Cambridge OL

Mathematics

CODE: (4024)

Chapter 40

Mensuration

Chapter 40 – MENSURATION

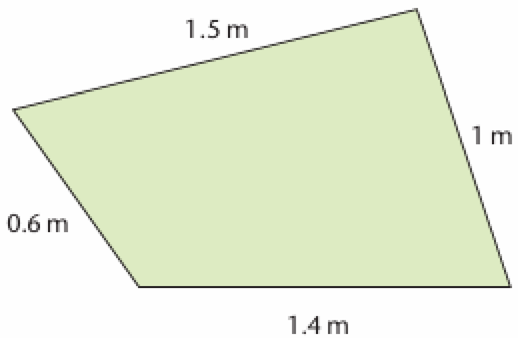
The perimeter of a 2-D shape

The perimeter of a shape is the distance all the way around the edge of the shape. Since the perimeter of a shape is a length, you must use units such as centimetres (cm), metres (m) or kilometres (km).

Example 40.1

Question

Find the perimeter of this shape.



Solution

Perimeter = 0.6 + 1.4 + 1 + 1.5 = 4.5 m

Note

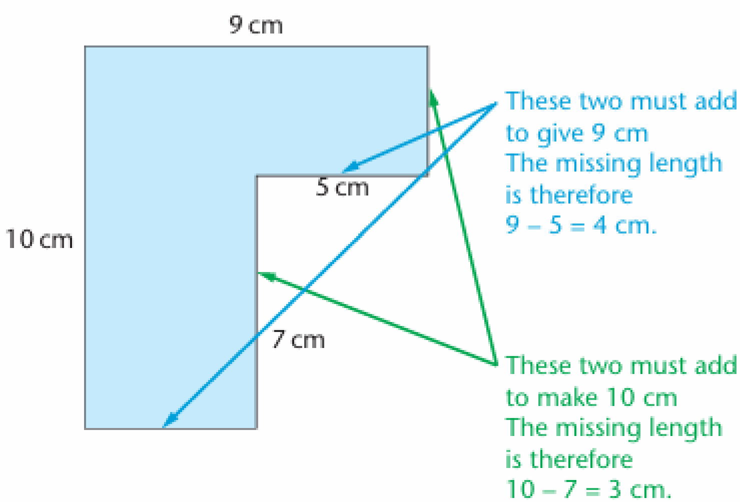
Although you don't have to give the units in your working, you must remember to give the units with your answer.

Sometimes not all of the lengths of the shape are given in the diagram. Before trying to find the perimeter, work out all the lengths.

Example 40.2

Question

Find the perimeter of this shape made from rectangles.



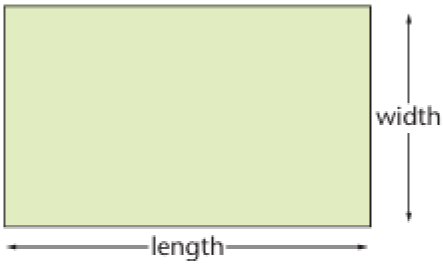
Solution

Perimeter = 9 + 10 + 4 + 7 + 5 + 3 = 38 cm

The area of a rectangle

The area of a two-dimensional shape is the amount of flat space inside the shape.
For rectangles of any size,

area of a rectangle = length × width



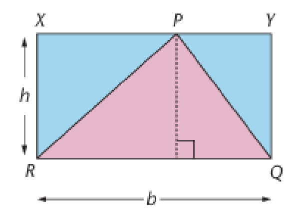
> **Note**
> Whenever you are giving an answer for an area, make sure you include the units.
> If the lengths are in centimetres, the area will be in cm^2.
> If the lengths are in metres, the area will be in m^2.

The area of a triangle

Triangle PQR has base length b and perpendicular height h.
Rectangle XYQR has the same base length and height.
The blue area is the same size as the pink area.

Area of the rectangle is bh, so the area of the triangle is $\frac{1}{2}bh$.

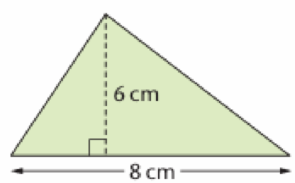
Area of a triangle $= \frac{1}{2} \times$ base $\times$ perpendicular height or $A = \frac{1}{2}bh$



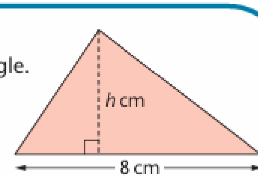
Example 40.3

Question

a Find the area of this triangle.



b The area of this triangle is 20 cm^2.
Find the perpendicular height of the triangle.



Solution

a Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 6$

$= 24\text{ cm}^2$

b Area $= \frac{1}{2}bh$

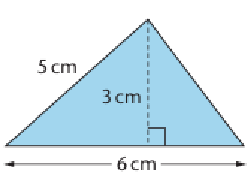
$20 = \frac{1}{2} \times 8 \times h$

$20 = 4h$

$h = 5$

So the height is 5 cm.

> **Note**
> Always use the perpendicular height of the triangle, never the slant height.
> In this triangle, area $= \frac{1}{2} \times 6 \times 3 = 9\text{ cm}^2$.



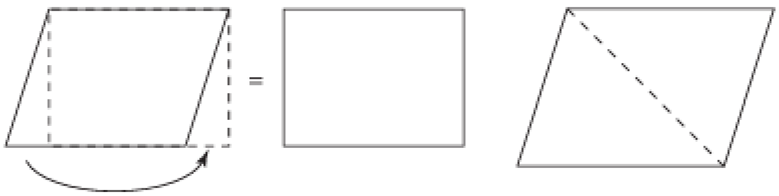
Remember that the units of area are always square units, such as square centimetres or square metres, written cm^2 or m^2.

When using the formula, you can use any of the sides of the triangle as the base, provided you use the perpendicular height that goes with it.

The area of a parallelogram

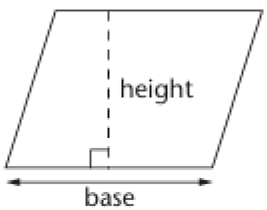
A parallelogram may be cut up and rearranged to form a rectangle or two congruent triangles

Area of a rectangle = base × height Area of each triangle = $\frac{1}{2}$ × base × height



Both these ways of splitting a parallelogram show how to find its area.

Area of a parallelogram = base × height

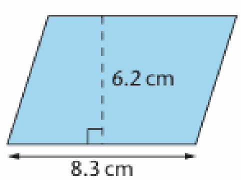


Note
Make sure you use the perpendicular height and not the sloping edge when finding the area of a parallelogram.

Example 40.4

Question

Find the area of this parallelogram.



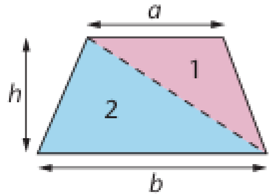
Solution

Area of a parallelogram = base × height
= 8.3 × 6.2
= 51.46 cm^2
= 51.5 cm^2 to 1 decimal place

Note
Always give your final answer to a suitable degree of accuracy, but don't use rounded answers in your working.

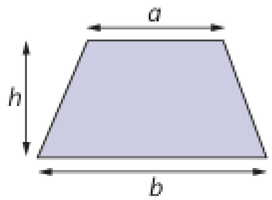
The area of a trapezium

A trapezium has one pair of opposite sides parallel.
It can be split into two triangles.



Area of triangle 1 $= \frac{1}{2} \times a \times h$

Area of triangle 2 $= \frac{1}{2} \times b \times h$



$$\text{Area of trapezium} = \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times (a + b) \times h$$
$$= \frac{1}{2}(a + b)h$$

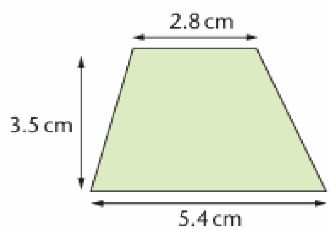
You can remember the formula in words or algebraically.

Area of a trapezium = half the sum of the parallel sides × the height
$= \frac{1}{2}(a + b)h$

Example 40.5

Question

Calculate the area of this trapezium.



Solution

Area of a trapezium $= \frac{1}{2}(a + b)h$
$= \frac{1}{2} \times (2.8 + 5.4) \times 3.5$
$= 14.35 \text{ cm}^2$
$= 14.4 \text{ cm}^2$ to 1 decimal place

Note

Use the brackets function on your calculator.
Without a calculator, remember to work out the brackets first.

Note

The plural of *trapezium* is *trapezia*.

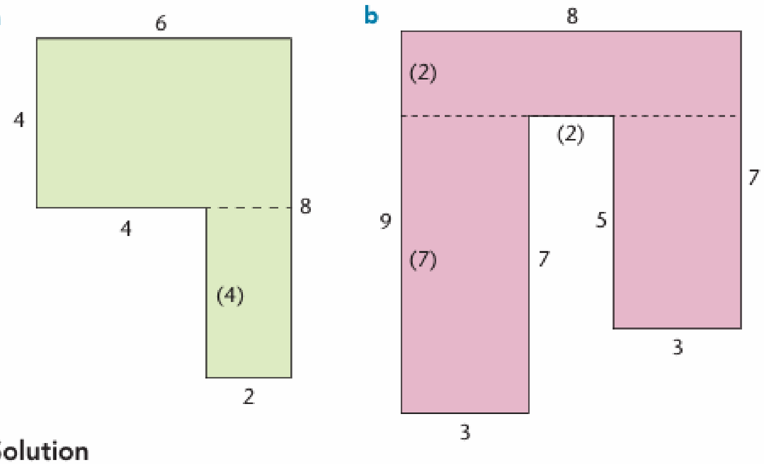
The area of shapes made from rectangles and triangles

A way to find the area of many compound shapes is to split them up into rectangles and triangles.

Example 40.6

Question

Work out the area of each these shapes. All lengths are in centimetres.



(The lengths marked in brackets were not all given in the question, but have been worked out. The dashed lines have been added as part of the answer.)

Solution

a The shape has been split into two rectangles by a horizontal dotted line. (It could have been split in a different way, by a vertical line.

Area = $6 \times 4 + 2 \times 4 = 24 + 8 = 32\,\text{cm}^2$

b The shape has been split into three rectangles.

Area = $8 \times 2 + 7 \times 3 + 5 \times 3 = 16 + 21 + 15 = 52\,\text{cm}^2$

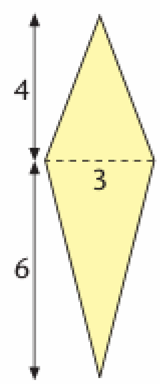
Note

A common error is to split the shape correctly, but then multiply the wrong numbers to get the area.

Example 40.7

Question

Work out the area of this shape. All lengths are in centimetres.



Solution

The shape has been split by the horizontal line into two triangles.

$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times 4 \times 3 + \tfrac{1}{2} \times 6 \times 3 \\ &= 6 + 9 \\ &= 15\,\text{cm}^2\end{aligned}$$

The circumference of a circle

The fact that the **circumference** of a circle is approximately three times the diameter has been known for thousands of years. Accurate calculations have found this number to hundreds of decimal places. The number is a never-ending decimal, and is denoted by the Greek letter π.

π is an irrational number. Your calculator has the number which π represents stored in its memory.

The formula for the circumference of a circle is

$$C = \pi d$$

If you know the radius of the circle instead of the diameter, use the fact that the diameter is double the radius, d = 2r.

An alternative formula for the circumference of a circle is

$$C = 2\pi r$$

Example 40.8

Question

A circle has a radius of 8 cm.

Find its circumference.

Solution

Circumference $= 2\pi r$

$= 2 \times \pi \times 8$

$= 50.265\ldots$

$= 50.3$ cm (to 1 d.p.)

Example 40.9

Question

A circle has a circumference of 20 m.

Find its diameter.

Solution

Circumference $= \pi d$

$20 = \pi d$

$d = 20 \div \pi$

$= 6.366\ldots$

$= 6.37$ m (to 2 d.p.)

In Example 40.10, the answer is given in terms of π.

Example 40.10

Question

Calculate the circumference of a circle with radius 6.5 cm.

Give your answer in terms of π.

Solution

Circumference $= 2\pi r$

$= 2 \times \pi \times 6.5$

$= 13\pi$ cm

Note

Make sure you can use the π key and the square key on your calculator.

The area of a circle

The formula to calculate the area, A, of a circle of radius r is

$$A = \pi r^2$$

Note

One of the most common errors is to mix up diameter and radius.

Every time you do a calculation, make sure you have used the right one.

Example 40.11

Question

The radius of a circle is 4.3 m.

Calculate the area of the circle.

Solution

$A = \pi r^2$

$= \pi \times 4.3^2$

$= 58.1\ \text{m}^2$ (to 1 d.p.)

Note

Make sure you can use the π key and the square key on your calculator.

Example 40.12

Question

The diameter of a circle is 18.4 cm.

Calculate the area of the circle.

Solution

$r = 18.4 \div 2$

$= 9.2\ \text{cm}$

$A = \pi r^2$

$= \pi \times 9.2^2$

$= 266\ \text{cm}^2$ (to the nearest cm^2)

Note

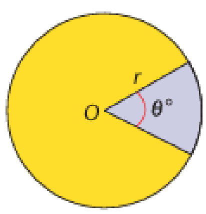
One of the most common errors is to mix up diameter and radius.

Every time you do a calculation, make sure you have used the right one.

Arc length and sector area

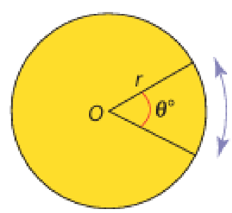
A sector is a fraction of a circle.

It is $\frac{\theta}{360}$ of the circle, where $\theta°$ is the sector angle at the centre of the circle.



Arc length $= \frac{\theta}{360} \times$ circumference

$= \frac{\theta}{360} \times 2\pi r$



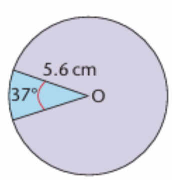
Sector area $= \frac{\theta}{360} \times$ area of circle

$= \frac{\theta}{360} \times \pi r^2$

Example 40.13

Question

Calculate the arc length and area of this sector.



Solution

Arc length $= \frac{\theta}{360} \times 2\pi r$

$= \frac{37}{360} \times 2\pi \times 5.6$

$= 3.62$ cm to 3 significant figures

Sector area $= \frac{\theta}{360} \times \pi r^2$

$= \frac{37}{360} \times \pi \times 5.6^2$

$= 10.1\ \text{cm}^2$ to 3 significant figures

Example 40.14

Question

Calculate the sector angle of a sector with arc length 6.2 cm in a circle with radius 7.5 cm.

Solution

Arc length $= \frac{\theta}{360} \times 2\pi r$

$6.2 = \frac{\theta}{360} \times 2\pi \times 7.5$

$\theta = \frac{6.2 \times 360}{2\pi \times 7.5}$

$= 47.4°$ to 3 significant figures.

Example 40.15

Question

A sector makes an angle of 54° at the centre of a circle.
The area of the sector is 15 cm².
Calculate the radius of the circle.

Solution

Sector area $= \frac{\theta}{360} \times \pi r^2$

$$15 = \frac{54}{360} \times \pi r^2$$

$$r^2 = \frac{15 \times 360}{54 \times \pi}$$

$$r^2 = 31.830\ldots$$

$$r = \sqrt{31.83\ldots}$$

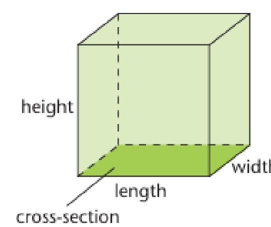
$= 5.64$ cm to 3 significant figures.

Note

You can rearrange the formula before you substitute, if you prefer.

The volume of a prism

A prism is a three-dimensional shape that has the same cross-section throughout its length. A cuboid is a prism with a rectangular cross-section.



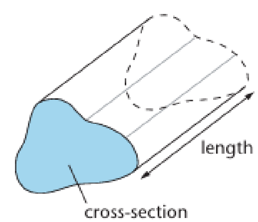
Volume of a cuboid = length × width × height

You can also think of this as

Volume of a cuboid = area of cross-section × height

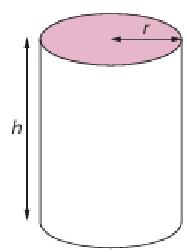
The general formula for the volume of a prism is

Volume of a prism = area of cross-section × length



Another important prism is the **cylinder**.

The cross-section of a cylinder is a circle, which has area πr^2.



Volume of a cylinder $= \pi r^2 h$

Example 40.16

Question

Calculate the volume of a cylinder with base diameter 15 cm and height 10 cm.

Solution

Radius of base $= \frac{15}{2} = 7.5$ cm.

Volume of a cylinder $= \pi r^2 h$

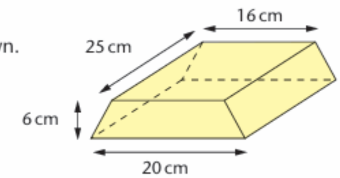
$= \pi \times 7.5^2 \times 10$

$= 1767$ cm³, to the nearest whole number.

Example 40.17

Question

A chocolate box is a prism with a trapezium as cross-section, as shown.
Calculate the volume of the prism.



Solution

Area of a trapezium $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(20 + 16) \times 6$

$= 108\,\text{cm}^2$

Volume of a prism = area of cross-section × length

$= 108 \times 25$

$= 2700\,\text{cm}^3$

Example 40.18

Question

A cylinder has volume $100\,\text{cm}^3$ and is 4.2 cm high.
Find the radius of its base.
Give your answer to the nearest millimetre.

Solution

Volume of a cylinder $= \pi r^2 h$

$100 = \pi \times r^2 \times 4.2$

$r^2 = \frac{100}{\pi \times 4.2}$

$= 7.578\ldots$

$r = \sqrt{7.578.}$

$= 2.752\ldots$

$= 2.8$ cm, to the nearest millimetre.

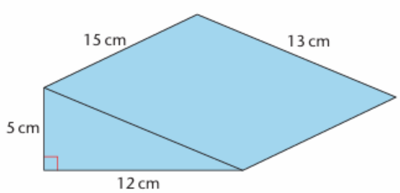
The surface area of a prism

To find the total surface area of a prism, you add up the surface area of all the individual surfaces.

Example 40.19

Question

Find the total surface area of this prism.



Solution

Area of end $= \frac{1}{2} \times 12 \times 5 = 30\,\text{cm}^2$

Area of other end $= 30\,\text{cm}^2$

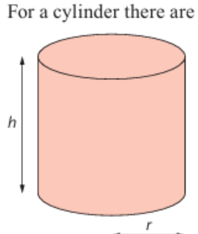
Area of base $= 12 \times 15 = 180\,\text{cm}^2$

Area of top $= 13 \times 15 = 195\,\text{cm}^2$

Area of back $= 5 \times 15 = 75\,\text{cm}^2$

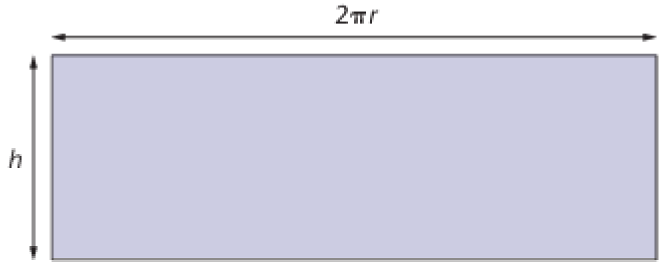
Total surface area $= 510\,\text{cm}^2$

For a cylinder there are three surfaces.



The two ends are circles and each have area πr^2.

If the cylinder were made out of paper, the curved surface would open out to a rectangle.



The length of the rectangle is the circumference of the cylinder, so is $2\pi r$.

The curved surface area is therefore $2\pi r \times h = 2\pi rh$.

Total surface area of a cylinder $= 2\pi rh + 2\pi r^2$

Example 40.20

Question

Calculate the total surface area of a cylinder with base diameter 15 cm and height 10 cm.

Solution

Radius of base $= \frac{15}{2} = 7.5$ cm

Area of two ends $= 2 \times \pi r^2 = 2 \times \pi \times 7.5^2 = 353.429\ldots$ cm^2

Curved surface area $= 2\pi rh = 2 \times \pi \times 7.5 \times 10 = 471.238\ldots$ cm^2

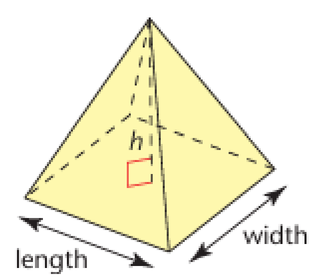
Total surface area $= 353.429\ldots + 471.238\ldots = 825$ cm^2, to the nearest whole number.

The volume of a pyramid, a cone and a sphere

Not all three-dimensional shapes are prisms. Some shapes have a cross-section which, though similar, decreases to a point.

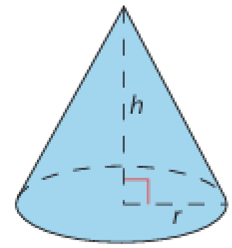
Such shapes include the pyramid and the cone. The volume of a pyramid or cone is given by

Volume $= \frac{1}{3} \times$ area of base $\times$ height



Because a cone has a circular base, for a cone with base radius r and height h,

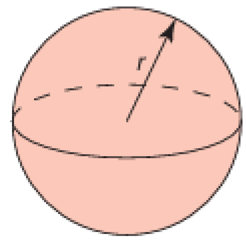
$$\text{Volume} = \frac{1}{3}\pi r^2 h$$



You also need to know about a different type of three-dimensional shape – the sphere.

For a sphere of radius r,

$$\text{Volume} = \frac{4}{3}\pi r^3$$

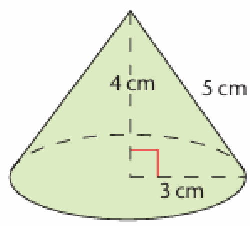


You do not need to learn these formulas but you do need to be able to use them.

Example 40.21

Question

Find the volume of this cone.



Solution

Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3^2 \times 4$

$= 37.7\text{ cm}^3$ to 3 significant figures.

Note

A hemisphere is half a sphere.

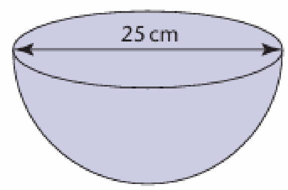
Example 40.22

Question

A bowl is in the shape of a hemisphere of diameter 25 cm.

How much water can the bowl hold?

Give your answer in litres.



Solution

Volume of hemisphere $= \frac{1}{2} \times$ volume of sphere

$= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 12.5^3$

$= 4090.6\ldots\text{ cm}^3$ 1 litre $= 1000\text{ cm}^3$

$= 4.09$ litres to 3 significant figures.

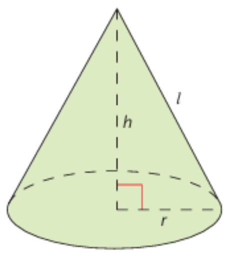
The surface area of a pyramid, a cone and a sphere

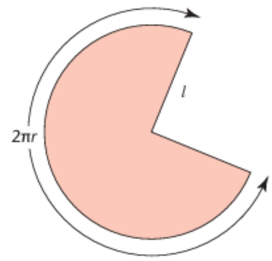
As with the surface area of a prism, the surface area of a pyramid, cone or sphere is the total area of all its surfaces. All the surfaces of a pyramid that have a base in the shape of a polygon are flat. Apart from the base, the faces are triangular.

To find the total surface area, you find the area of each face and add them together. A cone has a base in the shape of a circle and a curved surface.

The curved surface of a cone can be opened out to form a sector of a circle of radius l, where l is the slant height of the cone. The arc length of the sector is the circumference of the base of the cone.

Note
Make sure you distinguish between the perpendicular height, h, and the slant height, l, of a cone – and don't read l as 1!





Curved surface area of a cone = πrl

All of the surface of a sphere is curved.

The formula for the surface area of a sphere of radius r is

surface area of a sphere = $4\pi r^2$

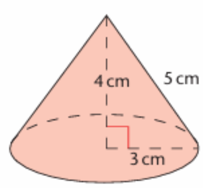
You do not need to learn these formulas but you do need to be able to use them.

Example 40.23

Question

This cone has a solid base.

Find the total surface area of the cone.



Solution

Curved surface area = πrl

$= \pi \times 3 \times 5 = 15\pi$

Area of base = πr^2

$= \pi \times 3^2 = 9\pi$

Total surface area = $15\pi + 9\pi$

$= 24\pi$

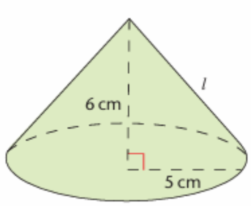
$= 75.4\,\text{cm}^2$ to 3 significant figures.

Note
Leaving the two areas in terms of π means that you are using exact values and avoids rounding errors.

Example 40.24

Question

Calculate the curved surface area of this cone.



Solution

First the slant height l must be found.

Using Pythagoras' theorem,

$l^2 = 5^2 + 6^2$

$= 61$

$l = \sqrt{61}$

Curved surface area = πrl

$= \pi \times 5 \times \sqrt{61}$

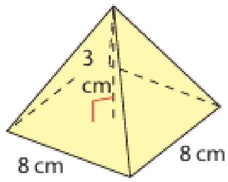
$= 123\,\text{cm}^2$ to 3 significant figures

Note
Leaving the value for l in surd form means that you are using an exact value and avoids rounding errors.

Example 40.25

Question

Calculate the surface area of this pyramid.



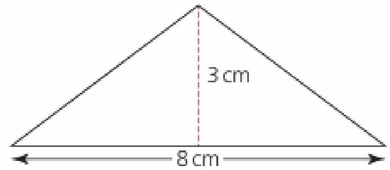
Solution

This is the section through midpoints of opposite sides of the base.

It is an isosceles triangle.

The height splits it into two right-angled triangles.

The hypotenuse of one of the right-angled triangles is the height of one of the triangular faces of the pyramid.



Using Pythagoras' theorem,

$\text{height}^2 = 4^2 + 3^2 = 25$

$\text{height} = 5$

Area of one triangular face $= \frac{1}{2} \times 8 \times 5 = 20\,\text{cm}^2$.

Total surface area of the pyramid $= 4 \times$ area of one triangular face $+$ the area of the square base

$= 4 \times 20 + 8 \times 8$

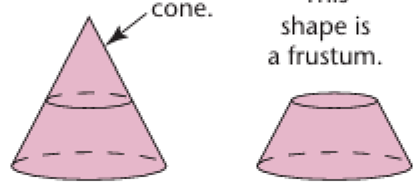
$= 144\,\text{cm}^2$

The area and volume of compound shapes

You have already met the areas of compound shapes made from rectangles and triangles.

In this section we look at examples of compound shapes made from other shapes you have met in this chapter.

The circle on the top of the frustum is in a plane parallel to the base.

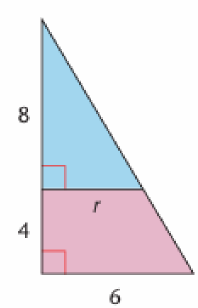


Volume of a frustum = volume of whole cone − volume of missing cone

Example 40.26

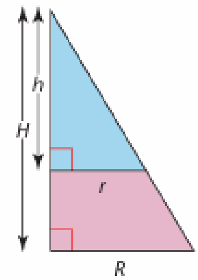
Question

Find the volume of the frustum remaining when a cone of height 8 cm is removed from a cone of height 12 cm and base radius 6 cm.



Solution

First, use similar triangles to find the base radius, r cm, of the cone which has been removed.



$\frac{r}{8} = \frac{6}{12}$

$r = 4$

Then find the volume of the frustum.

Volume of frustum = volume of whole cone − volume of missing cone

$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6^2 \times 12 - \frac{1}{3} \times \pi \times 4^2 \times 8$

= 318 cm² to 3 significant figures.

Example 40.27

Question

Calculate the curved surface area of the frustum in Example 40.26.

Solution

Slant height of complete cone $= \sqrt{6^2 + 12^2} = \sqrt{180}$

Curved surface area of complete cone $= \pi \times 6 \times \sqrt{180}$

Slant height of removed cone $= \sqrt{4^2 + 8^2} = \sqrt{80}$

Curved surface area of removed cone $= \pi \times 4 \times \sqrt{80}$

Curved surface area of frustum = curved surface area of whole cone − curved surface area of missing cone

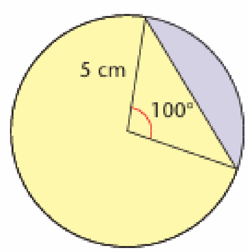
$= \pi \times 6 \times \sqrt{180} - \pi \times 4 \times \sqrt{80}$

140 cm² to 3 significant figures.

Example 40.28

Question

Calculate the area of the purple minor segment.



Solution

Area of minor segment = area of sector − area of triangle

Area of sector $= \frac{\theta}{360}\pi r^2$

$= \frac{100}{360} \times \pi \times 5^2$

$= 21.816\ldots \text{cm}^2$

Area of triangle $= \frac{1}{2}ab \sin C$

$= \frac{1}{2} \times 5^2 \times \sin 100°$

$= 12.310\ldots \text{cm}^2$

Area of minor segment $= 21.816\ldots - 12.310\ldots$

$= 9.51$ cm² to 3 significant figures.

Note

Write down more figures than you need in the working and round the final answer.

Using the calculator memory means you do not have to re-key the figures.

Note

When dealing with problems where you first have to work out how to solve them, follow these steps.

- Read the question carefully and plan.
  What do I know?
  What do I have to find?
  What methods can I apply?
- When you have finished, ask, 'Have I answered the question?'
  There may be one last step you have forgotten to do.

Key points

- The perimeter of a shape is the distance all the way around the edge of the shape.
- Area of a rectangle = length × width.
- Area of a triangle = $\frac{1}{2}$ × base × perpendicular height.
- Area of a parallelogram = base × perpendicular height.
- Area of a trapezium = half the sum of the parallel sides × the height.
- To find the area and perimeter of many compound shapes, split them up into simpler basic shapes.
- The circumference of a circle is πd or $2\pi r$, where d is the diameter and r is the radius.
- The area of a circle of radius r is πr^2.
- A sector is a fraction of a circle. Where $\theta°$ is the angle at the centre of the circle, this means that
  Arc length = $\frac{\theta}{360} \times 2\pi r$ and Sector area = $\frac{\theta}{360} \times \pi r^2$
- Area of minor segment = area of minor sector − area of triangle.
- Area of major segment = area of major sector + area of triangle, or area of circle − area of minor segment.
- Volume of a prism = area of cross-section × length.
- Volume of a cylinder of radius r and height $h = \pi r^2 h$.
- Curved surface area of a cylinder = $2\pi rh$.
- To find the total surface area of a solid, add the areas of all the individual surfaces.
- Volume of a sphere of radius $r = \frac{4}{3}\pi r^3$.
- Surface area of a sphere = $4\pi r^2$.
- Volume of a pyramid = $\frac{1}{3}$ × area of base × height.
- Volume of a cone of base radius r and height $h = \frac{1}{3}\pi r^2 h$.
- Curved surface area of a cone = πrl, where l is the slant height.
- Volume of a frustum of a cone = Volume of cone − volume of missing cone.

Note

You should know how to use the formulas for the volume of a sphere, pyramid and cone. You should also know how to use the formulas for the curved surface area of a cone and the surface area of a sphere. You do not need to memorise the formulas but you should be able to use them correctly.

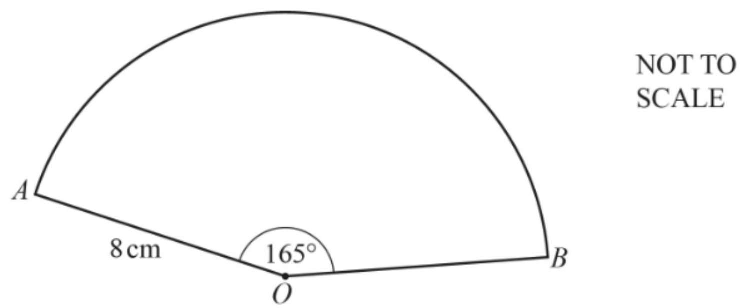
Revision questions

1.

The cross-section of a prism is an equilateral triangle of side 6cm.
The length of the prism is 20 cm.

Calculate the total surface area of the prism.

2.



The diagram shows a sector of a circle with centre O, radius 8 cm and sector angle 165°.
The surface area of a sphere is the same as the area of the sector.

Calculate the radius of the sphere.
[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

3.

A solid metal sphere with radius 6 cm is melted down and all of the metal is used to make a solid cone with radius 8 cm and height h cm.

i) Show that $h = 13.5$.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

[2]

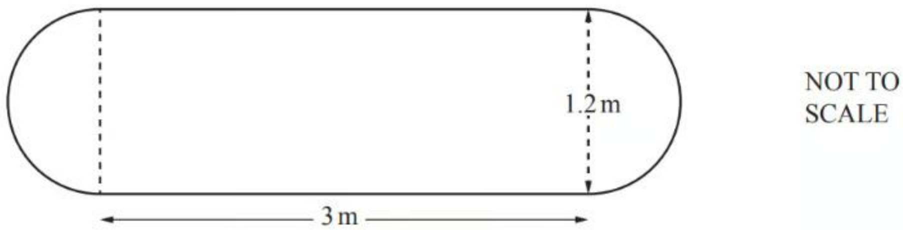
ii) Calculate the slant height of the cone.

.... cm [2]

iii) Calculate the curved surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]

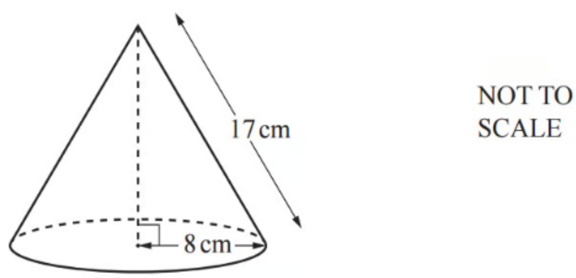
.... cm^2 [1]

4.



The diagram shows the surface of a garden pond, made from a rectangle and two semicircles.
The rectangle measures 3 m by 1.2 m.
Calculate the area of this surface.

5.



The diagram shows a solid cone.
The radius is 8 cm and the slant height is 17 cm.

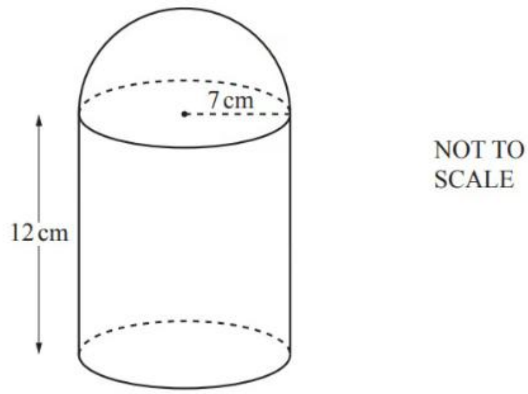
i) Calculate the curved surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.]

.... cm^2 [2]

ii) Calculate the volume of the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

.... cm^3 [4]

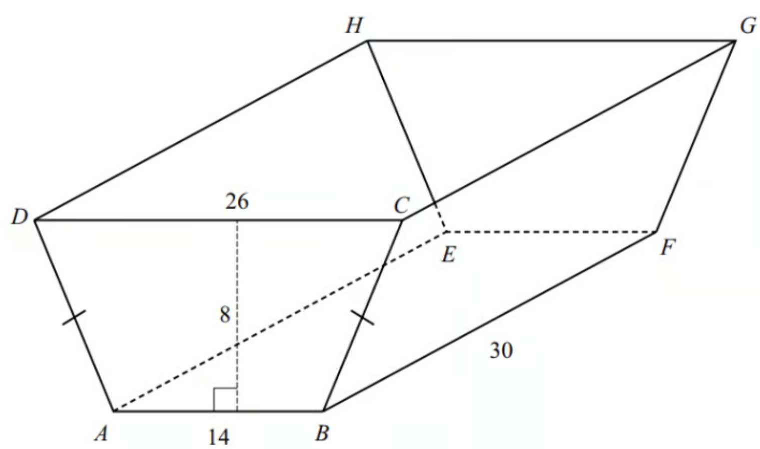
6.



The diagram shows a solid made from a cylinder and a hemisphere, both of radius 7cm.
The cylinder has length 12 cm.

Work out the total surface area of the solid.
Give your answer in terms of π.
[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$]

7.



The diagram shows a container in the shape of a prism with an open top.
The cross-section of the prism, $ABCD$, is a trapezium.
$AB = 14$ cm, $CD = 26$ cm and $BF = 30$ cm.
The height of the container is 8 cm.
The area of the cross-section is 160 cm²

Calculate the capacity of the container in litres.

8.

A solid cylinder has radius x cm and height $\frac{7x}{2}$ cm.

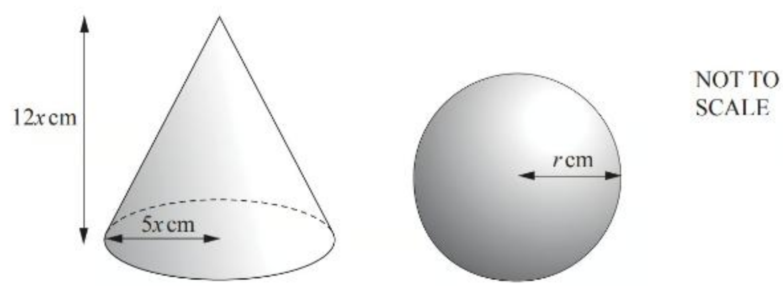
The surface area of a sphere with radius R cm is equal to the total surface area of the cylinder.

Find an expression for R in terms of x.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

9.

The diagram below shows a solid circular cone and a solid sphere.



The cone has radius $5x$ cm and height $12x$ cm.

The sphere has radius r cm.

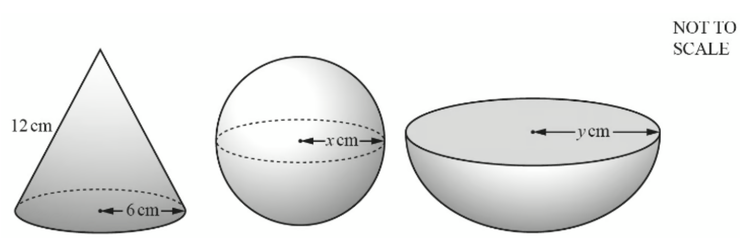
The cone has the same **total** surface area as the sphere.

Show that $r^2 = \frac{45}{2}x^2$.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.]

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

10.



The diagram shows three solids.

The base radius of the cone is 6 cm and the slant height is 12 cm.

Show that the total surface area of the cone is 108π cm^2.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.]