Cambridge OL

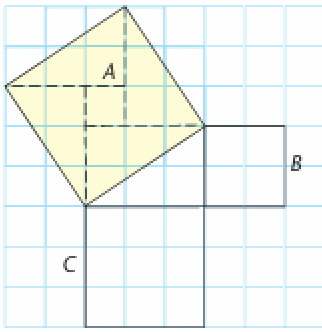
Mathematics

CODE: (4024)

Chapter 41

Pythagoras' theorem and trigonometry

Pythagoras' theorem



Look at square A.

Area of square A = areas of the four triangles + small square

$$= 4\left(\frac{1}{2} \times 2 \times 3\right) + 1$$

$$= 13$$

Area of square B + area of square $C = 4 + 9 = 13$

This is an example of the rule linking the areas of squares around a right angled triangle, known as Pythagoras' theorem.

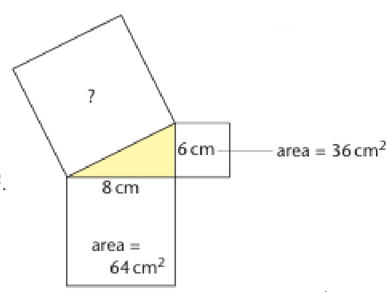
The largest square will always be on the longest side of the triangle – this is called the hypotenuse of the right-angled triangle

Pythagoras' theorem can be stated like this.

The area of the square on the hypotenuse = the sum of the areas of the squares on the other two sides.

Using Pythagoras' theorem

If you know the lengths of two sides of a right-angled triangle you can use Pythagoras' theorem to find the length of the third side.

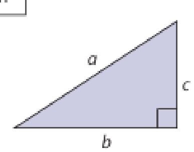


The unknown area = $64 + 36 = 100\,\text{cm}^2$.

This means that the sides of the unknown square have a length of $\sqrt{100} = 10\,\text{cm}$.

When using Pythagoras' theorem, you don't need to draw the squares – you can simply use the rule.

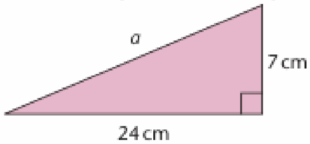
$$a^2 = b^2 + c^2$$



Example 41.1

Question

Find the length *a* in the diagram.



Solution

$a^2 = 7^2 + 24^2$

$a^2 = 49 + 576$

$a^2 = 625$

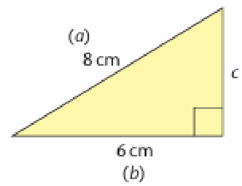
$a = \sqrt{625}$

$a = 25$ cm

Example 41.2

Question

Find the length *c* in the diagram.



Solution

$a^2 = b^2 + c^2$

$8^2 = 6^2 + c^2$

$64 = 36 + c^2$

$c^2 = 64 - 36$

$c = \sqrt{28}$

$c = 5.29$ cm (to 2 decimal places)

Using Pythagoras' theorem to solve problems

You can use Pythagoras' theorem to solve problems. It is a good idea to draw a sketch if a diagram isn't given. Try to draw it roughly to scale and mark on it any lengths you know.

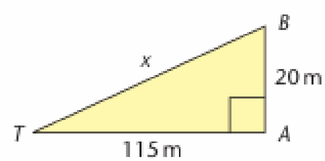
Example 41.3

Question

Tao is standing 115 m from a vertical tower.

The tower is 20 m tall.

Work out the distance from Tao directly to the top of the tower.



Solution

$x^2 = 115^2 + 20^2$

$= 13\,625$

$x = \sqrt{13625}$

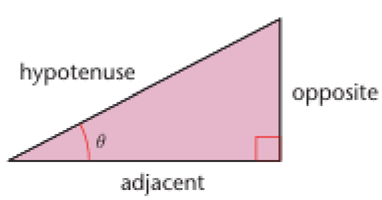
$= 116.7$ m (to 1 decimal place)

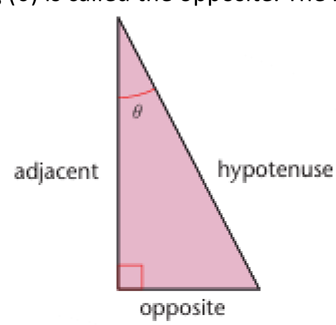
Tao is 116.7 m (to 1 decimal place) from the top of the tower.

Trigonometry

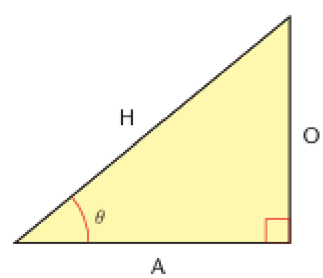
You already know that the longest side of a right-angled triangle is called the hypotenuse.

The side opposite the angle you are using (θ) is called the opposite. The remaining side is called the adjacent.





For a given angle θ °, all right-angled triangles with an angle θ ° will be similar (angles in each triangle of 90°, θ ° and (90 – θ °)). It follows that the ratios of the sides will be constant for that value of θ.



The ratio $\frac{\text{Opposite}}{\text{Hypotenuse}}$ is called the **sine** of the angle.

Note

Notice that the ratio of the lengths is written as a fraction, $\frac{\text{Opposite}}{\text{Hypotenuse}}$, rather than a ratio, Opposite : Hypotenuse.

Note

You need to learn the three ratios.

$\sin\theta = \frac{O}{H}$

$\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A}$

There are various ways of remembering these, but one of the most popular is to learn the 'word' 'SOHCAHTOA'.

Note

Label the sides in the order hypotenuse, opposite, adjacent.

To identify the opposite side, go straight out from the middle of the angle. The side you hit is the opposite.

You can shorten the labels to 'H', 'O' and 'A'.

θ is the Greek letter 'theta'.

Using the ratios 1

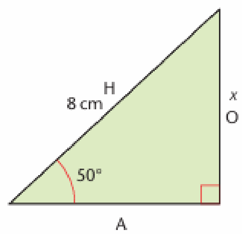
When you need to solve a problem using one of the ratios, you should follow these steps.

- Draw a clearly labelled diagram.
- Label the sides H, O and A.
- Decide which ratio you need to use.
- Solve the equation. In one type of problem you will encounter, you are required to find the numerator (top) of the fraction. This is demonstrated in the following examples.

Example 41.4

Question

Find the length marked x.



Solution

Since you know the hypotenuse (H) and want to find the opposite (O), you use the sine ratio.

$\sin 50° = \frac{O}{H}$

$\sin 50° = \frac{x}{8}$

$8 \times \sin 50° = x$ Multiply both sides by 8.

$x = 6.128\,35\ldots = 6.13$ cm correct to 3 significant figures.

Note

Press these keys on your calculator to find x.

8 × sin 5 0 =

Note

Make sure that your calculator is set to degrees.

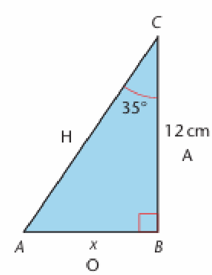
This is the default setting but, if you see 'rad' or 'R' or 'grad' or 'G' in the window, change the setting using the key labelled 'mode' or 'DRG' or 'set up'.

Example 41.5

Question

In triangle ABC, $BC = 12$ cm , angle $B = 90°$ and angle $C = 35°$.

Find the length AB.



Solution

Draw the triangle and label the sides.

Since you know the adjacent (A) and want to find the opposite (O), you use the tangent ratio.

$\tan 35° = \frac{O}{A}$

$\tan 35° = \frac{x}{12}$

$12 \times \tan 35° = x$ Multiply both sides by 12.

$x = 8.402\,49\ldots = 8.40$ cm correct to 3 significant figures.

Note

Press these keys on your calculator to find x.

1 2 × tan 3 5 =

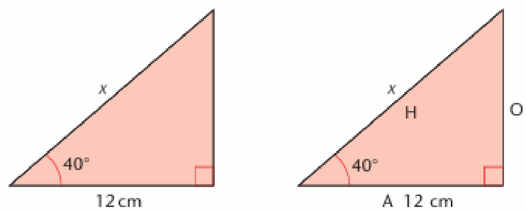
Using the ratios 2

In the second type of problem you will encounter, you are required to f ind the denominator (bottom) of the fraction. This is demonstrated in the following example.

Example 41.6

Question

Find the length marked x.



Solution

Since you know A and want to find H, you use the cosine ratio.

$\cos 40° = \frac{A}{H}$

$\cos 40° = \frac{12}{x}$

$x = \frac{12}{\cos 40°}$

$x = 15.66488\ldots = 15.7$ cm correct to 3 significant figures.

Note

Always look to see whether the length you are trying to find should be longer or shorter than the one you are given.

If your answer is obviously wrong, you have probably multiplied instead of divided.

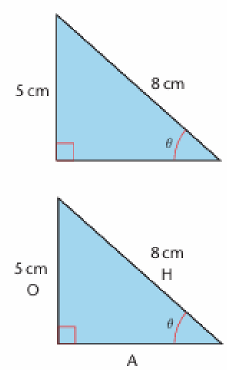
Using the ratios 3

In the third type of problem you will encounter, you are given the value of two sides and are required to find the angle. This is demonstrated in the following examples.

Example 41.7

Question

Find the angle θ.



Solution

This time, look at the two sides you know.

Since they are O and H, you use the sine ratio.

$\sin\theta = \frac{O}{H}$ $\sin\theta = \frac{5}{8}$

Work out $5 \div 8 = 0.625$ on your calculator and leave this number in the display.

Now use the $\sin^{-1}$ function (the inverse of sine).

With 0.625 still in the display, press SHIFT sin =, or the equivalent on your calculator.

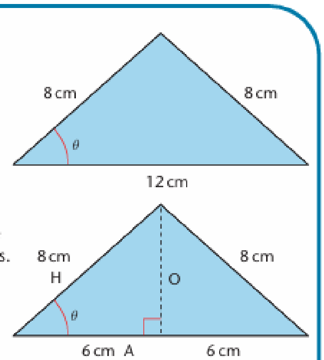
You should see 38.68218… .

So $\theta = 38.7°$ correct to 3 significant figures, or 39° correct to the nearest degree.

Example 41.8

Question

Find the angle θ.



Solution

Since this is an isosceles triangle, not a right-angled triangle, you need to use the fact that the line of symmetry splits an isosceles triangle into two equal right-angled triangles.

The sides you know are A and H so you use the cosine ratio.

$\cos\theta = \frac{A}{H} = \frac{6}{8}$

$\theta = \cos^{-1}\frac{6}{8}$

$\theta = 41.4°$ correct to 3 significant figures.

Note

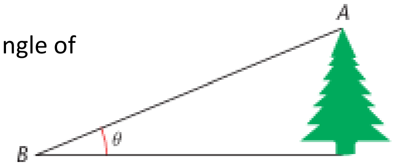
Example 41.8 shows how to deal with isosceles triangles.

You use the line of symmetry to split the triangle into two equal right-angled triangles.

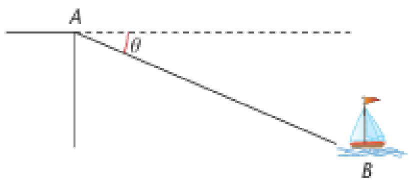
This works only with isosceles triangles, because they have a line of symmetry.

Angle of elevation and angle of depression

The angle that a line to an object makes above a horizontal plane is called the angle of elevation. The angle of elevation of the top of this tree, A, from B, is θ.



The angle a line makes to an object below a horizontal plane is called the angle of depression.



The angle of depression of the boat, B, from A, is θ.

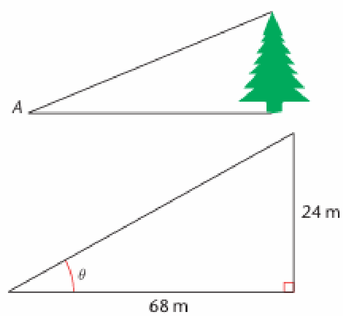
Note
Since alternate angles are equal, the angle of elevation of A from B equals the angle of depression of B from A.

Example 41.9

Question

The height of this tree is 24 m.
A is 68 m from the foot of the tree.

Find the angle of elevation of the top of the tree from A.



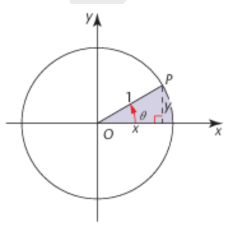
Solution

$\tan\theta = \frac{24}{68}$

$\theta = 19.4°$

The sine and cosine functions for obtuse angles

So far, you have only dealt with the sine and cosine functions for right-angled triangles, so the angles have all been acute. However, your calculator will give you the sine and cosine of any angle.



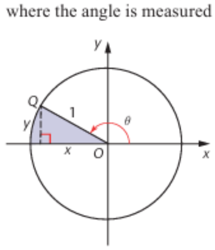
In this diagram you can see that for an acute angle

$\cos\theta = \frac{x}{1}$ so $x = \cos\theta$

$\sin\theta = \frac{y}{1}$ so $y = \sin\theta$

So P has coordinates $(\cos\theta, \sin\theta)$.

For other angles, the trigonometric functions are defined in a similar way, where the angle is measured anticlockwise from the x-axis.



By symmetry, Q has coordinates $(-\cos\theta, \sin\theta)$.

So

$\sin\theta = \sin(180 - \theta)$

$\cos\theta = -\cos(180° - \theta)$

By symmetry, $\sin\theta = y = \sin(180° - \theta)$
and $\cos\theta = x = -\cos(180° - \theta)$.

So for an obtuse angle θ, $\sin\theta$ is equal to $\sin(180° - \theta)$
and $\cos\theta$ takes the same value as $\cos(180° - \theta)$ but is negative.

Example 41.10

Question

You are given that sin 40° = 0.6428 and cos 40° = 0.7660.
Without using your calculator, write down the values of

a sin 140°

b cos 140°.

Solution

a sin 140° = sin (180° − 40°) = sin 40° = 0.6428

b cos 140° = cos (180° − 40°) = −cos 40° = −0.7660

Check these values by finding sin 140° and cos 140° directly on your calculator.

Example 41.11

Question

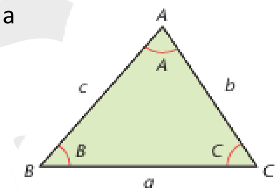
Solve the equation sin x = 0.8 for 0° ≤ x ≤ 180°.

Solution

Using a calculator gives x = 53.13°, so this is one solution.
But sin x is also positive for values of x between 90° and 180°.
So another solution is 180° − 53.13° = 126.87°.
So x = 53.13° or 126.87°

Non-right-angled triangles

All the trigonometry that you have learned so far has been based on finding lengths and angles in right-angled triangles. However, many triangles are not right-angled. You need a method to find lengths and angles in these other triangles.

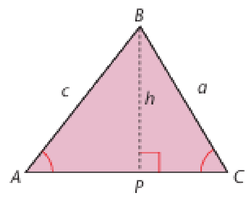


For simplicity, we shall use a single letter to represent each side and each angle of the triangle, a capital letter for an angle and a lowercase letter for a side. The side opposite an angle takes the same, lowercase letter, as shown in the diagram above.

There are two rules for dealing with non-right-angled triangles; they are called the sine rule and the cosine rule. Learn the formulas so that you can use them easily but you do not need to memorise them.

The sine rule

The sine rule is derived from the trigonometry in right-angled triangles that you have already met.



Use triangle BCP to find an expression for h in terms of a and C.

$\frac{h}{a} = \sin C$ so $h = a \sin C$

Use triangle ABP to find an expression for h in terms of c and A.

$\frac{h}{c} = \sin A$ so $h = c \sin A$

Equating the expressions for h gives

$h = a \sin C = c \sin A$

Dividing both sides by ($\sin A \sin C$) gives

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Note
You do not need to be able to prove the sine rule.

Similarly, by drawing a perpendicular from point C to side AB we can show that

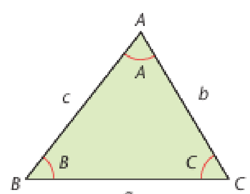
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

So the full sine rule is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

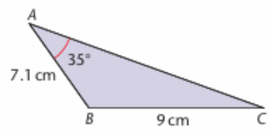
or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



To use the sine rule you must know one side and the angle opposite it. You also need to know one other angle or one other side.

Example 41.13



Question

Find C.

Solution

Since you are finding an angle, use the formula with angles on top.

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{7.1} = \frac{\sin 35^\circ}{9}$$

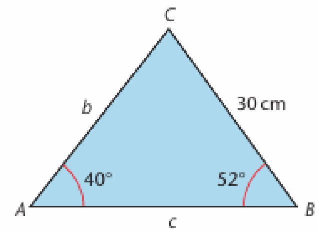
$$\sin C = \frac{\sin 35^\circ}{9} \times 7.1$$

$$\sin C = 0.4524\ldots$$

$$C = \sin^{-1}(0.4524\ldots)$$

$$C = 26.9^\circ \text{ to 1 decimal place}$$

Example 41.12



Question

Find these.

a Length b **b** Length c

Solution

a Since you are finding a length, use the formula with lengths on top.

Choose pairs of angles and opposite sides where three of the four values are known and substitute into the formula.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 52^\circ} = \frac{30}{\sin 40^\circ}$$

$$b = \frac{30}{\sin 40^\circ} \times \sin 52^\circ$$

$$b = 36.8 \text{ cm to 1 decimal place}$$

b Before you can find c, you need to find angle C.

$$C = 180 - (40 + 52)$$

$$C = 88^\circ$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 88^\circ} = \frac{30}{\sin 40^\circ}$$

$$c = \frac{30}{\sin 40^\circ} \times \sin 88^\circ$$

$$c = 46.6 \text{ cm to 1 decimal place}$$

Note

Although you could use the pair b and B instead of a and A, you should, whenever possible, use values that are given rather than values that have been calculated.

The ambiguous case of the sine rule

Construct accurately triangle ABC, given that AC = 7 cm, CB = 5 cm and angle A is 30°. There are two possible triangles that you can draw with this information.

Using the sine rule to calculate angle B, with $a = 5$ cm, $b = 7$ cm and $A = 30°$.

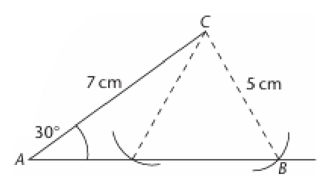
$$\frac{\sin B}{7} = \frac{\sin 30}{5}$$

$$\sin B = 7 \times \frac{\sin 30}{5}$$

$$= 0.7$$

$$B = 44.4 \text{ or } (180 - 44.4)$$

$$= 44.4^\circ \text{ or } 135.6^\circ$$

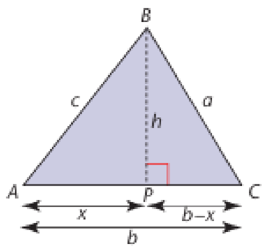


In this triangle, the obtuse angle also gives a possible triangle, since 135.6° + 30° is less than 180. Then C = 14.4°. Using B = 44.4° gives C = 105.6°. In triangles you have met in the past, this did not happen.

The ambiguous case can only happen when you are finding an angle opposite the longest side you are given. This is because the largest angle in a triangle is opposite the longest side.

The smallest angle in a triangle is opposite the shortest side. Use these facts with the sine rule to help you check whether you need to consider the ambiguous case.

The cosine rule



To derive the cosine rule, you combine right-angled trigonometry with Pythagoras' theorem.

Use Pythagoras' theorem for triangle *BPA*.

$h^2 + x^2 = c^2$ so $h^2 = c^2 - x^2$.

Use Pythagoras' theorem for triangle *BPC*.

$h^2 + (b - x)^2 = a^2$ so $h^2 = a^2 - (b - x)^2$.

Equate the two expressions for h^2 $\quad h^2 = a^2 - (b - x)^2 = c^2 - x^2$

Expand the bracket and simplify $\quad a^2 - (b^2 - 2bx + x^2) = c^2 - x^2$

$$a^2 - b^2 + 2bx - x^2 = c^2 - x^2$$

$$a^2 - b^2 + 2bx = c^2$$

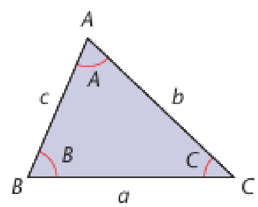
Rearrange $\quad a^2 = c^2 + b^2 - 2bx$

But $\frac{x}{c} = \cos A$ so $x = c\cos A$ $\quad$ giving $\quad a^2 = c^2 + b^2 - 2bc\cos A$

This is the cosine rule.

Note
You do not need to be able to prove the cosine rule.

Changing the letters and rearranging gives the six formulas for the cosine rule.

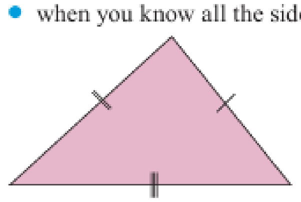


$$a^2 = c^2 + b^2 - 2bc\cos A$$
$$b^2 = c^2 + a^2 - 2ca\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

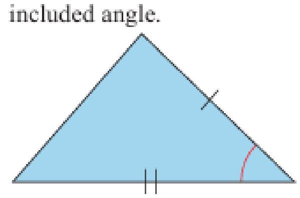
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Use the cosine rule

- when you know all the sides



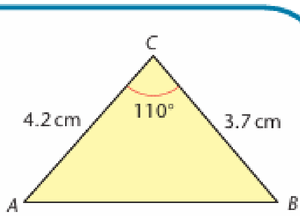
- when you know two sides and the included angle.



Example 41.14

Question

Find *c*.



Solution

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 3.7^2 + 4.2^2 - (2 \times 3.7 \times 4.2 \times \cos 110°)$

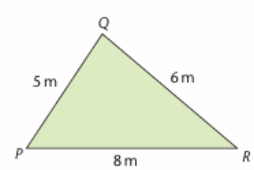
$c^2 = 41.959\ldots$

$c = 6.48$ cm to 2 decimal places

Example 41.15

Question

Find *R*.



Solution

$\cos R = \frac{p^2 + q^2 - r^2}{2pq}$

$\cos R = \frac{6^2 + 8^2 - 5^2}{2 \times 6 \times 8}$

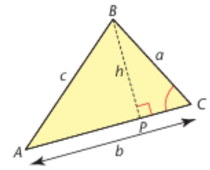
$\cos R = 0.78125$

$R = \cos^{-1}(0.78125)$

$R = 38.6°$ to 1 decimal place

The general formula for the area of any triangle

To find the area of a triangle when you do not know the perpendicular height, you can derive a formula from the trigonometry of right-angled triangles.



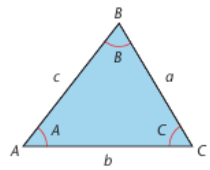
Area of triangle $ABC = \frac{1}{2} \times b \times h$

But $\frac{h}{a} = \sin C$ so $h = a \sin C$

Substituting this expression in the formula for the area of a triangle gives

Area of triangle $ABC = \frac{1}{2}ab \sin C$

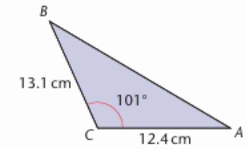
$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$
$$= \frac{1}{2}bc \sin A$$
$$= \frac{1}{2}ca \sin B$$



Example 41.16

Question

Find the area of the triangle shown.



Solution

Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 13.1 \times 12.4 \times \sin 101°$

$= 79.7\,\text{cm}^2$ to 1 decimal place

Finding lengths and angles in three dimensions

You can find lengths and angles of three-dimensional objects by identifying right-angled triangles within the object and using Pythagoras' theorem or trigonometry.

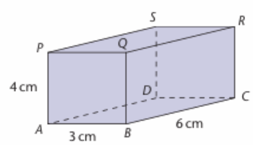
Example 41.17

Question

A rectangular box measures 4 cm by 3 cm by 6 cm.

Calculate these.

a AC **b** AR **c** Angle RBC **d** Angle ARC



Solution

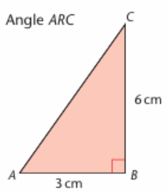
a Identify the right-angled triangle required and draw a sketch.

$AC^2 = AB^2 + BC^2$

$AC^2 = 3^2 + 6^2$

$AC^2 = 45$

$AC = 6.71$ cm to 2 decimal places



Note

Don't use 6.71^2 as this has been rounded.

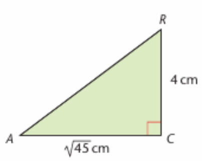
You need AC^2 and this is 45.

b $AR^2 = AC^2 + RC^2$

$AR^2 = 45 + 16$

$AR^2 = 61$

$AR = 7.81$ cm to 2 decimal places



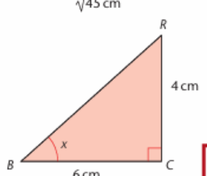
Note

When there is a choice of formulas to use, use the one which contains as many given values as possible. You may have calculated a value incorrectly.

c $\tan x = \frac{4}{6}$

$x = \tan^{-1}\frac{4}{6}$

$x = 33.7°$ to 1 decimal place



Note

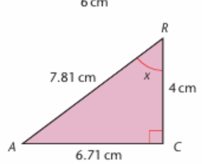
You can't avoid using a calculated value here.

Using 6.71 will give an answer correct to 3 significant figures but it is good practice to use the more accurate value, $\sqrt{45}$, and round at the end.

d $\tan x = \frac{\sqrt{45}}{4}$

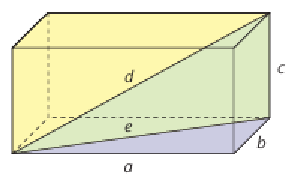
$x = \tan^{-1}\frac{\sqrt{45}}{4}$

$x = 59.2°$ to 1 decimal place



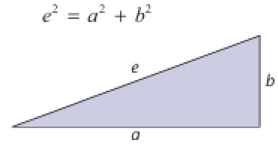
In Example 41.17, notice how you used the result of part **a** to work out the result of part **b**.

You can use this method to derive a general formula for the length of the diagonal of a cuboid measuring a by b by c.

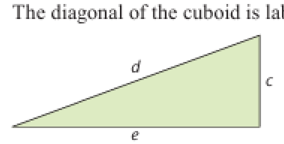


The diagonal of the rectangular base is labelled e.

$$e^2 = a^2 + b^2$$



The diagonal of the cuboid is labelled d.



$d^2 = e^2 + c^2$

But $e^2 = a^2 + b^2$

so

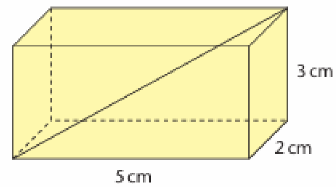
$d^2 = a^2 + b^2 + c^2$

$d = \sqrt{a^2 + b^2 + c^2}$

Example 41.18

Question

Find the length of the diagonal of this cuboid.



Solution

The length of the diagonal of a cuboid = $\sqrt{a^2+b^2+c^2}$.

In this cuboid a = 5 cm, b = 2 cm and c = 3 cm.

Length of diagonal = $\sqrt{5^2+2^2+3^2}$

$= \sqrt{38}$

= 6.2 cm to 1 decimal place

Example 41.19

Question

A tree, TC, is 20 m north of point A.

The angle of elevation of the top of the tree, T, from A, is 35°.

Point B is 30 m east of point A.

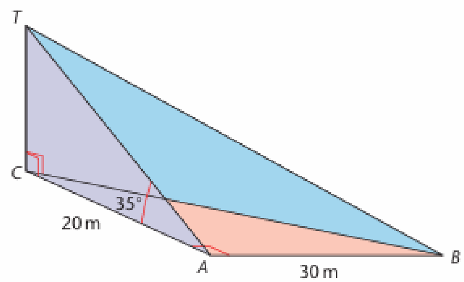
A, B and C are on horizontal ground.

Calculate

a the height of the tree, TC

b the length BC

c the angle of elevation of T from B.



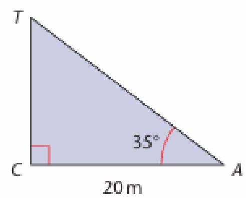
Solution

First, draw a diagram.

a $\tan 35° = \frac{TC}{20}$

$TC = 20 \tan 35°$

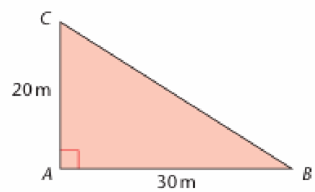
TC = 14.0 m to 1 decimal place



b $BC^2 = 20^2 + 30^2$

$BC^2 = 1300$

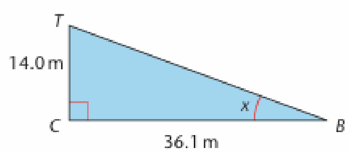
BC = 36.1 m to 1 decimal place



c $\tan x = \frac{14.0\ldots}{36.1\ldots}$

$x = \tan^{-1}\left(\frac{14.0\ldots}{36.1\ldots}\right)$

x = 21.2° to 1 decimal place



Example 41.20

Question

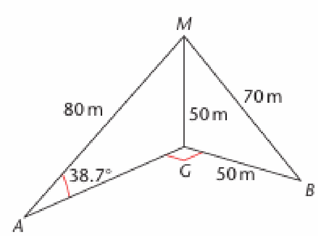
A mast, MG, is 50 m high.

It is supported by two ropes, AM and BM, as shown.

ABG is horizontal.

Other measurements are shown on the diagram.

Is the mast vertical?



Solution

If $M\hat{G}A$ is a right angle then $\sin 38.7° = \frac{50}{80}$.

$\sin^{-1}\left(\frac{50}{80}\right) = 38.68\ldots$

So $M\hat{G}A$ is a right angle.

If $M\hat{G}B$ is a right angle then the side lengths of triangle MGB will fit Pythagoras' theorem.

$50^2 + 50^2 = 5000$

$\sqrt{5000} = 70.7\ldots$

MB is shorter than this, so $M\hat{G}B$ is not a right angle.

The mast leans towards B ($M\hat{G}B < 90°$).

The angle between a line and a plane

To find the angle between a line and a plane, you need to identify the correct angle. The shortest distance between a point and a plane is the perpendicular distance to it. Use this fact to identify a right-angled triangle. The hypotenuse will be the given line. One side will be a line in the plane. The angle you need is the angle between these lines. In this diagram, the angle between the line XY and the plane ABCD is the angle YXP.

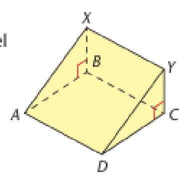
Example 41.21

Question

For this triangular prism, sketch the triangle and label the angle between the line and the plane given.

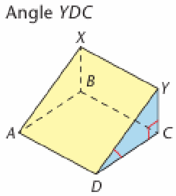
a DY and $ABCD$ **c** AY and $BCYX$

b AY and $ABCD$ **d** BY and ABX

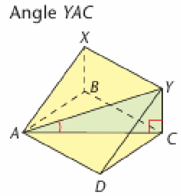


Solution

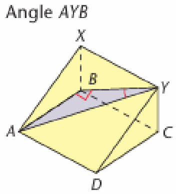
a Angle YDC



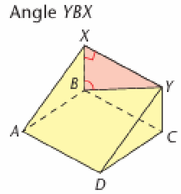
b Angle YAC



c Angle AYB



d Angle YBX



Example 41.22

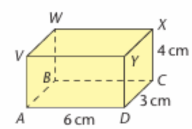
Question

$ABCDVWXY$ is a cuboid.

Calculate the angle between the following lines and planes.

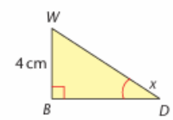
a DW and $ABCD$

b DW and $ABVW$



Solution

a Angle BDW is the required angle.

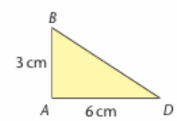


First, find length BD.

$BD^2 = 6^2 + 3^2$

$BD^2 = 45$

$BD = \sqrt{45}$ cm

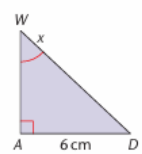


$\tan x = \frac{4}{\sqrt{45}}$

$x = \tan^{-1}\frac{4}{\sqrt{45}}$

$x = 30.8°$ to 1 decimal place

b Angle DWA is the required angle.



First, find length WA.

$WA^2 = 4^2 + 3^2$

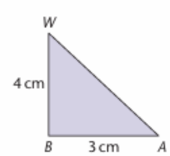
$WA^2 = 25$

$WA = 5$ cm

$\tan x = \frac{6}{5}$

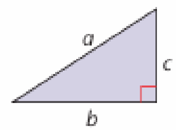
$x = \tan^{-1}\frac{6}{5}$

$x = 50.2°$ to 1 decimal place



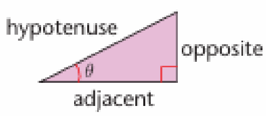
Key points

- The angle that a line to an object makes above a horizontal plane is called the angle of elevation.
- The angle that a line to an object makes below a horizontal plane is called the angle of depression.
- The shortest distance between a point and a line is the perpendicular distance.
- In right-angled triangles
  - Use Pythagoras' theorem when you know two sides and need to find the third.
    $a^2 = b^2 + c^2$



  - Use trigonometry when you need to find an angle or you need to find a side when you know one side and an angle.

$$\sin\ \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \cos\ \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \tan\ \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$



- In triangles without right angles
  - For obtuse angles, $\sin\ \theta = \sin\ (180 - \theta)$ and $\cos\ \theta = -\cos\ (180 - \theta)$.
  - The sine rule is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

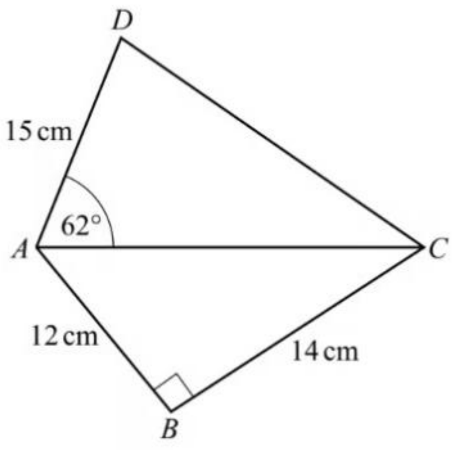
    Use the sine rule to find lengths and angles when you know
    - any two angles and a side
    - two sides and an angle opposite one of these.
  - The cosine rule is
    $c^2 = a^2 + b^2 - 2ab\cos C$ or $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$
    Use the cosine rule
    - to find the third side when you know two sides and the angle between them
    - to find an angle when you know all three sides.
  - The area of a triangle is given by Area $= \dfrac{1}{2}ab\sin C$.
- In three dimensions
  - The length of the diagonal of a cuboid with dimensions a, b and c is $\sqrt{a^2 + b^2 + c^2}$.
  - To solve problems in three dimensions, identify a right-angled triangle containing the length or angle you need to find. Draw a separate sketch of this triangle. Then, use Pythagoras's theorem and/or trigonometry, as needed.

Note

You do not need to memorise the formulas for the sine and cosine rule and the area of a triangle but it is a good idea to learn them so that you can use them easily.

Revision questions

1.

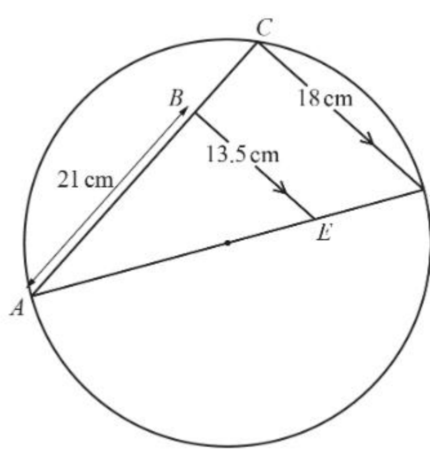


NOT TO SCALE

The diagram shows a quadrilateral, $ABCD$, formed from two triangles, ABC and ACD.
ABC is a right-angled triangle.

Calculate angle BAC.

2.

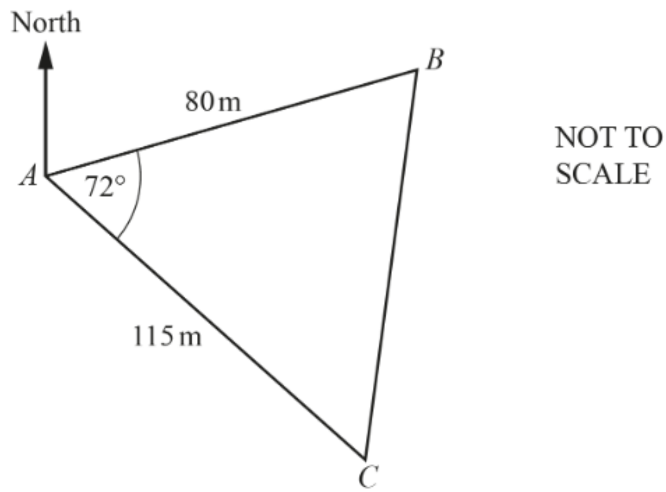


NOT TO SCALE

C lies on a circle with diameter AD.
B lies on AC and E lies on AD such that BE is parallel to CD.
AB = 21 cm, CD = 18 cm and BE = 13.5 cm.

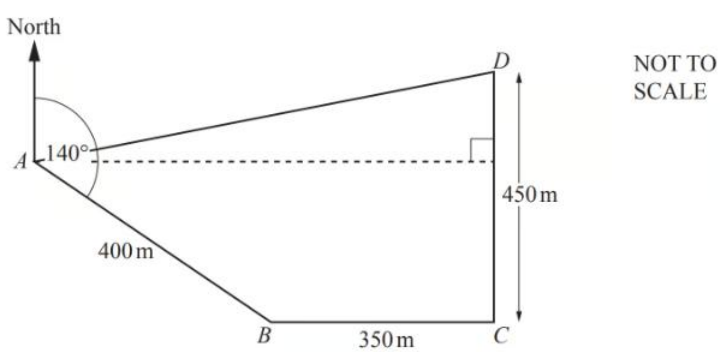
Work out the radius of the circle.

3.



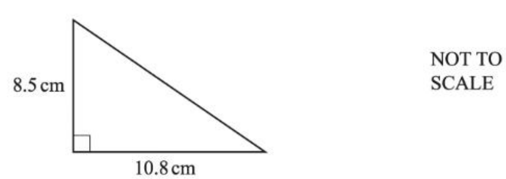
The diagram shows the positions of three points A, B and C in a field.
Calculate the shortest distance from point B to AC.

4.



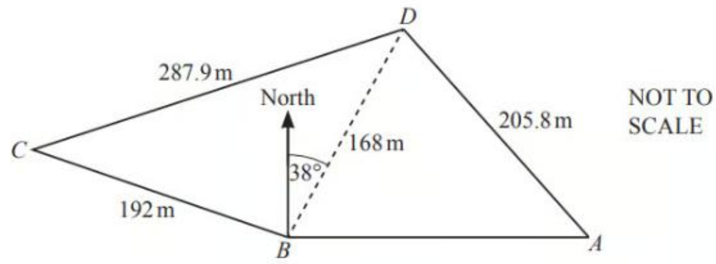
The diagram shows a field $ABCD$.
The bearing of B from A is 140°.
C is due east of B and D is due north of C.
$AB = 400$m, $BC = 350$m and $CD = 450$m.
Calculate the distance from D to A.

5.



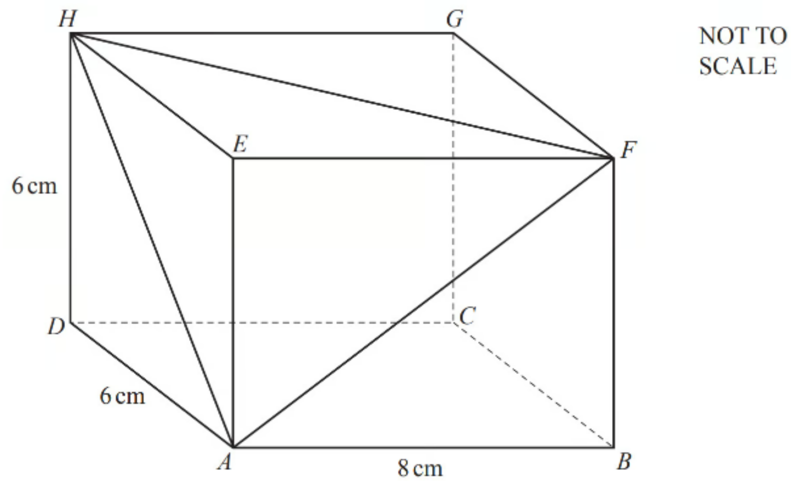
The diagram shows a right-angled triangle.
Calculate the perimeter.

6.



The diagram shows a field, *ABCD*, on horizontal ground.
BC = 192 m, *CD* = 287.9 m and *BD* = 168 m.
Calculate angle *CBD* and show that it rounds to 106.0°, correct to 1 decimal place.

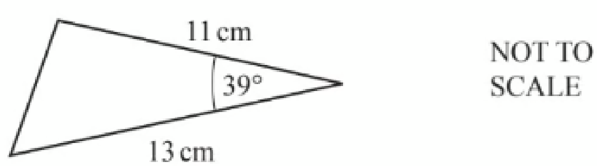
7.



The diagram shows a cuboid.
AB = 8 cm, *AD* = 6 cm and *DH* = 6 cm.

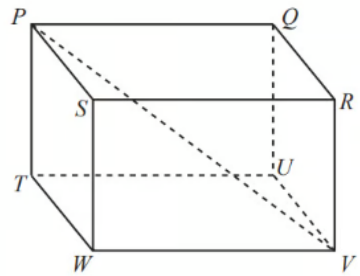
Calculate angle *HAF*.

8.



Calculate the area of the triangle.

9.



NOT TO SCALE

The diagram shows a cuboid $PQRSTUVW$.

$PV = 17.2$ cm

The angle between the line PV and the base $TUVW$ of the cuboid is $43°$.

Calculate PT.

10.

The diagram shows a cuboid $ABCDEFGH$.

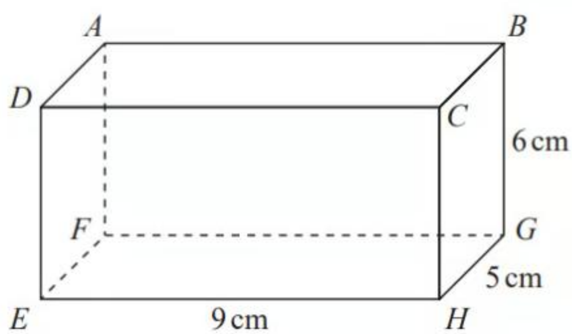


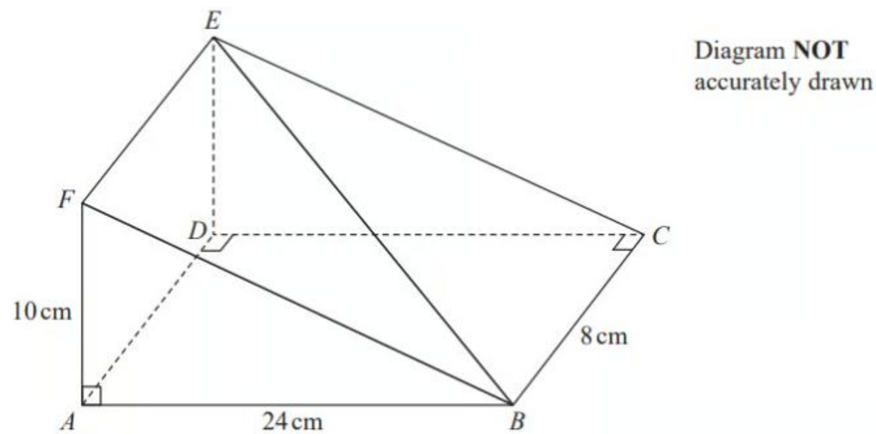
Diagram **NOT** accurately drawn

$EH = 9$ cm, $HG = 5$ cm and $GB = 6$ cm.

Work out the size of the angle between AH and the plane $EFGH$.
Give your answer correct to 3 significant figures.

11.

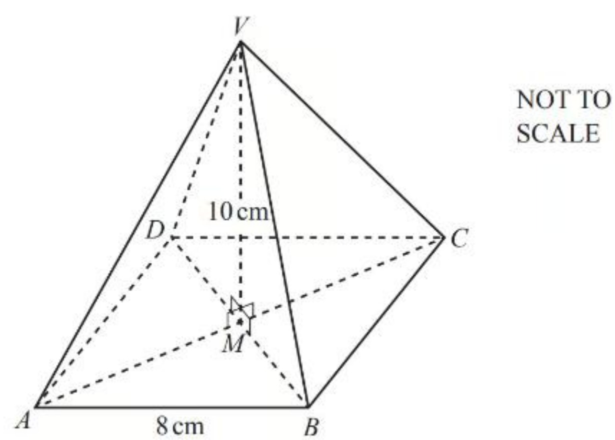
The diagram shows a triangular prism.



$AF = 10$ cm, $AB = 24$ cm and $BC = 8$ cm.
Angle FAB = angle ADC = angle $BCD = 90°$

Work out the size of the angle between the line BE and the plane $ABCD$.
Give your answer correct to 1 decimal place.

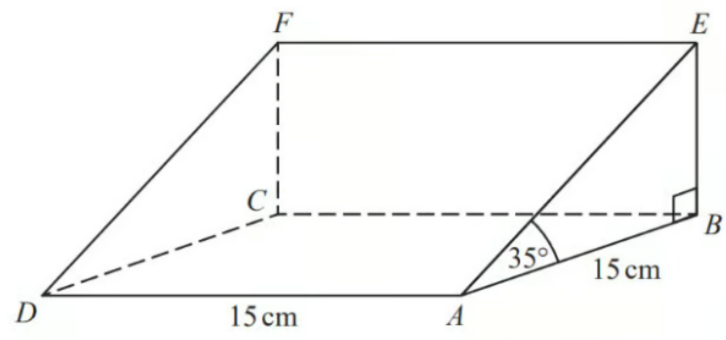
12.



The diagram shows a pyramid with a square base $ABCD$ of side length 8cm.
The diagonals of the square, AC and BD, intersect at M.
V is vertically above M and $VM = 10$ cm.
Calculate the angle between VA and the base.

13.

The diagram shows a triangular prism.



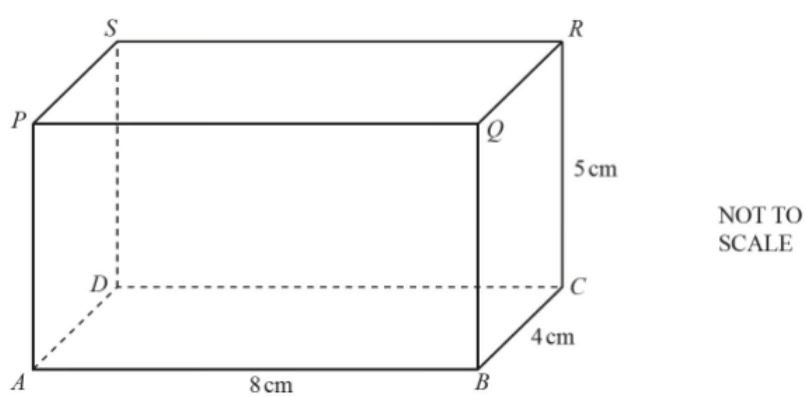
The base, $ABCD$, of the prism is a square of side length 15 cm.
Angle ABE and angle CBE are right angles.
Angle $EAB = 35°$

M is the point on DA such that

$$DM:MA = 2:3$$

Calculate the size of the angle between EM and the base of the prism.
Give your answer correct to 1 decimal place.

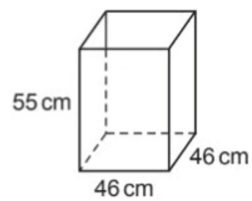
14.



The diagram shows a cuboid.
$AB = 8$ cm, $BC = 4$ cm and $CR = 5$ cm.
Calculate the angle between the diagonal AR and the plane $BCRQ$.

15.

Alvin has a crate in the shape of a cuboid.
The crate is open at the top.
The internal dimensions of the crate are 46cm long by 46cm wide by 55cm high.



Alvin has a stick of length 95cm.
Alvin places the stick in the crate so that the shortest possible length extends out above the top of the crate.

Calculate the length of the stick that extends out of the crate.

16.

The diagram shows a solid pyramid $ABCDE$ with a horizontal base.

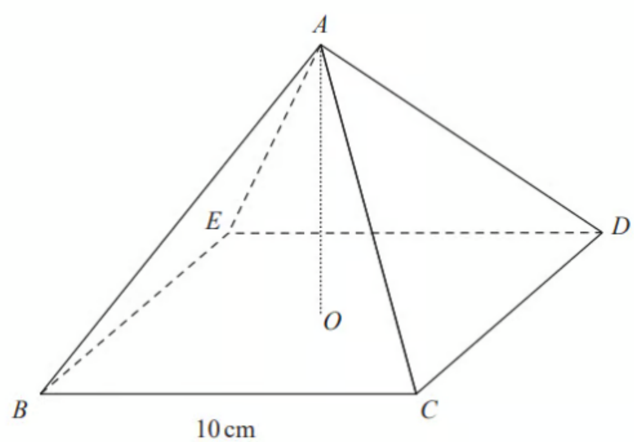


Diagram **NOT** accurately drawn

The base, $BCDE$, of the pyramid is a square of side 10 cm.

The vertex A of the pyramid is vertically above the centre O of the base so that
$AB = AC = AD = AE$

The **total** surface area of the pyramid is 360 cm^2

Work out the size of the angle between AC and the base $BCDE$.
Give your answer correct to 3 significant figures.