

Cambridge OL

Mathematics

CODE: (4024)

Chapter 26

Proportion



Proportion

In Chapter 11 you solved problems involving proportion. When two quantities are in direct proportion, as one quantity increases, so does the other.

When two quantities are in **indirect proportion**, as one quantity increases, the other decreases.

Proportion as a formula

Example 26.1

Question

Find the formula connecting x and y .

x	5	15
y	3	9

Solution

You can see that $y \propto x$.

So the formula will be $y = kx$.

When $x = 5$, $y = 3$.

Substituting these values in the equation gives $3 = k \times 5$, so $k = \frac{3}{5}$.

This gives the formula $y = \frac{3}{5}x$.

Note

You can easily tell whether the proportion is direct or inverse – in direct proportion, both variables change in the same way, either up or down; in inverse proportion, when one variable goes up, the other will go down.

Example 26.2

Question

Find the formula connecting x and y .

x	4	20
y	10	2

Solution

$$y \propto \frac{1}{x}$$

$$\text{So } y = k \times \frac{1}{x} \text{ or } y = \frac{k}{x}.$$

When $x = 4$, $y = 10$.

Substituting these values in the formula gives $10 = \frac{k}{4}$ so $k = 10 \times 4 = 40$.

This gives the formula $y = \frac{40}{x}$.

Check the formula works for the other pair of values in the table: $y = \frac{40}{x} = \frac{40}{20} = 2$, which is correct.

Other types of proportion

This is the method you use for finding the formula for the proportion. It is the same method you used previously.

- First write the proportionality using the \propto sign. For example, $y \propto x^2$.
- Replace the \propto sign with $=$ and multiply the right-hand side by k .
For example, $y = kx^2$.
- Substitute a known pair of values for x and y .
- Solve the equation to find k .
- Rewrite the formula using the found value of k .

Example 26.3

Question

y is proportional to x^2 and when $x = 5$, $y = 10$.

- a Find a formula connecting x and y . b Find the value of y when $x = 15$.

Solution

a $y \propto x^2$

So $y = kx^2$

$10 = k \times 5^2$

Substitute $x = 5$, $y = 10$.

$k = \frac{10}{25} = \frac{2}{5}$

Solve for k .

$y = \frac{2}{5}x^2$

Substitute for k in the formula.

b $y = \frac{2}{5} \times 15^2 = 90$

Substitute $x = 15$ into the formula.

Note

There are different ways of talking about proportion. 'y varies directly as x^2 ' means the same as 'y is proportional to x^2 '.

Example 26.4

Question

y is inversely proportional to x^2 and when $x = 5$, $y = 10$.

- a Find a formula connecting x and y .
b Find the value of y when $x = 10$.
c Find the value of x when $y = 1000$.

Solution

a $y \propto \frac{1}{x^2}$

So $y = k \times \frac{1}{x^2} = \frac{k}{x^2}$

$10 = \frac{k}{5^2}$

Substitute $x = 5$, $y = 10$.

$k = 10 \times 25 = 250$

Solve for k .

$y = \frac{250}{x^2}$

Substitute for k in the formula.

b $y = \frac{250}{10^2} = 2.5$

Substitute $x = 10$ into the formula.

c $1000 = \frac{250}{x^2}$

Substitute $y = 1000$ into the formula.

$1000x^2 = 250$

$x^2 = \frac{250}{1000}$

$x^2 = \frac{1}{4}$

$x = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}$

Note

For inverse proportion you can go straight to the fraction version with k as the numerator.

Example 26.5

Question

$y \propto \frac{1}{x^2}$ and $y = 7$ when $x = 7$.

Find y when $x = 14$.

Solution

Although you can use the formula method, this is quicker.

x has been **multiplied** by 2.

Since y is **inversely** proportional to x^2 , y will be **divided** by 2^2 .

$y = \frac{7}{2^2} = 1.75$

Example 26.6

Question

y varies inversely as the square root of x .

When $x = 4$, $y = 12$.

Find the value of y when $x = 9$.

Solution

First, find the formula connecting x and y .

$y \propto \frac{1}{\sqrt{x}}$

So $y = k \times \frac{1}{\sqrt{x}} = \frac{k}{\sqrt{x}}$

$12 = \frac{k}{\sqrt{4}}$

Substitute $x = 4$, $y = 12$.

$k = 12 \times 2 = 24$

Solve for k .

$y = \frac{24}{\sqrt{x}}$

Substitute for k in the formula.

$y = \frac{24}{\sqrt{9}} = 8$

Substitute $x = 9$ into the formula.

Example 26.7

Question

y is inversely proportional to x^2 .

When x takes a certain value, y is 18.

Find the value of y when x is multiplied by 3.

Solution

x is **multiplied** by 3.

Since y is **inversely** proportional to x^2 , y will be **divided** by 3^2 .

$y = \frac{18}{3^2} = 2$

Key points

- The symbol \propto means 'is proportional to'.
- For direct proportion, $y \propto x$, $y \propto x^2$, $y \propto x^3$, $y \propto \sqrt{x}$, $y \propto \sqrt[3]{x}$.
- For indirect proportion, $y \propto \frac{1}{x^2}$, $y \propto \frac{1}{x}$, and so on.
- Proportion can be expressed as a formula. For instance, $y \propto x^2$, has the formula $y = kx^2$. The value of k can be found by substituting a pair of corresponding values for x and y .

Revision questions

1.

$$y \propto \frac{1}{\sqrt{x}}$$

When $y = 8, x = 4$.

Find y when $x = 49$.

2.

y is directly proportional to the square root of x .

When $x = 9, y = 6$.

Find y when $x = 25$.

3.

y is inversely proportional to x .

When $x = 9, y = 8$.

Find y when $x = 6$.

4.

R is directly proportional to the **cube** of p .

When $p = 2, R = 24$.

Find the formula for R in terms of p .

5.

y is directly proportional to $(x - 4)$.

When $x = 16, y = 3$.

Find y in terms of x .

6.

 y is inversely proportional to x^3 .When $x = 2$, $y = 0.5$.Find y in terms of x .

7.

 y is directly proportional to $(x - 1)^2$.When $x = 3$, $y = 24$.Find y when $x = 6$.

8.

 y is proportional to $(x - 1)^2$.Given that $y = 18$ when $x = 4$, find y when $x = 6$.

9.

 m is inversely proportional to the square of $(p - 1)$.When $p = 4$, $m = 5$.Find m when $p = 6$.

10.

 y is inversely proportional to the square root of $(x + 1)$.When $x = 8$, $y = 2$.Find y when $x = 99$.

11.

 y is inversely proportional to $(x + 3)^2$ When $x = 2$, $y = 8$.Find y when $x = 7$.