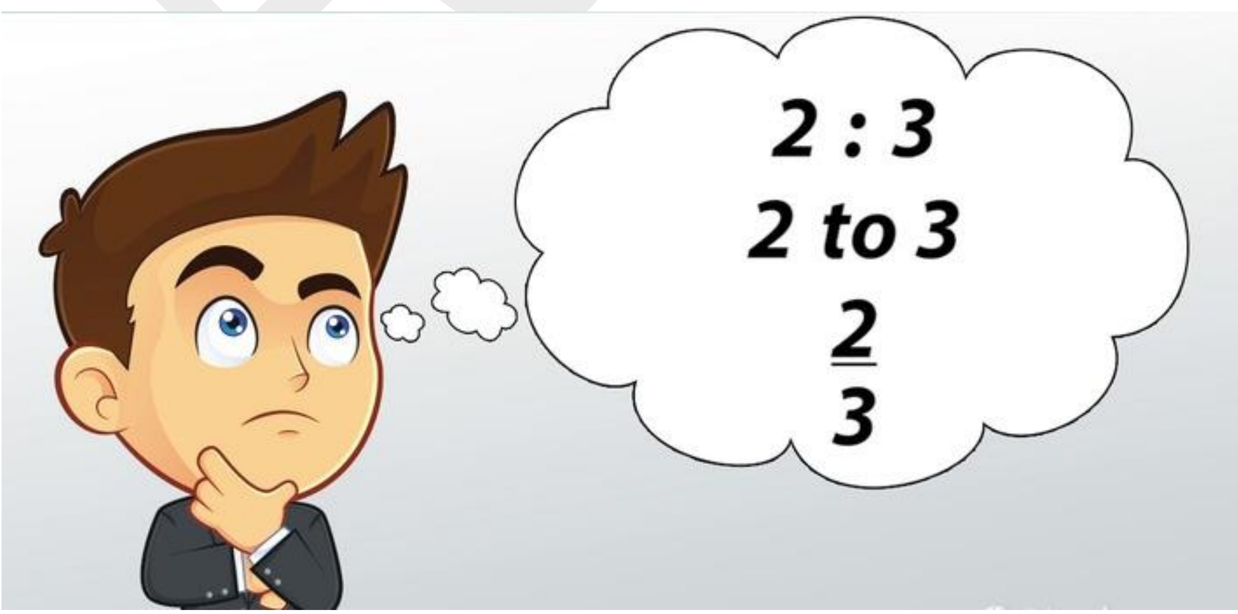
# Cambridge OL

# Mathematics

CODE: (4024)

Chapter 11 and 12

# Ratio and proportion and Rates



# Ratio

## Simplifying ratios

When something is divided into a number of parts, ratios are used to compare the sizes of those parts.

For example, if ten sweets are divided between two people so that one person gets six sweets and the other person gets four sweets, the sweets have been divided in the ratio 6: 4.

As with fractions, ratios can be simplified.

### Example 11.1

**Question**

Write each of these ratios in its simplest form.

**a** 1 millilitre : 1 litre

**b** 1 kilogram : 200 grams

**Note**

'In its simplest form' means simplify as much as possible.

When the units are the same, you do not need to include them in the ratio.

**Solution**

**a** 1 millilitre : 1 litre = 1 millilitre : 1000 millilitres  Same units.
= 1 : 1000

**b** 1 kilogram : 200 grams
= 1000 grams : 200 grams  Same units.
= 5 : 1  Divide each by 200.

## Writing a ratio in the form 1: n

It is sometimes useful to have a ratio with 1 on the left.

A common scale for a scale model is 1: 24.

The scale of a map or enlargement is often given as 1: n.

To change a ratio to this form, divide both numbers by the number on the left.

### Example 11.2

**Question**

Write these ratios in the form 1 : *n*.

**a** 2 : 5  **b** 8 mm : 3 cm  **c** 25 mm : 1.25 km

**Solution**

**a** 2 : 5 = 1 : 2.5  Divide each side by 2.

**b** 8 mm : 3 cm = 8 mm : 30 mm  Write each side in the same units.
= 1 : 3.75  Divide each side by 8.

**c** 25 mm : 1.25 km = 25 : 1 250 000  Write each side in the same units.
= 1 : 50 000  Divide each side by 25.

**Note**

1 : 50 000 is a common map scale. It means that 1 cm on the map represents 50 000 cm, or 500 m, on the ground.

## Using ratio to find an unknown quantity

Sometimes you know one of the quantities in the ratio, but not the other.

If the ratio is in the form 1: n, you can work out the second quantity by multiplying the first quantity by n. You can work out the first quantity by dividing the second quantity by n.

### Example 11.3

**Question**

A map is drawn to a scale of 1 cm : 2 km.

**a** On the map, the distance between Amhope and Didburn is 5.4 cm. What is the real distance in kilometres?

**b** The length of a straight railway track between two stations is 7.8 km. How long is this track, in centimetres, on the map?

**Solution**

**a** The real distance, in kilometres, is twice as large as the map distance, in centimetres.
So multiply by 2.
$2 \times 5.4 = 10.8$
Real distance = 10.8 km

**b** The map distance, in centimetres, is half of the real distance, in kilometres.
So divide by 2.
$7.8 \div 2 = 3.9$
Map distance = 3.9 cm

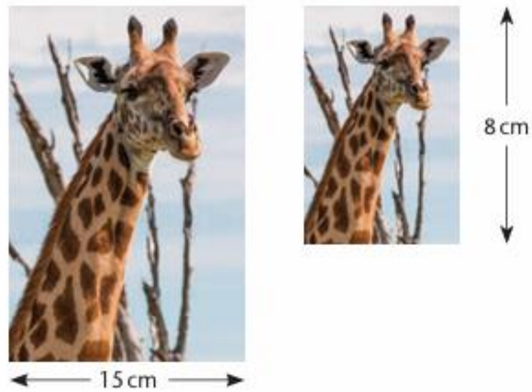
## Example 11.4

**Question**

Two photos are in the ratio 2 : 5.

**a** What is the height of the larger photo?

**b** What is the width of the smaller photo?




**Solution**

**a** The multiplier for the ratio of the heights is 4, since for the smaller photo $2 \times 4 = 8$. Use the same multiplier for the larger photo.

$$\begin{array}{c} 2:5 \\ \times 4 \downarrow \quad \downarrow \times 4 \\ 8:20 \end{array}$$

Height of the larger photo = 20 cm.

**b** The multiplier for the ratio of the widths is 3, since for the larger photo $5 \times 3 = 15$. Use the same multiplier for the smaller photo.

$$\begin{array}{c} 2:5 \\ \times 3 \downarrow \quad \downarrow \times 3 \\ 6:15 \end{array}$$

Width of the smaller photo = 6 cm.

### Dividing a quantity in a given ratio

Use this method to divide a quantity in a given ratio:

- find the total number of shares by adding the parts of the ratio
- divide the quantity by the total number of shares to find the multiplier
- multiply each part of the ratio by the multiplier.

## Example 11.5

**Question**

To make fruit punch, orange juice and grapefruit juice are mixed in the ratio 5 : 3.

Jo wants to make 1 litre of punch.

**a** How much orange juice does she need, in millilitres?

**b** How much grapefruit juice does she need, in millilitres?

**Solution**

$5 + 3 = 8$ First, work out the total number of shares.

$1000 \div 8 = 125$ Convert 1 litre to millilitres and divide by 8 to find the multiplier.

A table is often helpful for this sort of question.

| | Orange | Grapefruit |
|---|---|---|
| Ratio | 5 | 3 |
| Amount | $5 \times 125 = 625$ ml | $3 \times 125 = 375$ ml |

**a** She needs 625 ml of orange juice.

**b** She needs 375 ml of grapefruit juice.

**Note**

To check your answers, add the parts together.

Together they should equal the total quantity.

For example, 625 ml + 375 ml = 1000 ml ✔

# Proportion

## Direct proportion

In direct proportion, both quantities increase at the same rate.

For example, if you need 200 g of flour to make 10 cupcakes, you need 400 g of flour to make 20 cupcakes. The quantity of flour is multiplied by 2. The number of cupcakes is multiplied by 2.

Both quantities are multiplied by the same number, called the multiplier. To find the multiplier, you divide both pair of quantities.

So a quick way to solve this type of problem is to find the multiplier and use it to find the unknown quantity.

### Example 11.6

**Question**

A car uses 20 litres of fuel when making a journey of 160 kilometres.
How many litres of fuel would it use to make a similar journey of 360 kilometres?

**Solution**

You could solve this problem using ratios, but here is a method just using proportion.
Write the distance travelled in the two journeys as a fraction.
You want to find the fuel needed for a 360 kilometre journey, so make 360 the numerator and 160 the denominator.
Cancel the fraction so that it is in its simplest form.

$\frac{360}{160} = \frac{9}{4}$

$20 \times \frac{9}{4} = 45$ Multiply the amount of fuel needed for the first journey by the multiplier.

45 litres of fuel would be used to make a journey of 360 kilometres.

## Inverse proportion

In inverse proportion, as one quantity increases, the other decreases. In such cases, you need to divide by the multiplier, rather than multiply.

### Example 11.7

**Question**

Three excavators can dig a hole in 8 hours.
How long would it take four excavators to dig the hole?

**Solution**

Clearly, as more excavators are to be used, the digging will take less time.
The multiplier is $\frac{4}{3}$.

$8 \div \frac{4}{3} = 6$ Divide the known time by the multiplier to find the unknown time.

It will take four excavators 6 hours to dig the hole.

### Key points

- Simplify ratios by dividing by common factors whenever you can.
- To simplify ratios involving measures, both quantities must be in the same unit.
- If a quantity is in the form $1 : n$, you can work out the second quantity by multiplying the first quantity by $n$. You can work out the first quantity by dividing the second quantity by $n$.
- To find an unknown quantity, each part of the ratio must be multiplied by the same number, called the multiplier.
- To divide a quantity in a given ratio, first find the total number of shares, then divide the total quantity by the total number of shares to find the multiplier. You can then multiply each part of the ratio by the multiplier.
- To solve proportion problems, use a pair of known values to find the multiplier. Then, use the multiplier to find the unknown quantity.

# Chapter 12 - Rates

## Common measures of rate

In this section, you will look at rate of pay and rate of flow.

**Example 12.1**

**Question**

Nila has a part-time job. Her rate of pay is $18.70 per hour.

How much does she earn in a week where she works for 15 hours 30 minutes?

**Solution**

Number of hours worked = 15.5

Earnings = $\$18.70 \times 15.5$

$= \$289.85$

Rate of flow is a compound measure linking volume or mass with time.

$$\text{Average rate of flow} = \frac{\text{volume or mass}}{\text{total time taken}}$$

Rates of flow are measured in units such as kilograms per hour (kg/h) or litres per hour (l/h).

**Example 12.2**

**Question**

It takes half an hour to fill a 1000 litre tank.

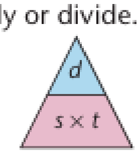
What is the average rate of flow?

**Solution**

Rate of flow $= \frac{1000\text{l}}{0.5\text{h}} = 2000$ l/h

**Note**

You may find the d.s.t. triangle helpful. Cover up the quantity you are trying to find. What is left shows you how to multiply or divide.




## Speed

Speed is a compound measure linking distance and time.

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

The units of your answer will depend on the units you begin with. Speed has units of 'distance per time', for example, km/h.

The formula for speed can be rearranged to find the distance travelled or the time taken for a journey.

$$\text{Distance} = \text{speed} \times \text{time} \qquad \text{Time} = \frac{\text{distance}}{\text{speed}}$$

## Applying other measures of rate

There are several other situations in which you need to use a rate. The same basic principles can be applied. The formulas for these will be given in the question. Here are some examples.

### Pressure

Pressure is a compound measure linking force and area.

$$\text{Pressure} = \frac{\text{force}}{\text{area}}$$

Usually, force is measured in newtons (N). When the area is given in square metres ($m^2$), the pressure is measured in $N/m^2$.

### Density

Another example of a compound measure is density, which links mass and volume.

$$\text{Density of a substance} = \frac{\text{mass}}{\text{volume}}$$

It is measured in units such as grams per cubic centimetre ($g/cm^3$)

**Example 12.4**

**Question**

A heavy box is placed on a table.
The base of the box is a rectangle measuring 50 cm by 40 cm.
The box exerts a force of 120 N on the table.
Calculate the pressure exerted on the table, in $N/m^2$.

**Solution**

Area $= 0.5 \times 0.4 = 0.2\,m^2$

Force $= 120\,N$

Pressure $= \dfrac{\text{force}}{\text{area}}$

$= \dfrac{120}{0.2}$

$= 600\,N/m^2$

**Note**

Remember to change the units from cm to m.

**Example 12.5**


**Question**

The density of gold is $19.3\,g/cm^3$.
Calculate the mass of a gold bar with a volume of $30\,cm^3$.

**Solution**

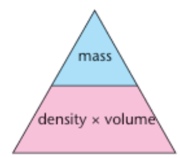
Density $= \dfrac{\text{mass}}{\text{volume}}$

so mass = density × volume.

The mass of the gold bar is density × volume $= 19.3 \times 30 = 579\,g$.

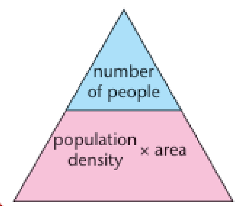
**Note**

As with speed, there is a triangle that can be helpful with these questions.




**Note**

Once again, a triangle can be helpful with these questions.




### Population density

Population density is another example of a compound measure. It gives an idea of how heavily populated an area is. It is measured as the number of people per square kilometre.

$$\text{Population density} = \frac{\text{number of people}}{\text{area}}$$

**Example 12.6**

**Question**

In a small town, 300 people live in an area of $2.4\,km^2$.
Find the population density of the town.

**Solution**

Population density $= \frac{300}{2.4} = 125$ people/$km^2$

**Key points**

- Average rate of flow $= \dfrac{\text{volume or mass}}{\text{total time taken}}$
  This is an example of one of the basic rates that you should know.
- Average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$
  You are expected to know this relationship and to use it to solve problems.
- Other examples of compound measures include:
  - pressure (measured in units such as $N/m^2$)
  - density (measured in units such as $g/cm^3$)
  - population density (measured in units such as people/$km^2$).

  You do not need to memorise the formula for these compound measures but you should be able to use them correctly.

# Revision questions

1) $y \propto \dfrac{1}{\sqrt{x}}$

When $y = 8, x = 4$.

Find $y$ when $x = 49$.

2) $y$ is directly proportional to the square root of $x$.
When $x = 9$, $y = 6$.

Find $y$ when $x = 25$.

3) $y$ is inversely proportional to $x$.
When $x = 9$, $y = 8$.

Find $y$ when $x = 6$.

4) $R$ is directly proportional to the **cube** of $p$.
When $p = 2$, $R = 24$.

Find the formula for $R$ in terms of $p$.

5) $y$ is directly proportional to $(x - 4)$.
When $x = 16$, $y = 3$.

Find $y$ in terms of $x$.

6) $y$ is inversely proportional to $x^3$.
When $x = 2$, $y = 0.5$.

Find $y$ in terms of $x$.

7) $y$ is directly proportional to $(x-1)^2$.
When $x=3, y=24$.

Find $y$ when $x=6$.

8) y is proportional to $(x-1)^2$.

Given that y = 18 when $x = 4$, find $y$ when $x = 6$.

9) $m$ is inversely proportional to the square of $(p-1)$.
When $p=4,\ m=5$.

Find $m$ when $p=6$.

10) $y$ is inversely proportional to the square root of $(x+1)$.
When $x=8, y=2$.
Find $y$ when $x=99$.

11) $y$ is inversely proportional to $(x+3)^2$
When $x=2, y=8$.

Find $y$ when $x=7$.

12) $y$ is inversely proportional to the square of $(x+2)$.
When $x=3, y=2$.

Find $y$ when $x=8$.