Cambridge OL

Mathematics

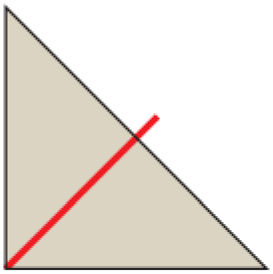
CODE: (4024)

Chapter 36 and Chapter 37

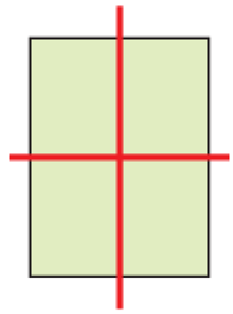
Symmetry and Angles

Line symmetry

A line of symmetry divides a shape into two identical parts.
A shape may have more than one line of symmetry.



The triangle has one line of symmetry.

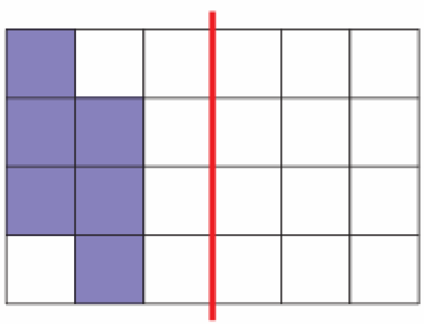


The rectangle has two lines of symmetry.

Example 36.1

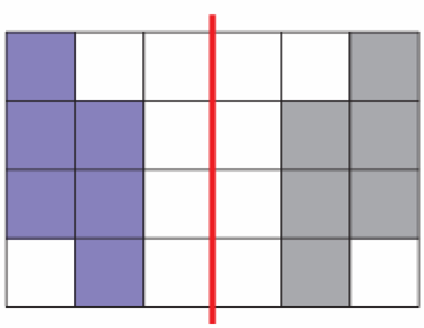
Question

Complete the pattern so that L is a line of symmetry.



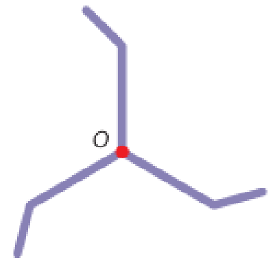
L

Solution



The image is the same distance from the line of symmetry as the original shape, but on the opposite side of the line.

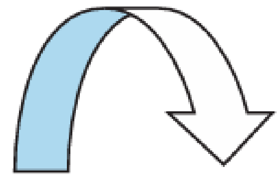
Rotational symmetry



This shape has rotational symmetry of order 3 about centre O.

There are three positions where the shape fits onto the original shape in one complete turnabout centre O.

The order of rotational symmetry is the number of times that a shape fits onto itself in one complete turn. This shape has no rotational symmetry.

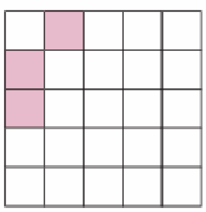


There is only one position where the shape fits onto the original in one complete turn. It has rotational symmetry of order 1.

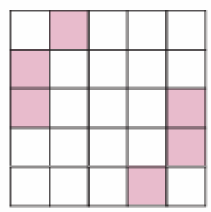
Example 36.2

Question

Shade three more squares in this shape so that the pattern has rotational symmetry of order 2.



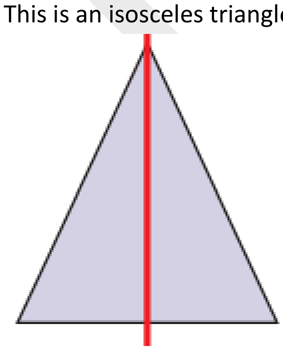
Solution



You can check that your diagram is correct by rotating it through a half turn.

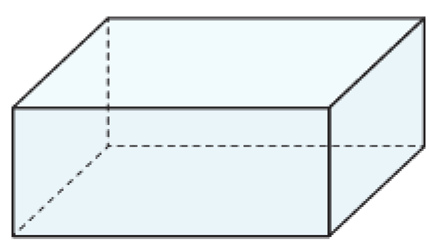
Symmetry properties of shapes and solids

This is an isosceles triangle.

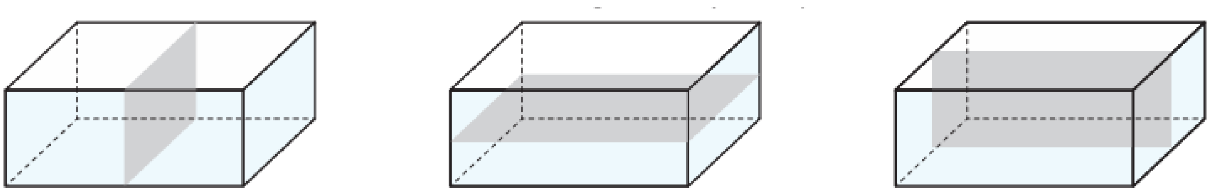


It has one line of symmetry and no rotational symmetry.

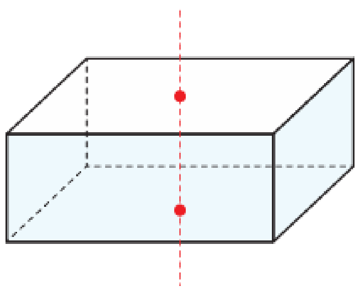
These symmetry properties are the same for any isosceles triangle. This is a cuboid.



A **plane of symmetry** divides a solid into two identical parts. The cuboid has three planes of symmetry.



The cuboid has rotational symmetry of order 2 about the axis shown.



The cuboid has three axes of rotational symmetry. When describing the symmetry of a shape, remember to describe both rotational and line symmetry.

Example 36.3

Question

Describe the symmetry of

a a parallelogram

b a square-based pyramid

c a regular nine-sided polygon. Use this to find the size of each interior angle of the polygon.

Solution

a A parallelogram has no lines of symmetry and rotational symmetry of order 2.

b A square-based pyramid has four planes of symmetry and one axis of rotational symmetry of order 4.

c A regular nine-sided polygon has nine lines of symmetry and rotational symmetry of order 9.
By symmetry: exterior angle = $360 \div 9 = 40°$; interior angle = $180 - 40 = 140°$.

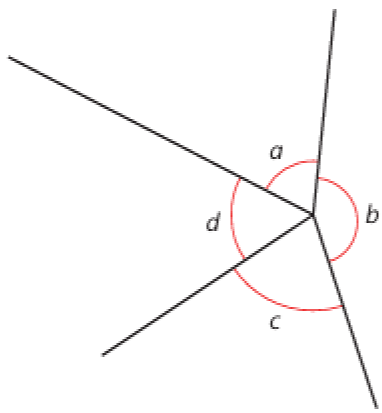
Key points

- In two dimensions, a figure has line symmetry if it reflects onto itself in a line.
- A two-dimensional shape has rotational symmetry if it will rotate about a centre onto itself in more than one position. The number of ways it fits is the order of rotational symmetry.
- In three dimensions, a shape has plane symmetry if it reflects onto itself in a plane.
- A three-dimensional shape has rotational symmetry if it will rotate about an axis onto itself in more than one position. The number of ways it fits is the order of rotational symmetry.

Chapter 37 – Angles

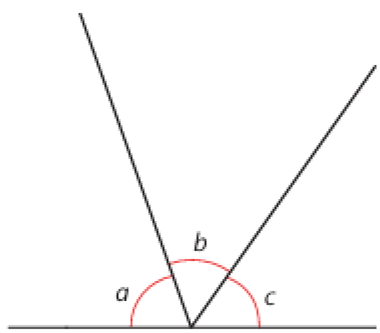
Angles formed by straight lines

The sum of the angles at a point is 360°.



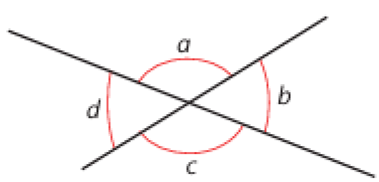
$a + b + c + d = 360°$

The sum of the angles on a straight line is 180°.



$a + b + c = 180°$

When two lines cross, vertically opposite angles are equal.



$a = c$ and $b = d$

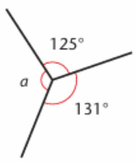
Example 37.1

Question

Work out the size of angles *a*, *b*, *c*, *d* and *e*.

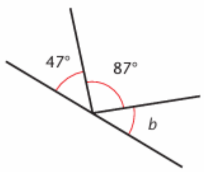
Give a reason for each answer.

a

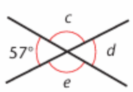


> **Note**
> The diagrams are not drawn accurately, so do not try to measure the angles.

b



c



> **Note**
> Remember to give a reason for your answer.
> State the angle fact that you have used.

Solution

a $a + 125° + 131° = 360°$ — The sum of the angles at a point is 360°.

$a + 256° = 360°$

$a = 104°$

b $47° + 87° + b = 180°$ — The sum of angles on a straight line is 180°.

$134° + b = 180°$

$b = 46°$

c $c + 57° = 180°$ — The sum of the angles on a straight line is 180°.

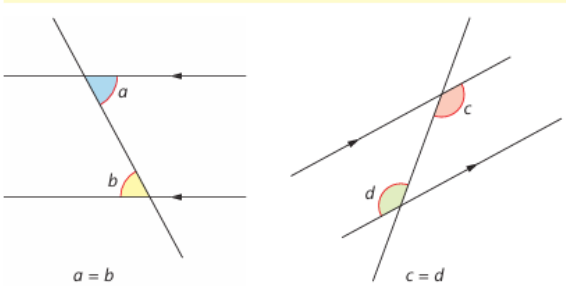
$c = 123°$

$d = 57°$ — Vertically opposite angles are equal.

$e = 123°$ — Vertically opposite angles are equal.

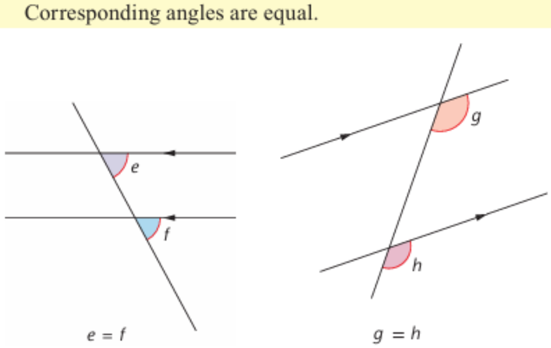
Angles formed within parallel lines

Alternate angles are equal.



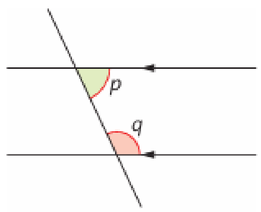
> **Note**
> Thinking of a Z-shape may help you to remember that alternate angles are equal. Alternatively, turn the page round and look at the diagrams upside down and you will see the same shapes!

Corresponding angles are equal.

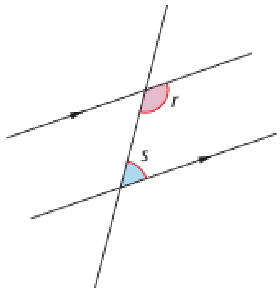


> **Note**
> Thinking of an F-shape or a translation may help you to remember that corresponding angles are equal.

Co-interior angles (sometimes called **allied angles**) are **supplementary**, which means that they add up to 180°.



$p + q = 180°$

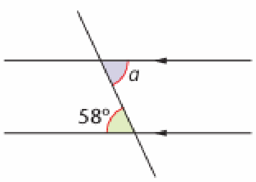


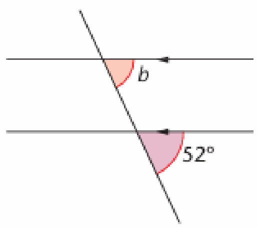
$r + s = 180°$

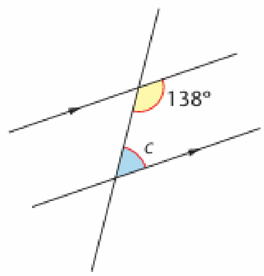
Example 37.2

Question

Find the size of each of the lettered angles in these diagrams, giving your reasons.







Solution

$a = 58°$ Alternate angles are equal.

$b = 52°$ Corresponding angles are equal.

$c = 42°$ Co-interior angles add up to 180°.

Note

Remembering a C-shape or using facts about angles on a straight line may help you to remember that co-interior angles add up to 180°.

Note

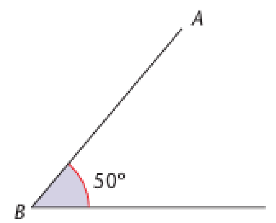
Single letters are often used for angles, as in this chapter so far.

Sometimes, the ends of a line are labelled with letters.

These can be used to name the angles.

In the diagram, angle $ABC = 50°$.

This may also be written $A\hat{B}C = 50°$.



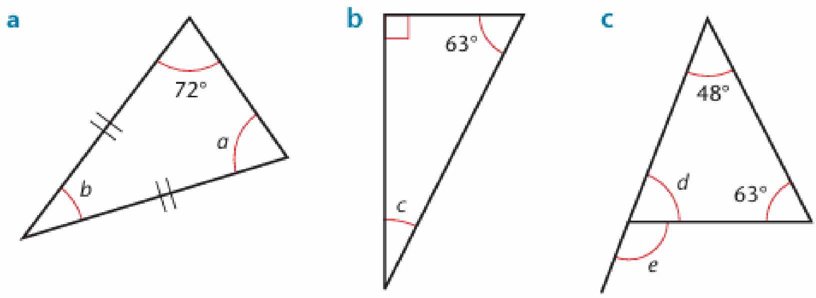
Example 37.3

Question

Work out the sizes of the angles in these diagrams.

Sides the same length are marked with two short lines.

Give reasons for your answers.



Solution

a $a = 72°$ Equal angles in an isosceles triangle.

$b + 72° + 72° = 180°$ Angles in a triangle add up to 180°.

$b + 144° = 180°$

$b = 36°$

b $c + 63° = 90°$ Other angles in a right-angled triangle add up to 90°, or angles in a triangle add up to 180°.

$c = 27°$

c $d + 48° + 63° = 180°$ Angles in a triangle add up to 180°.

$d + 111° = 180°$

$d = 69°$

$e + 69° = 180°$ The sum of angles on a straight line is 180°.

$e = 111°$

Note

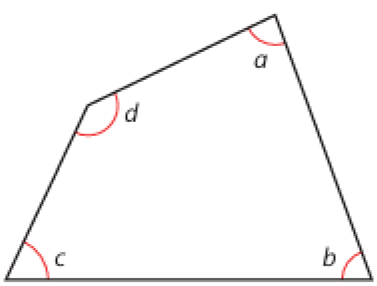
Angle $e = 111°$

$= 63° + 48°$

So the exterior angle of a triangle is equal to the sum of the two opposite interior angles.

The angles in a quadrilateral

The angles in a quadrilateral add up to 360°



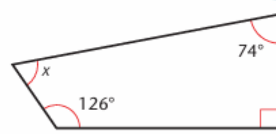
$$a + b + c + d = 360°$$

Example 37.4

Question

Work out the size of angle x.

Give a reason for your answer.



Solution

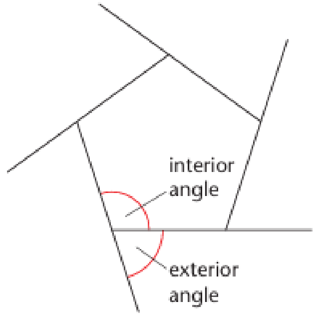
$x = 360° - (126° + 90° + 74°)$ Angles in a quadrilateral add up to 360°.

$x = 70°$

Note

Sides the same length are marked with two short lines.

Unknown angles marked with the same letter in a diagram are equal in size.



The angles in a polygon

The sum of the exterior angles of any convex polygon is 360°.

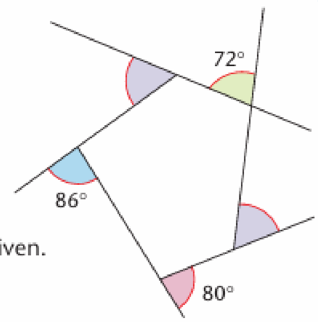
At each vertex, the interior and exterior angles make a straight line.

For any convex polygon, at any vertex: interior angle + exterior angle = 180°.

Example 37.5

Question

The two unlabelled exterior angles of this pentagon are equal.
Find their size.



Solution

First, find the sum of the angles that are given.

72° + 80° + 86° = 238°

The sum of all the exterior angles is 360°.

The sum of the remaining two angles = 360° – 238° = 122°.

So each angle is 122 ÷ 2 = 61°.

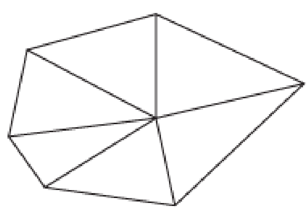
The two angle facts above can be used to find the sum of the interior angles of a polygon with n sides (and n angles).

At each vertex of a convex polygon, the interior angle + the exterior

Sum of (interior + exterior angles) for the polygon = 180° × n

But the sum of the exterior angles is 360°

For an n-sided convex polygon,
the sum of the interior angles = $(180n - 360°) = 180° (n - 2)$



Another way you can find the sum of the interior angles of any polygon is to divide it into triangles.

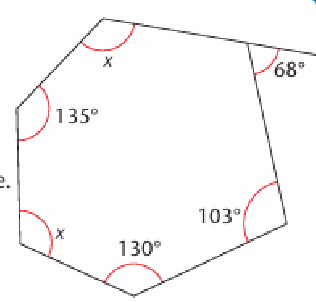
The sum of the angles in all the triangles = $180° \times n$

The sum of the angles at the centre = 360° and this must be subtracted.

Example 37.6

Question

Find the value of x in this hexagon.



Solution

First, find the size of the missing interior angle.

Interior angle + exterior angle = 180°

Interior angle = 180° – 68°

= 112°

Next, find the sum of the interior angles.

Sum of interior angles = $112° + 103° + 130° + 135° + 2x$

$= 480° + 2x$

The sum of the interior angles of a hexagon = $180°(n - 2)$

$= 180° \times 4$

$= 720°$

Therefore

$480° + 2x = 720°$

$2x = 720° - 480°$

$2x = 240°$

$x = 120°$

Example 37.7

Question

Find the interior angle of a regular hexagon.

Solution

For a regular hexagon, the exterior angle = $360° \div 6 = 60°$.

So the interior angle = $180° - 60° = 120°$.

Example 37.8

Question

Find the number of sides of a regular polygon with an interior angle of 144°.

Solution

If the interior angle = 144°, the exterior angle = $180° - 144° = 36°$.

The sum of the exterior angles = 360°.

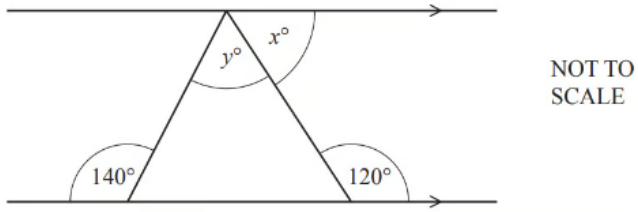
Therefore the number of sides = $360° \div 36° = 10$.

Key points

- Angles around a point add up to 360°.
- Angles on a straight line add up to 180°.
- Where two lines intersect, opposite angles are equal.
- Alternate angles in parallel lines are equal and form a Z-shape.
- Corresponding angles in parallel lines are equal and form an F-shape.
- Co-interior angles in parallel lines add up to 180° and form a C-shape.
- Angles in a triangle add up to 180°.
- Angles in a quadrilateral add up to 360°.
- The exterior angles of a polygon add up to 360°.
- At any vertex of a convex polygon, the interior angle and the exterior angle add up to 180°.
- Angles in an n-sided convex polygon add up to $180° \times (n - 2)$.

Revision questions

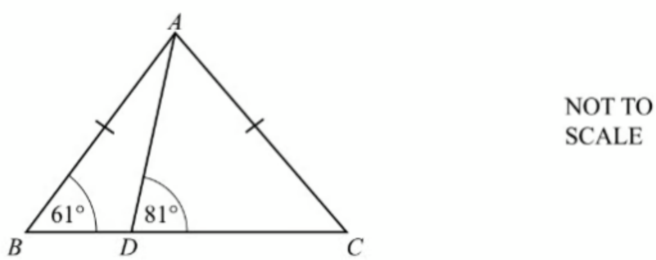
1.



The diagram shows a triangle drawn between a pair of parallel lines.
Find the value of x and the value of y.

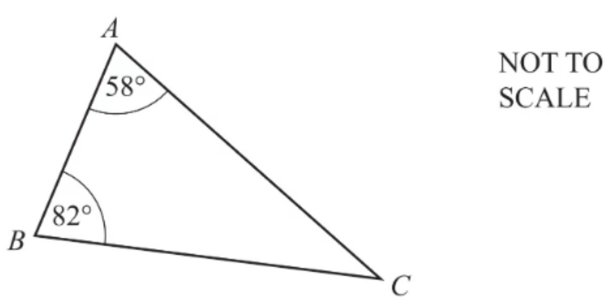
2.

The diagram shows two triangles, ABD and ADC.



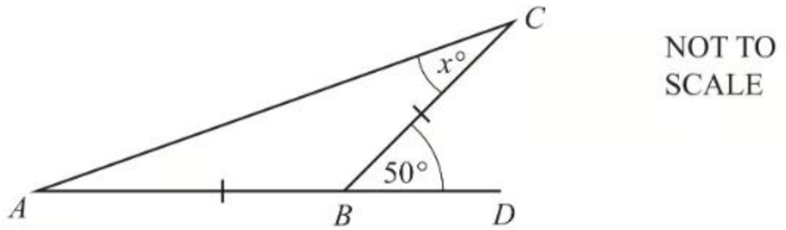
BDC is a straight line, $AB = AC$, angle $ABD = 61°$ and angle $ADC = 81°$.
Work out angle DAC.

3.



The diagram shows triangle ABC.
The triangle is reflected in the line BC to give a quadrilateral ABDC.
Write down the mathematical name of the quadrilateral ABDC.

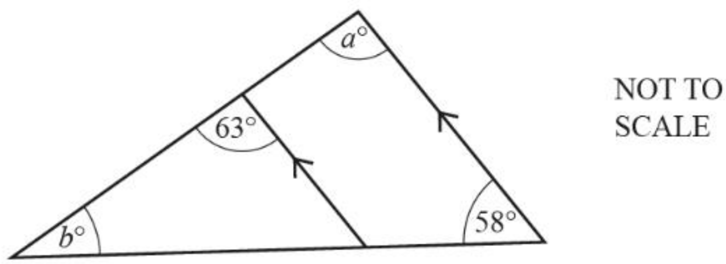
4.



$AB = BC$ and ABD is a straight line.

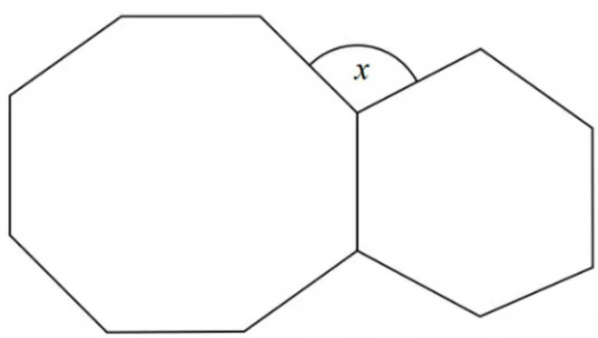
Find the value of x.

5.



Complete the statements.

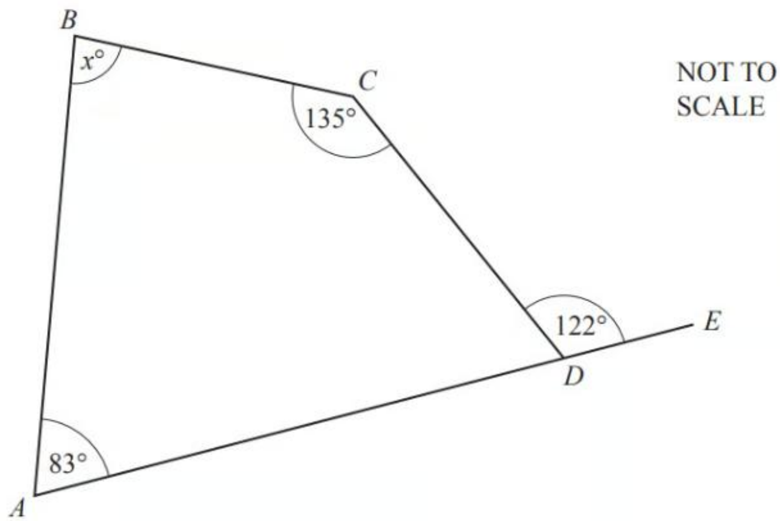
6.



The diagram shows a regular octagon and a regular hexagon.

Find the size of the angle marked x.
You must show all your working.

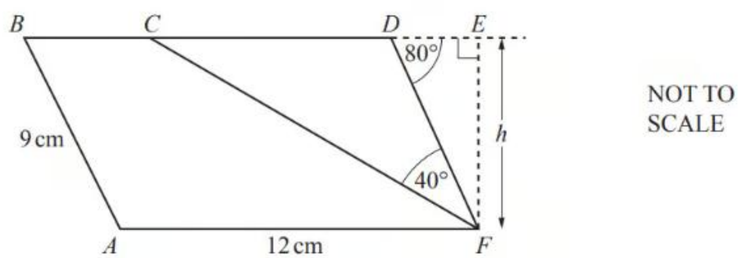
7.



The diagram shows quadrilateral $ABCD$ with AD extended to E.
Angle $BCD = 135°$, angle $BAD = 83°$ and angle $CDE = 122°$.

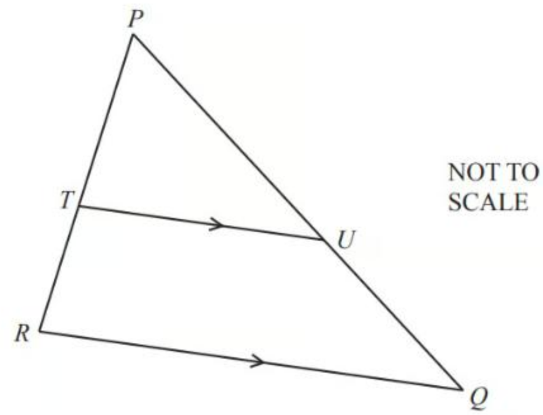
Find the value of x.

8.



ABDF is a parallelogram and *BCDE* is a straight line.
AF = 12cm, *AB* = 9 cm, angle *CFD* = 40° and angle *FDE* = 80°.
Explain why triangle *CDF* is isosceles.

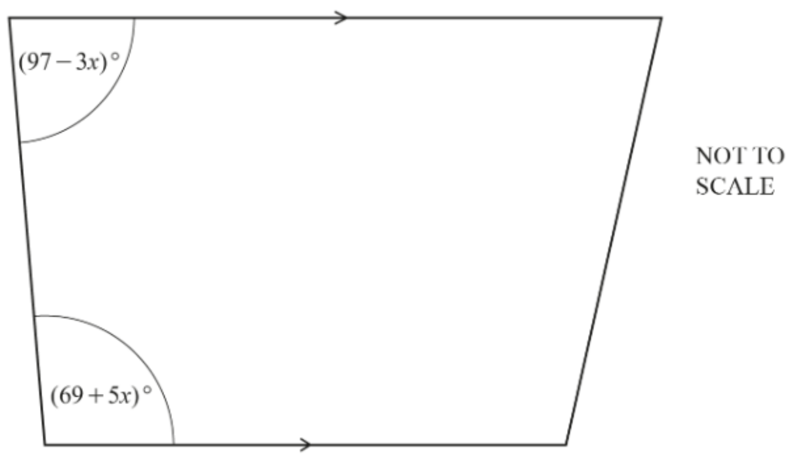
9.



PQR is a triangle.
T is a point on *PR* and *U* is a point on *PQ*.
RQ is parallel to *TU*.
Explain why triangle *PQR* is similar to triangle *PUT*.
Give a reason for each statement you make.

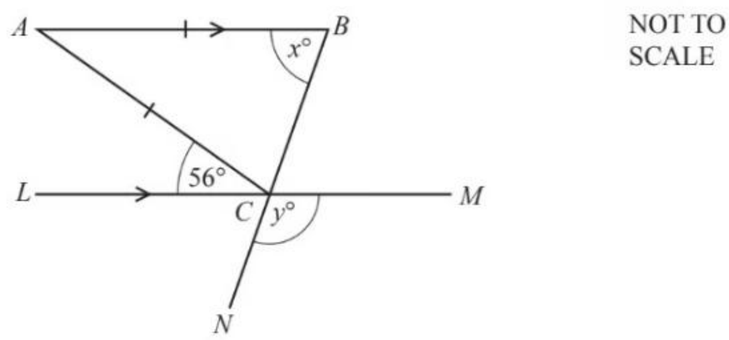
10.

The diagram shows a trapezium.



Work out the value of x.

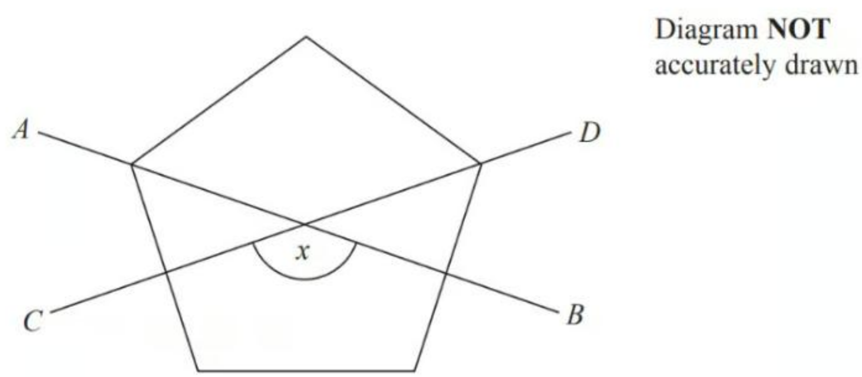
11.



The diagram shows an isosceles triangle ABC with $AB = AC$.
LCM and BCN are straight lines and LCM is parallel to AB.
Angle $ACL = 56°$.

Find the value of x and the value of y.

12.



The diagram shows a regular pentagon.
AB and CD are two of the lines of symmetry of the pentagon.

Work out the size of the angle marked x.
You must show all your working.

13.

$ABCDEFGHI$ is a regular 9–sided polygon.

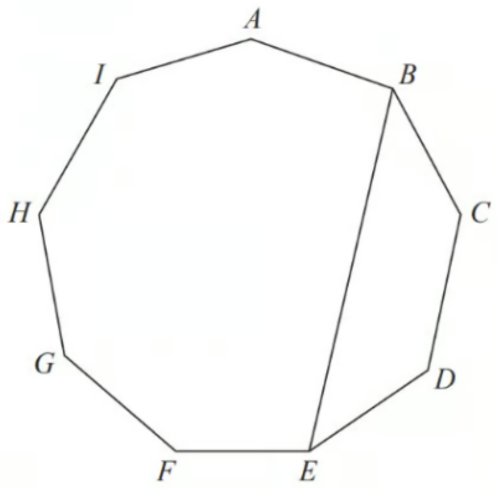
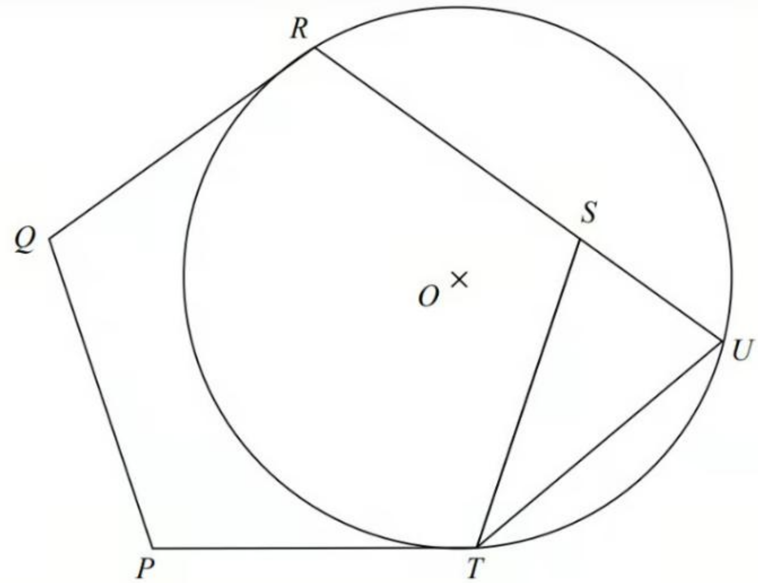


Diagram **NOT** accurately drawn

The vertices B and E are joined with a straight line.

Work out the size of angle BEF.
You must show how you get your answer.

14.



$PQRST$ is a regular pentagon.
R, U and T are points on a circle, centre O.
QR and PT are tangents to the circle.
RSU is a straight line.

Prove that $ST = UT$.