Cambridge OL

Mathematics

CODE: (4024)

Chapter 34 and Chapter 35

Scale drawings and Similarity

Chapter 34 - SCALE DRAWINGS

Scale drawings and maps

A scale drawing is exactly the same shape as the original drawing but is different in size.

Large objects are scaled down in size so that.

Example 34.1

Question

Here is a scale drawing of a truck.

The scale of the drawing is 1 cm to 2 m.

a How long is the truck?

b Will the truck go safely under a bridge 4 m high?

c The truck driver is 1.8 m tall.
How high will he be on the scale drawing?

Solution

a Measure the length of the truck on the scale drawing.
Length of truck on the drawing = 4 cm.
As 1 cm represents 2 m, multiply the length on the drawing by 2 and change the units.
Length of truck in real life = $4 \times 2 = 8$ m.

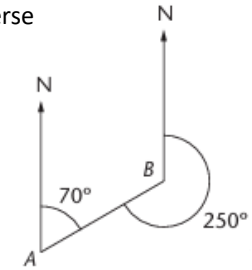
b Height of truck on the drawing = 2.5 cm.
Height of truck in real life = $2.5 \times 2 = 5$ m.
So the truck will not go under the bridge.

c To change from measurements in real life to measurements on the drawing, you have to divide by 2 and change the units.
Height of driver in real life = 1.8 m.
Height of driver on the drawing = $1.8 \div 2 = 0.9$ cm.

Bearings

Bearings are used to describe direction.
They are measured clockwise from north. Bearings are always given from some fixed point.

In the diagram, the bearing of B from A is 070°. The bearing of A from B is called the reverse bearing. In this case,



$$\begin{aligned}\text{the bearing of } A \text{ from } B &= \text{the bearing of } B \text{ from } A + 180° \\ &= 070° + 180° \\ &= 250°\end{aligned}$$

Example 34.2

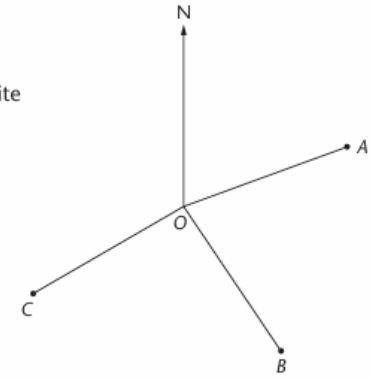
Question

Use a protractor to measure and write down each of these bearings.

a *A* from *O*

b *B* from *O*

c *C* from *O*



Solution

a 070°

b 146°

c 240°

If you have a circular protractor you can measure all of these directly.

If you only have a semi-circular protractor then for part **c** you need to measure the obtuse angle and subtract it from 360°.

Note

Make sure you use the clockwise scale.

Note

If the bearing is greater than 180°, the reverse bearing is found by subtracting 180°.

Note

Bearings must have three figures, so if the angle is less than 100° you must put a zero in front of the figures.

Example 34.3

Question

A, *B* and *C* are three towns.

B is 20 km from *A*, on a bearing of 085° from *A*.

C is 15 km from *B*, on a bearing of 150° from *B*.

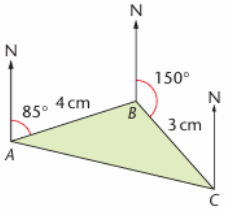
a Make a scale drawing showing the three towns.

Use a scale of 1 cm to 5 km.

b **i** How far is *A* from *C*?

ii What is the bearing of *A* from *C*?

c Town *D* is due west of *C*. What is the bearing of *D* from *C*?



Solution

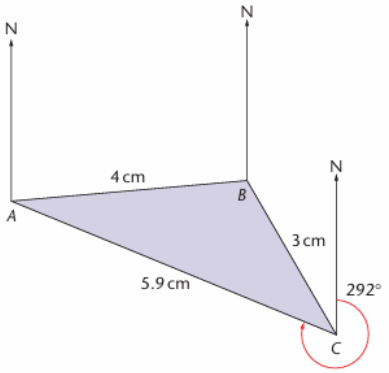
a Make a sketch and label the angles and lengths for the final diagram.

Then draw the diagram, starting far enough down the page to make sure it will all fit in.

b **i** *CA* measures 5.9 cm on the scale drawing, so the real distance is $5.9 \times 5 = 29.5$ km.

ii The bearing is 292°.

c 270°



Note

The directions north, east, south and west are at right angles to each other.

Key points

- The scale of a drawing or map tells you what every 1 unit represents; for example, every 1 cm represents 4 km.
- A scale drawing is exactly the same shape as the original object but is different in size.
- Bearings describe direction. A bearing is a three-figure angle measured clockwise from north.
- You can measure the bearing of one point from another using a protractor.
- You can make a scale drawing containing bearings and interpret it in context.

Chapter - 35 SIMILARITY

Similar shapes

In mathematics the word ‘similar’ has a very exact meaning. It does not mean ‘roughly the same’ or ‘alike’.

For two shapes to be similar, each shape must be an exact enlargement of the other.

Although the scale factor for both pairs of sides is 2, the shapes are not similar because corresponding angles are not the same.

For two shapes to be similar

- all corresponding sides must have proportional lengths
- all corresponding angles must be equal.

Because the lengths of three sides define a unique triangle, for two triangles to be similar, only one of the tests here needs to be made.

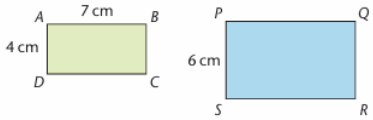
If you can establish that the angles are the same, you can conclude that the triangles are similar and carry out calculations to find the lengths of the sides

Calculating lengths in similar shapes

Example 35.1

Question

The rectangles *ABCD* and *PQRS* are similar.
Find the length of *PQ*.



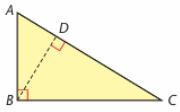
Solution

Since the widths of the rectangles are 6 cm and 4 cm, the scale factor is $6 \div 4 = 1.5$.
Length of $PQ = 7 \times 1.5$
$= 10.5$ cm

Example 35.2

Question

In the triangle, angle *ABC* = angle *BDC* = 90°, *AB* = 6 cm, *BC* = 8 cm and *BD* = 4.8 cm.



a Explain why triangles *ABC* and *BDC* are similar.
b Calculate the length of *DC*.

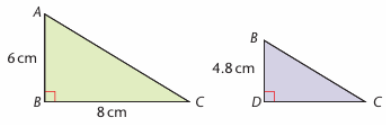
Note

When explaining why two triangles are similar, look for reasons why the angles are equal. Usual reasons include: ‘opposite angles are equal’, ‘alternate angles are equal’ and ‘the angle is in both triangles (common angles)’.

Solution

a In the triangles *ABC* and *BDC*, angle *ABC* = angle *BDC* = 90° (given).
The angle at C is in both triangles.
Since the angle sum of a triangle is 180°, the third angles must be equal.
So, since all the corresponding angles are equal, the triangles are similar.

b First redraw the triangles so they are the same way round as each other.



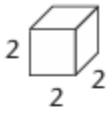
Note

It is always easier to spot the corresponding sides if the shapes are the same way round.
It is worth spending time redrawing the shapes separately and the same way round, and marking the lengths on the new diagram.

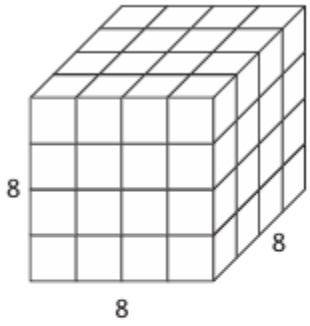
Since *AB* = 6 cm and *BD* = 4.8 cm the scale factor = $4.8 \div 6 = 0.8$.
$CD = 8 \times 0.8$
$= 6.4$ cm

The areas and volumes of similar shapes

This cube has a volume of $8\,cm^3$.



This cube has a volume of $512\,cm^3$.



The lengths of the small cube have been enlarged by scale factor 4.

The volume has been enlarged by scale factor 64.

Since there are three dimensions for volume, and each dimension has been enlarged by scale factor 4, the volume scale factor = 4^3.

Similarly, consider the area of the face of each cube.

For the small cube, the area is $4\,cm^2$.

For the large cube, the area is $64\,cm^2$.

The area has been enlarged by scale factor 16.

There are two dimensions for area, so the area scale factor is 4^2.

For mathematically similar shapes

- area scale factor = (length scale factor)^2
- volume scale factor = (length scale factor)^3.

Example 35.3

Question

A model aircraft is made to a scale of 1 : 50.
The area of the wing on the model is $18\,cm^2$.
What is the area of the wing on the real aircraft?

Solution

Length scale factor = 50
Area scale factor = 50^2
Area of real wing = 18×50^2
= $45\,000\,cm^2$
= $4.5\,m^2$

Note

Remember that
1 m = 100 cm
$1\,m^2 = 10\,000\,cm^2$
$1\,m^3 = 1\,000\,000\,cm^3$

Example 35.4

Question

A jug holding 50 cl is 12 cm high.
A similar jug holds 2 litres.
What is its height?

Solution

50 cl = 0.5 litres

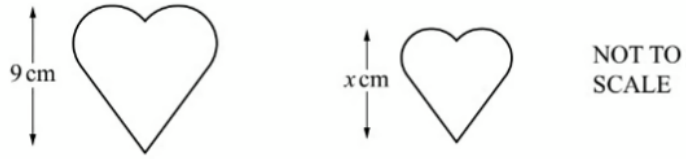
Volume scale factor = $\frac{2}{0.5} = 4$

Length scale factor = $\sqrt[3]{4}$

Height of larger jug = $12\,cm \times \sqrt[3]{4}$
= 19.0 cm to 1 decimal place

Revision questions

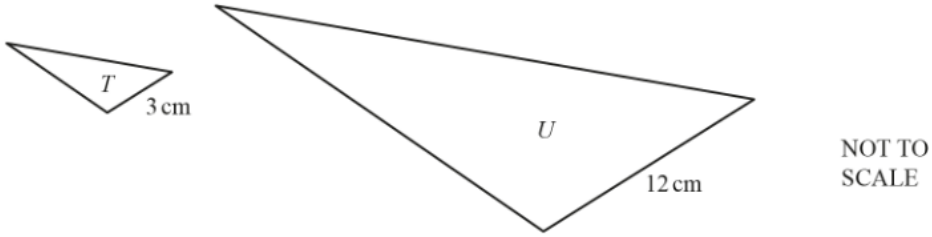
1.



The two shapes are mathematically similar.
The area of the larger shape is 36 cm^2 and the area of the smaller shape is 25 cm^2.
The height of the larger shape is 9 cm and the height of the smaller shape is x cm.

Find the value of x.

2.

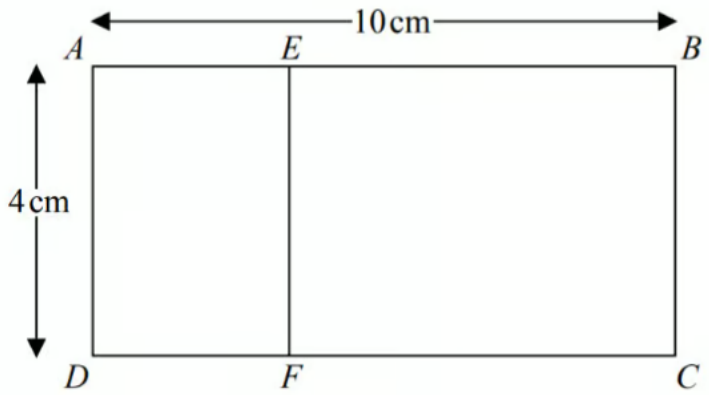


The diagram shows two mathematically similar triangles, *T* and *U*.
Two corresponding side lengths are 3 cm and 12 cm.
The area of triangle *T* is 5 cm^2.

Find the area of triangle *U*.

3.

Rectangle $ABCD$ is mathematically similar to rectangle $DAEF$.

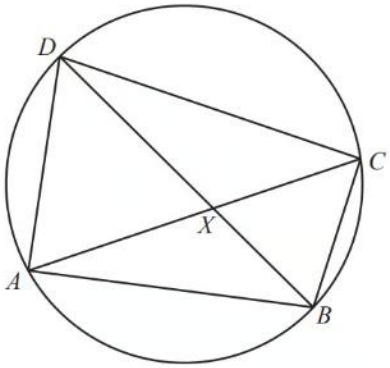


$AB = 10$ cm.

$AD = 4$ cm.

Work out the area of rectangle $DAEF$.

4.



NOT TO SCALE

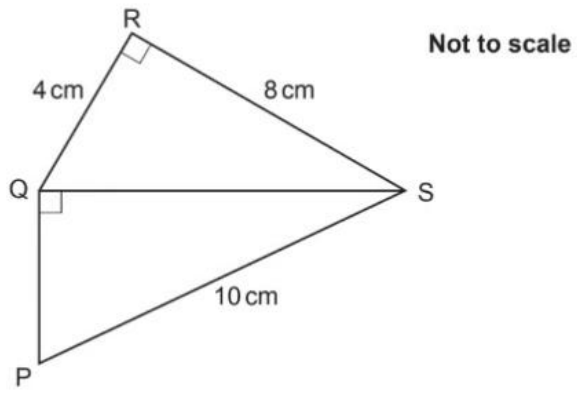
The diagonals of the cyclic quadrilateral $ABCD$ intersect at X.

Explain why triangle ADX is similar to triangle BCX.

Give a reason for each statement you make.

5.

The diagram below shows two right-angled triangles.



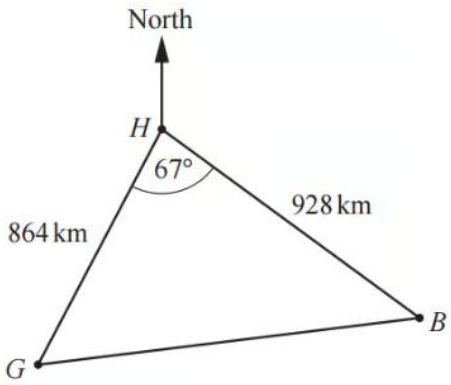
Prove that triangles PQS and QRS are similar.

6. In triangle *ABC*, *BC* = 7.6 cm and *AC* = 6.2 cm.

Using a ruler and compasses only, construct triangle *ABC*.
Leave in your construction arcs.
The side *AB* has been drawn for you.

7.

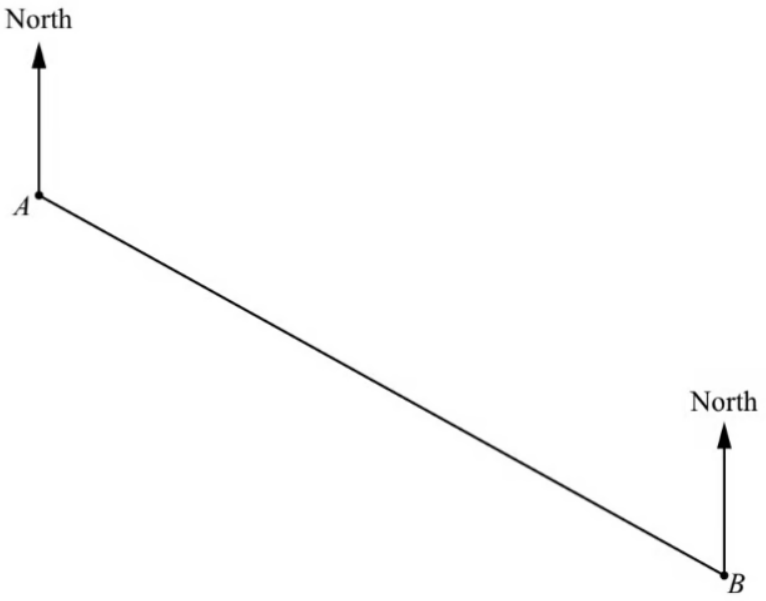
The diagram shows the positions of three cities, Geneva (*G*), Budapest (*B*) and Hamburg (*H*).



NOT TO SCALE

The bearing of Budapest from Hamburg is 133°.
Find the bearing of Hamburg from Budapest.

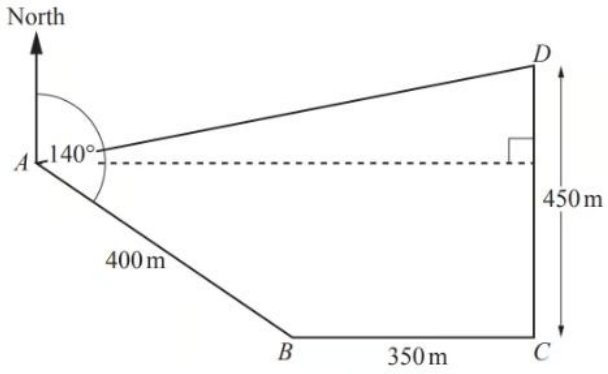
8.



Two towns, *A* and *B*, are shown on a map.
The scale of the map is 1 cm to 3 km.

Find the actual distance between *A* and *B*.

9.



NOT TO SCALE

The diagram shows a field $ABCD$.
The bearing of B from A is 140°.
C is due east of B and D is due north of C.
$AB = 400$m, $BC = 350$m and $CD = 450$m.
Find the bearing of D from B.

11.

The scale of a map is 1 : 10 000 000.
On the map, the area of Slovakia is 4.9 cm^2.

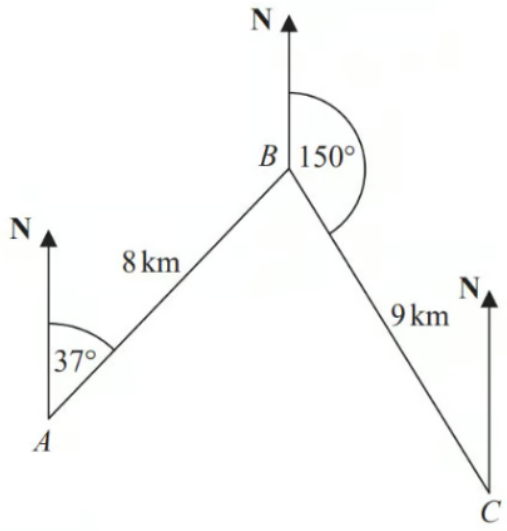
Calculate the actual area of Slovakia.
Give your answer in square kilometres.

12.

On a map with scale 1 : 25000, the area of a lake is 33.6 square centimetres.
Calculate the actual area of the lake, giving your answer in square kilometres.

13.

The diagram shows the positions of three towns, Acton (A), Barston (B) and Chorlton (C).



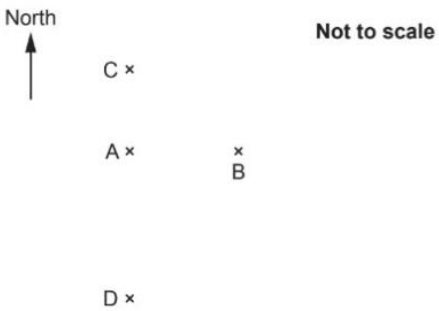
Barston is 8 km from Acton on a bearing of **037°**.
Chorlton is 9km from Barston on a bearing of **150°**.

Find the bearing of Chorlton from Acton.
Give your answer correct to 1 decimal place.
You must show all your working.

14.

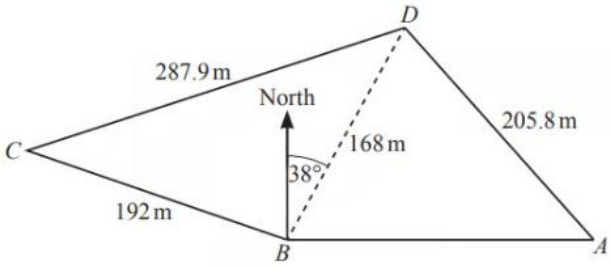
A, B, C and D are four towns.

B is 25 kilometres due East of A.
C is 25 kilometres due North of A.
D is 45 kilometres due South of A.



Work out the bearing of B from C.

15.



The diagram shows a field, $ABCD$, on horizontal ground.
$BC = 192$ m, $CD = 287.9$ m, $BD = 168$ m and $AD = 205.8$ m.

Angle $CBD = 106°$.

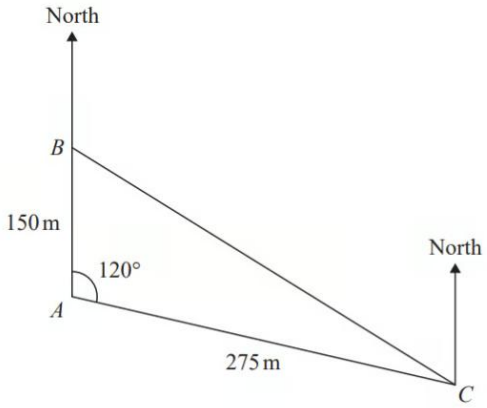
i) The bearing of D from B is 038°.

Find the bearing of C from B.

ii) A is **due east** of B.

Calculate the bearing of D from A.

16. The diagram shows the positions of three ships, A, B and C.



Ship B is due north of ship A.

The bearing of ship C from ship A is 120°

Calculate the bearing of ship C from ship B.
Give your answer correct to the nearest degree.

17.

A , B and C are three towns.

The bearing of B from A is 105°
The bearing of C from B is 230°

The distance of C from A is 180 km.
The distance of C from B is 95 km.

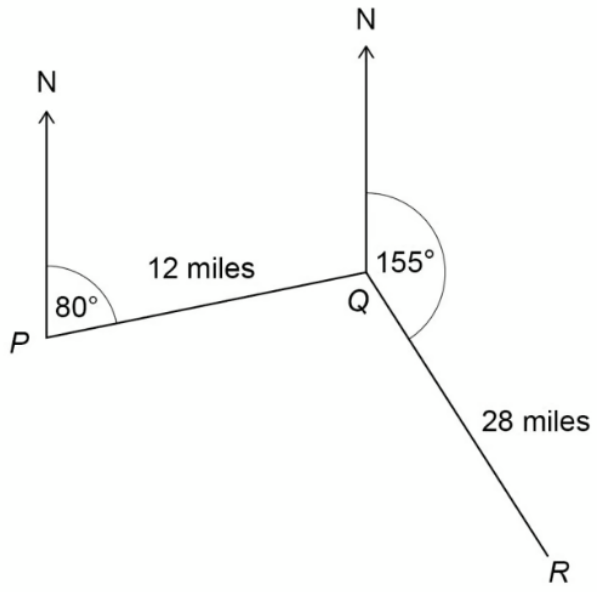
Calculate the distance of B from A.
Give your answer correct to 3 significant figures.

18.

A ship sails from P to Q and then from Q to R.

Q is 12 miles from P, on a bearing of 080°

R is 28 miles from Q, on a bearing of 155°



Work out the direct distance from P to R.