

Cambridge OL

Mathematics

CODE: (4024) Chapter 13 and chapter 14 percentages and using calculator





Fractions, decimals and percentages

Since 'per cent' means 'out of 100', the fraction equivalent to a percentage is found by writing the percentage as a fraction with a denominator of 100.

So
$$13\% = \frac{13}{100}$$
, $5\% = \frac{5}{100}$, $140\% = \frac{140}{100}$, $2.3\% = \frac{2.3}{100}$, etc.

Sometimes you need to write equivalent fractions to give a fraction in its simplest form.

So
$$13\% = \frac{13}{100}$$
, but $5\% = \frac{1}{20}$, $140\% = 1\frac{2}{5}$, $2.3\% = \frac{23}{1000}$, etc.

The decimal equivalents can be found by carrying out the division shown in the fraction.

So
$$13\% = \frac{13}{100} = 0.13, 5\% = \frac{5}{100} = 0.05, 140\% = \frac{140}{100} = 1.4$$

 $2.3\% = \frac{2.3}{100} = 0.023$

Percentage of a quantity

Using fractions

This is often the best method to use when calculating percentages without a calculator. When you are calculating simple percentages such as 10%, 20% or 5%, first find 10% and then multiply or divide the result to find the percentage you want.

Example 13.1		
Question		
a Find 15% of \$240.	b Find 31% of \$6.80.	
Solution		
a 10% of \$240 = 240 ÷ 10	b 10% of 680c = 680 + 10	
= \$24	= 68c	
5% of \$240 = half of \$24	30% of $680 = 68 \times 3$	
= \$12	= 204	
So 15% of \$240 = \$24 + \$12	So 30% of \$6.80 = 204c	
= \$36	1% of 680c = 680 + 100 = 6.8c	
	So 31% of \$6.80 = 204c + 6.8c	
	= 210.8c	
	= \$2.11 (to the nearest cent)	

Sometimes it is easier to use the facts that $25\% = \frac{1}{4}$ and $50\% = \frac{1}{2}$ to work out the percentage of a quantity.

	Example 13.2			
	Question a Find 25% of \$72.	b	Find 75% of \$90.	
	Solution			
	a 25% of \$72 = 72 ÷ 4	b	50% of \$90 = 90 ÷ 2	
	= \$18		= \$45	
			25% of \$90 = \$45 ÷ 2	
			= \$22.50	
			So 75% of \$90 = \$45 + \$22.50	
			= \$67.50	
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Using decimals

To calculate a percentage of an amount, multiply the amount by the decimal equivalent of the percentage.



Example 13.3		
Question a Find 37% of \$48.	ь	Find 2.4% of \$36000.
Solution a 37% of \$48 = \$48 × 0.37 = \$17.76	ь	2.4% of \$36000 = \$36000 × 0.024 = \$864

Expressing one quantity as a percentage of another

To express one quantity as a percentage of another, first write a fraction with the first quantity as the numerator of a fraction and the second quantity as the denominator. Then multiply the fraction by 100.



Percentage change

Percentage increases and decreases are worked out as percentages of the original amount, not the new amount. Percentage profit or loss is worked out as a percentage of the cost price, not the selling price.

Percentage change = $\frac{\text{change}}{\text{original amount}} \times 100$

Example 13.5

Question

An art dealer buys a painting for \$45 and sells it for \$72. What percentage profit is this?

Solution

Profit = \$72 - \$45 = \$27

Percentage profit = $\frac{27}{45} \times 100 = 60\%$

Example 13.6

Question

The value of a computer drops from \$1200 to \$700 in a year. What percentage decrease is this?

Solution

Decrease in value = 1200 - 700 = 500Percentage decrease = $\frac{500}{1200} \times 100 = 41.7\%$ to 1 decimal place

Percentage increase and decrease

To find the amount after a percentage increase, work out the increase and add it on to the original amount.



To find the amount after a percentage decrease, work out the decrease and subtract it from the original amount.

Example 13.7			
Question An engineer receives an i Her salary was \$ 24000 b What is her new salary?	crease of 3% in her annual fore the increase.	salary.	
Solution 3% = 0.03 Increase = \$24000 × 0.03 New salary = \$24000 + 5	= \$720 Find the i 720 = \$24720 Add the i	increase. increase to the original amount.	
Example 13.8			
Question In a sale all prices are red Find the sale price of an a	iced by 15%. rticle previously priced at \$	17.60.	
Solution 15% = 0.15			
17.60 × 0.15 = 2.64 \$17.60 - 2.64 = \$14.96	Find the decrease. Subtract the decrease fron	n the original amount.	

Percentage increase and decrease using a multiplier

You can find a quantity after a percentage increase or decrease in one step using a multiplier.

Example 13.9

Question

Amir's salary is \$17000 per year. He receives a 3% increase. Find his new salary.

Solution

Amir's new salary is 103% of his original salary. So the multiplier is 1.03. New salary = $17000 \times 1.03 = 17510$.

Example 13.10

Question

In a sale all prices are reduced by 15%. Rushna buys a tablet in the sale. The original price was \$155. Calculate the sale price.

Solution

A 15% decrease is the same as 100% - 15% = 85%. So the multiplier is 0.85. Sale price = $$155 \times 0.85 = 131.75



Finding the original quantity

To increase a quantity by 20%, you calculate new amount = original amount × 1.20.

It follows that original amount = new amount ÷ 1.20.



A **salary** is a fixed amount of money you earn each month or year from your job. It is paid monthly. What you earn is called your **gross pay.**

Tax and other amounts may be deducted from your **gross** pay.

He also earns 5% commission on the sales he makes each In May his sales are \$7500. What is his gross pay for May? Solution

a Jenny's monthly gross pay = \$8500 + 12 = \$708.333... = \$708.33 b Basic monthly pay = \$9600 + 12 = \$800 Commission = \$7500 × 0.05 = \$375 Total pay for May = \$800 + \$3375 = \$1175

The amount actually received by the employee is often called the take-home pay.

Some employees, particularly sales staff, are paid a basic wage or salary plus a **commission**, which is usually a percentage of their sales.

Discount, profit and loss

A discount is a reduction in the price of something. This is often a percentage of the original price but can be a fixed amount.



Solution

Rashid will pay 100 - 15 = 85% of the original price

Use your preferred method to find a percentage of an amount.

Rashid pays \$38 × 0.85 = \$32.30

Percentage profit and loss are always worked out as a percentage of the price paid by the seller.

Example 13.15

Question

A shopkeeper buys potatoes at \$1.40 per kilogram. He sells them for \$1.80 per kilogram. Find his percentage profit.

Solution

Solution

1 decimal place

Profit = \$1.80 - \$1.40 = \$0.40 Percentage profit = $\frac{0.40}{1.40} \times 100 = 28.6\%$ to 1 decimal place

Example 13.16

Question

Françoise buys a car for \$9500. A year later she sells it for \$8300. Calculate her percentage loss.

Repeated percentage change

Earlier in this chapter, you were shown a single-step method for increasing a quantity by a percentage.

For example, to increase a quantity by 18%, you multiply by 1.18. Similarly, to increase a quantity by 7%, you multiply by 1.07.

This method is particularly useful when calculating repeated increases.

Compound and simple interest

When you invest money in a bank, you may receive interest. There are two types of interest. Simple interest is calculated as a percentage of the original amount.

If you invest \$4000 for 2 years in an account that pays 5% simple interest annually, you will receive \$4000 × 0.05 = \$200 at the end of the first year and the same again each year.

By contrast, compound interest is added to the original amount so that the amount of interest paid each year increases.

Compound interest is just one example of repeated percentage change, and you can calculate the new amount in the same way as you calculate other repeated percentage changes.

However, because it has such wide application in everyday life, it has its own formula for working out the value of the investment after a given time.

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Example 13.17

Loss = \$9500 - \$8300 = \$1200

Percentage loss = $\frac{1200}{9500} \times 100 = 12.6\%$ to

Question Due to inflation, prices increase by 5% per year. An item costs \$12 now What will it cost in 2 years' time? Solution A 5% increase means that the new amount is 100 + 5 = 105% of the old amount each year To find 105%, you multiply by 1.05. In 1 year the price will be $12 \times 1.05 = 12.60$. In 2 years the price will be $12.60 \times 1.05 = 13.23$. Alternatively, this calculation can be worked out as \$12 × 1.05 × 1.05 = \$13.23 \$12 × 1.05² = \$13.23

Example 13.18

Question Selena invests \$4000 and receives 5% interest a year which is added to the amount each year. How much is her investment worth in total after a 1 year b 6 years? Note Solution On a calculator, the calculation a After one year the investment is worth $4000 \times 1.05 = £4200$. for part **b** can be done using the b After 6 years the investment is worth power key. $4000\times1.05\times1.05\times1.05\times1.05\times1.05\times1.05\times1.05$ This is usually labelled x[®] but $=4000 \times (1.05)^{6}$ may be labelled y^x or x^y o





In this formula

- A is the value of the investment after the given time
- P is the amount invested and is called the principal
- r is the percentage rate of interest (usually per year but it can be per month)
- n is the time (usually in years) for which the money is invested. You can use

either the method used in Example 13.18 or the formula above.

Example 13.19

Question

Salim has \$4000 to invest for 5 years. The bank offers him two options.

- 3.5% simple interest or
- 3.25% compound interest
- Which is the better option?
- How much more will his investment be worth with the better option?

Solution

3.5% simple interest Interest for one year = $0.035 \times 4000 = 140 Interest for 5 years = $5 \times $140 = 700 Value of investment at the end of 5 years = \$4000 + \$700 = \$47003.25% compound interest The multiplier for a 3.25% increase is 1.0325. Value of investment at the end of 5 years = $$4000 \times 1.0325^5 = 4693.65 (to the nearest cent). Alternatively, using the formula

$$A = P \left(1 + \frac{r}{100} \right)^n$$

$$=4000 \times \left(1 + \frac{3.25}{100}\right)^5$$

= \$4693.65 (to the nearest cent)

So 3.5% simple interest is the better option.

Difference = \$4700 - \$4693.65 = \$6.35 (to the nearest cent)

Key points

- A percentage is equivalent to a fraction with the percentage as the numerator and 100 as the denominator. To find the equivalent decimal, divide out the fraction.
- To find a percentage of a quantity, multiply the quantity by the fraction or decimal equivalent to the percentage.
- To find one quantity as a percentage of another, write a fraction with the first quantity as numerator and the second as denominator. Then, multiply by 100.
- Percentage change = $\frac{\text{change}}{\text{original amount}} \times 100.$
- To find a quantity after a percentage increase, add the percentage to 100% and use the equivalent decimal as a multiplier.
- To find a quantity after a percentage decrease, subtract the percentage from 100% and use the equivalent decimal as a multiplier.
 Simple interest paid on a sum of money is calculated on the original
- sum.Compound interest is added to the original investment.

$$A = P \left(1 + \frac{r}{100}\right)^n$$

- A is the value of the investment at any time
 P is the amount invested
 r is the percentage rate of interest
 n is the number of years the money is invested.
 To find the original quantity, given the result of an increase or
- To find the original quantity, given the result of an increase or decrease, divide by the multiplier.

$$A = P \left(1 + \frac{r}{100} \right)^n$$

►



Chapter 14 – using calculator

Order of operations

You should know that this is the correct order of operations when carrying out any calculation:

- First work out anything in brackets.
- Then work out any powers (such as squares or square roots).
- Then do any multiplication or division.
- Finally do any addition or subtraction. A scientific calculator should follow this order of operations. Using the correct order of operations, $2 + 3 \times 4 = 14$.
 - Enter the calculation $2 + 3 \times 4$ on your calculator and check that

you get the answer 14.

When a calculation is written as a fraction, you will need to use brackets to ensure that the calculator divides the result of the calculation in the numerator by the result of the calculation in the denominator.

Using the correct order of operations, $\frac{9+3}{4+2} = 2$.

On a calculator you will need to enter $(9 + 3) \div (4 + 2)$

for this calculation. Check that you get the answer 2 when you do this.

A scientific calculator has function keys that can be used in calculations.

You need to find out what these keys look like on your calculator.

Your calculator will have keys for squares, cubes, square roots and cube roots.

You should make sure that you can use these keys in calculations.





Standard form on your calculator

You can do calculations involving standard form on your calculator, using Check which of these your calculator has.



In following work $\mathbf{x10}^{x}$ is used, but if your calculator has \mathbf{EXP} use that instead.



Checking accuracy

When you have solved a problem, it is useful to check whether the answer is sensible. When the problem is set in a real-life context you may be able to use your experience to help you check.

For example, if the question is about a shopping bill, think about whether the total you have found is realistic. If the question is not set in context, you need to use other methods to check your answer.



You have checked answers by rounding the numbers in the calculation to one significant figure and finding an estimate for the result. There are also other methods that you can use.

Using number facts

Look at these calculations.

$40 \times 5 = 200$	$40 \times 0.5 = 20$
$40 \times 2 = 80$	$40 \times 0.8 = 32$
$40 \times 1.1 = 44$	$40 \times 0.9 = 36$

You can see that when 40 is multiplied by a number greater than 1, the result is greater than 40.

When 40 is multiplied by a number less than 1, the result is smaller than 40. Look at these calculations.

$40 \div 5 = 8$	$40 \div 0.5 = 80$

- $40 \div 2 = 20$ $40 \div 0.8 = 50$
- $40 \div 1.1 = 36.36...$ $40 \div 0.9 = 44.44...$

You can see that when 40 is divided by a number greater than 1, the result is smaller than 40.

When 40 is divided by a number less than 1, the result is greater than 40.

These results can be applied to any multiplication or division. Starting with any positive number,

•multiplying by a number greater than 1 gives a result that is larger than the starting number

•multiplying by a number between 0 and 1 gives a result that is smaller than the starting number

• dividing by a number greater than 1 gives a result that is smaller than the starting number

• dividing by a number between 0 and 1 gives a result that is larger than the starting number.

Using inverse operations

Inverse operations can be used to check the result of a calculation.

You know that subtraction is the inverse of addition and that division is the inverse of multiplication. You can use this to check the result of a calculation.

Work out 6.9 \div 75 on your calculator. The answer is 0.092. You can check this answer using the multiplication 0.092 \times 75.

The answer to the multiplication is 6.9, so the answer to the division was correct.

Finding the square root is the inverse of finding the square of a number. You can use these facts to check the result of a calculation.



Example 14.4 Question Explain how you can tell the answer to each of these calculations is wrong. a 297 + 2.8 = 10.6 to 3 significant figures b 752 + 24 = 18 048 c √35 = 9.52 to 2 decimal places Solution a Rounding to 1 significant figure gives 300 + 3 = 100, so the answer 10.6 is too small. b 752 is divided by a number greater than 1, so the result should be less than 752 but 18 048 is greater than 752. c 6² = 36, so √35 must be less than 6, but 9.52 is greater than 6. Alternatively, 9² = 81, so √35 must be much less than 9.

Interpreting the calculator display

When a calculation involves units, you need to make sure that you use the same units for all of the quantities and that you give the correct units for your answer.

It is also important that you think about the accuracy needed when you give your answer. If the answer is not exact, think about what degree of accuracy is sensible.

Interpreting the calculator display When a calculation involves units, you need to make sure that you use the same units for all of the quantities and that you give the correct units for your answer.

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Example 14.5				
Q a	Question a \$18 is shared equally between 10 people. How much does each person receive?			
ь So	\$18 is shared equally between	13 people. How much does each person receive?		
а	18 + 10 = 1.8	The calculator answer is 1.8.		
b	$18 \div 13 = 1.384615$	The answer is not exact.		
	Each person receives \$1.38.	Round the answer to 2 decimal places.		

Calculating with time

When a calculation involves time, you need to make sure that you use your calculator correctly.

Entering time on a calculator

There are 60 minutes in an hour, so a time of 2 hours 30 minutes is the same as $2\frac{1}{2}$ hours.

You enter 2.5 on your calculator.

If you need to enter a time such as 1 hour 12 minutes, first convert the minutes to a fraction of an hour.

12 minutes is $\frac{12}{60}$ of an hour or, as a decimal, 0.2 hour.

So 1 hour 12 minutes is 1.2 hours.

You can use the same method to enter times given in minutes and seconds on your calculator.

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Interpreting the result of a time calculation

Time calculations often require answers to be given in hours and minutes or minutes and seconds. If you have used a calculator, then the answer will be a decimal which you need to convert to the correct units. Some conversions can be done mentally.

Hours, minutes and seconds on a calculator

Some calculators have a or "key.

You can use this for working with time on your calculator.

When you do a calculation and the answer is a time in hours, but your calculator shows a decimal, you change it to hours and minutes by pressing the o''' key and then the key.

To convert back to a decimal, press the o' " key again.

To enter a time of 8 hours 32 minutes on your calculator, press this sequence of keys:



The display should look like this:

8°35°0

You may wish to experiment with this key and learn how to use it to enter and convert times.

Fractions on a calculator

You need to be able to calculate with fractions without a calculator. However, when a calculator is allowed you can

use the fraction key.

The display will look like this. _ _ _ 를

This is the sequence of keys to press to do the calculation $\frac{2}{5} + \frac{1}{2}$:

2 = 5 > + 1 = 2 =

The display should look like this

This is the sequence of keys to press to enter a mixed number, such as $2\frac{3}{2}$, into your calculator:

-9-10-

2 SHIFT = 3 V 5 =

Your display will look like this

You can also cancel a fraction on your calculator.

When you press **B** = **1 2** =, you should see:

When you press = again, the display changes to $\frac{2}{3}$

When you do calculations with fractions on your calculator, it will automatically give the answer as a fraction in its simplest form.

Similarly, if you enter an improper fraction into your calculator and press the = key, the calculator will automatically change it to a mixed number.

You may wish to experiment with calculations using the fraction key on your calculator. The fraction key will look like this $\boxed{\blacksquare}$ or like this $\boxed{\blacksquare}^{b}/_{c}$

The following instructions are for a calculator with a fraction key that looks like this .

To enter a fraction such as $\frac{2}{5}$ into your calculator, you need to press this sequence of keys:



Example 14.6

Question

- a Convert 8 minutes 36 seconds into minutes
- **b** Write 6.4 hours in hours and minutes.

Solution

- 8 minutes 36 seconds is 8³⁶/₆₀ minutes.
- $36 \div 60 = 0.6$ Convert the fraction to a decimal using division. 8 minutes 36 seconds is 8.6 minutes.
- b $0.4 \times 60 = 24$ Multiply the decimal by 60 to convert to minutes. 0.4 hour is 24 minutes
- 6.4 hours is 6 hours 24 minutes.

Key points

- Use some method of estimating the answer as a check.
- Not all calculators are the same. Make sure you know how to use yours.
- Take care to correctly enter and interpret values representing time or money.

Revision questions

1. In 2018, Gretal earned \$32 000. She paid tax of 24% on these earnings. Work out the amount she paid in tax in 2018.

2. Kristian receives \$72.Kristian spends 45% of his \$72 on a computer game.Calculate the price of the computer game.

3. Ria is going to buy a caravan.
The total cost of the caravan is £7000 plus VAT at 20%.
Ria pays a deposit of £3000
She pays the rest of the total cost in 6 equal monthly payments.
Work out the amount of each monthly payment.

4. In 2018, Gretal earned \$32 000.In 2019, Gretal's earnings increased by 7%.Work out her earnings in 2019.

5. A bag contains 54 red marbles and some blue marbles. 36% of the marbles in the bag are red. Find the number of blue marbles in the bag.

6. Roberto buys a toy for \$5.00. He then sells it for \$4.60. Calculate his percentage loss.

7.

Use your calculator to work out $\sqrt{1-(\sin 33^{\circ})^2}$.

8. Work out

$$\frac{2.5 \times \sin 43^{\circ}}{8.2^2 - 50.5}$$

Give your answer correct to 3 significant figures.

9. Use your calculator to work out $19.42^2 - \sqrt[3]{1006} \div 4.95$

Write down your full calculator display.

10. Use your calculator to work out $\frac{29^2 - 4.6}{\sqrt{35 - 1.9^3}}$ Write down all the figures on your calculator display.