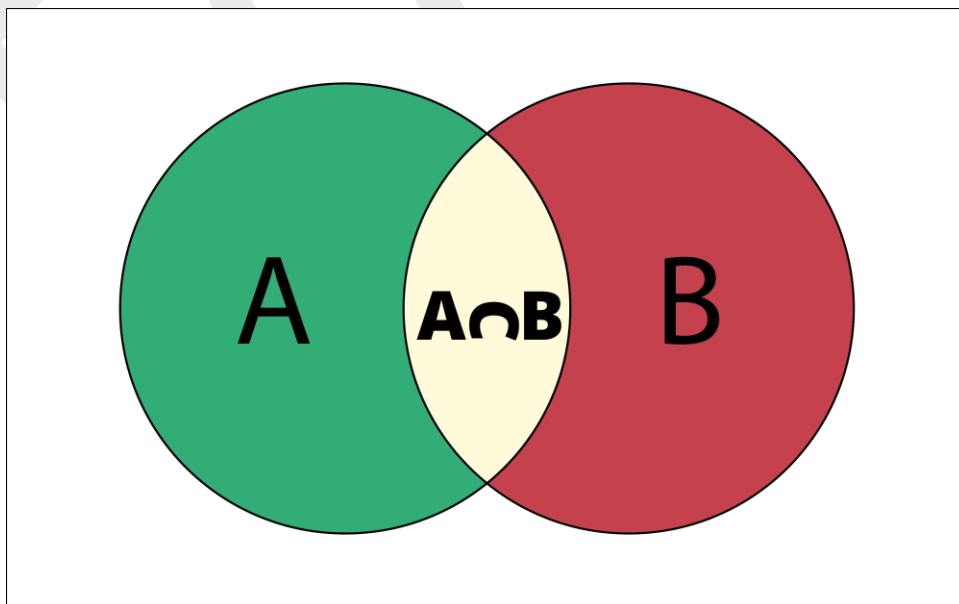# Cambridge OL

Mathematics

CODE: (4024)

Chapter 02

Sets

The definition of a set

- A set is a collection of numbers, shapes, letters, points or other objects.

- They form a set because they fulfil certain conditions.

- The notation for a set is a pair of curly brackets: {…}.

- The individual members of a set are called **elements**.

- A set can be defined by giving a rule which satisfies all the elements, or by giving a list of the elements.

The universal set

- All the sets in Exercise 2.1 are **finite sets**. This means that they have a fixed number of elements.

- Sets can also have an infinite number of elements.

- Most of the sets you will be dealing with will be finite sets.

- Sometimes we make sure we are dealing with a finite set by defining a **universal set.**

- A universal set is a set from which – for a particular situation – all other sets will be taken.

- Similarly, if you define the universal set as positive integers less than 12, then the set in question 2 could have been defined as {prime numbers}.

Notation

The symbol for a universal set is $\mathscr{E}$

The symbol $\in$ means 'is an element of'.

The symbol $\notin$ means 'is not an element of'.

So　　　$3 \in$ {prime numbers}

and　　　$4 \notin$ {prime numbers}.

Venn diagrams

- Venn diagrams are a way of showing sets and the relationships between sets.

- Venn diagrams were introduced in 1880 by John Venn.

- In a Venn diagram, the universal set is shown by a rectangle. Other sets are drawn as circles or ovals within the rectangle.

- The diagrams here show three typical Venn diagrams.

- Venn diagrams are not restricted to two sets; they can have three or more sets.

- Sometimes Venn diagrams need to be drawn and the elements of the sets filled in.
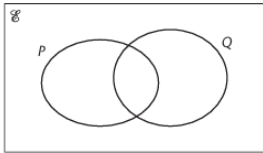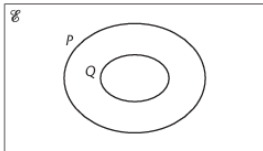
This Venn diagram shows two sets, P and Q, where there are some elements that are in both sets.
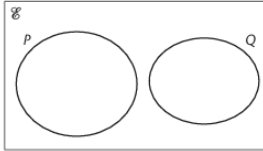
This Venn diagram shows two sets, P and Q, where there are no elements that are in both sets.

Sets P and Q are **disjoint**.

This Venn diagram shows two sets, P and Q, where all the elements of set Q are also in set P.

Set Q is a **subset** of set P.

Example 2.1

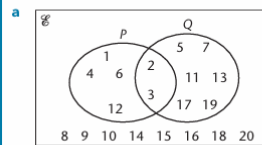$\mathscr{E}$ = {integers from 1 to 20}   P = {factors of 12}   Q = {prime numbers}

Question

a Draw a Venn diagram to show the elements of the universal set and its subsets P and Q.
b List the elements that are in both set P and set Q.

Solution

a

Note
Do not forget to fill in the elements of $\mathscr{E}$ that are in neither P nor Q.

b 2, 3

We do not put an element in a Venn diagram more than once.

The relationship between sets

- The intersection of two sets, P and Q, is all the elements that are in both set P and set Q.

- You can write this as P ∩ Q.

- The **union** of two sets, P and Q, is all the elements that are in set P or set Q or both.

- You can write this as P ∪ Q.

- If a set has no elements, it is called the empty set.

- The symbol for the empty set is ∅.

- It may seem a trivial idea, but it is quite important when dealing with some sets.

The complement of a set

A' means those elements of the universal set which are not in set A.

It is called the complement of A.

Subsets

A set with 4 elements has 16 or 2^4 subsets.

In general, a set with n elements has 2^n subsets.

Example 2.2

Question

On separate copies of the diagram, shade these sets.
a P'
b (P ∩ Q)'
c (P ∪ Q)'

Solution

a

b

c
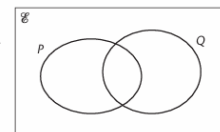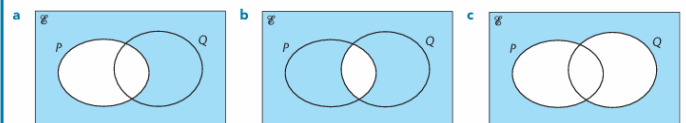
Example 2.3

Question

Find all the subsets of P = {a, b, c, d}.

Solution

Subsets with one element:	{a}, {b}, {c}, {d}
Subsets with two elements:	{a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}
Subsets with three elements:	{a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}

In addition to these 14 subsets,

the set P itself is regarded as a subset: {a, b, c, d}

the empty set also is regarded as a subset: Ø

This makes 16 subsets in total.

Notation

The symbol $\subseteq$ means 'is a subset of'.

So $A \subseteq B$ means set A is a subset of set B. It includes the possibility that set A could be the set B itself or the empty set.

The symbol $\not\subseteq$ means 'is not a subset of'. So $A \not\subseteq B$ means set A is not a subset of set B.

Problem solving with Venn diagrams

If A is a set, then n(A) means the number of elements in set A.

When you are using a Venn diagram to solve a problem, you can write the number of elements in the subsets rather than filling in all the elements.

Example 2.4

Question

There are 32 students in a class.

They can choose to study history (*H*), or geography (*G*), or both or neither:

- 18 study history
- 20 study geography
- 8 study both history and geography.

a Draw a Venn diagram to show this information.

b Find the number of students who study neither history nor geography.

Solution

a Since 8 students study both,

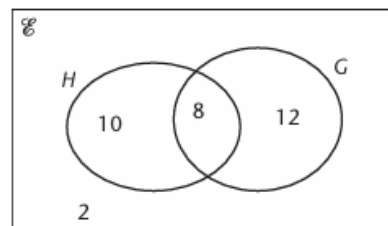10 students study history but not geography

12 students study geography but not history.

b The number of students who study history, or geography or both is

10 + 8 + 12 = 30.

So the number of students who study neither history nor geography is

32 − 30 = 2.

Alternative ways to define a set

If a set is finite, you can define it by listing its elements.

However, if a set is infinite, you cannot list all the elements, so you define it by giving the rule used to form it.

There are various ways to do this. For example,

$A = \{x : x \text{ is a natural number}\}$

$B = \{(x, y) : y = 2x + 1\}$

$C = \{x : 2 \leqslant x \leqslant 5\}$

Three-set problems

Example 2.5

Question

In a sixth form of 200 students, three of the subjects students can study are mathematics (M), technology (T) and psychology (P).

110 students study mathematics, 85 study technology and 70 study psychology

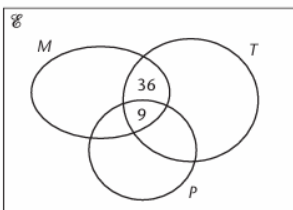45 study mathematics and technology   19 study technology and psychology

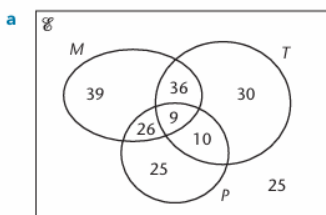35 study mathematics and psychology   9 study all three subjects.

▶

a Copy and complete this Venn diagram.
b Find
  i   the number of students who study none of the three subjects
  ii $n(M \cap T \cap P')$
  iii $n[M \cap (T \cup P)]$.

Solution

a

b i $200 - (39 + 36 + 9 + 26 + 30 + 10 + 25) =$
       $200 - 175 = 25$
   ii 36
   iii $36 + 9 + 26 = 71$

Key points
- A set can be defined by giving a rule or description, or by listing the elements; for example, $\{1, 2, 3, 4\}$, $\{(x, y): y = mx + c\}$.
- Venn diagrams can be used to represent sets and their elements.
- Set notation:

Universal set	$\mathscr{E}$
Empty set	$\varnothing$
Number of elements in set A	$n(A)$
Is an element of	$\in$
Is not an element of	$\notin$
Complement of set A	A'
A is a subset of B	$A \subseteq B$
A is not a subset of B	$A \nsubseteq B$
Union of A and B	$A \cup B$
Intersection of A and B	$A \cap B$

Revision questions

1) 4024/12/M/J/23/Q19

(a) In the Venn diagram, shade the region represented by $(A \cap B') \cup (B \cap C')$
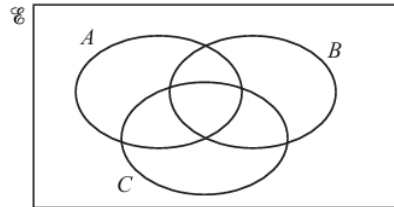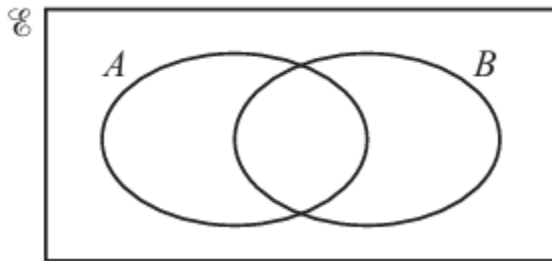

(b) One morning 50 people visit a library.

• 35 of them borrow a book.

• 12 of them use a computer.

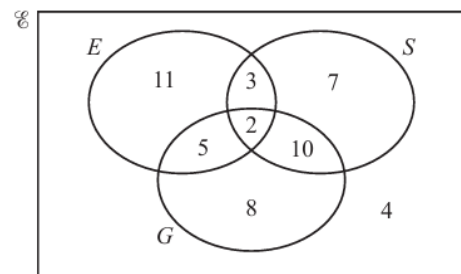• 8 of them do not borrow a book and do not use a computer.

Using a Venn diagram, or otherwise, find the number of people who use a computer but do not borrow a book.

2) 4024/11/O/N/22/Q9

(a) In the Venn diagram, shade the region represented by A∩ B

(b) This Venn diagram shows information about the number of students who study English (E), Spanish (S) and German (G).

i) Find the number of students who study English and German but not Spanish.

(ii) Find n (GUS)'

3) (a) The sets A and B are shown on the Venn Diagram in the answer space.
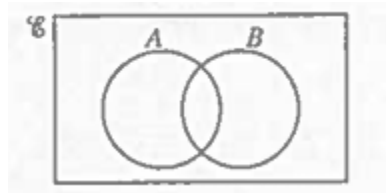The element y is such that y € A and y € B. On the diagram, write y in the correct region.

(b) ξ= {x: x is an integer and 1 ≤ x ≤ 8}.
P= {x: x > 5}.
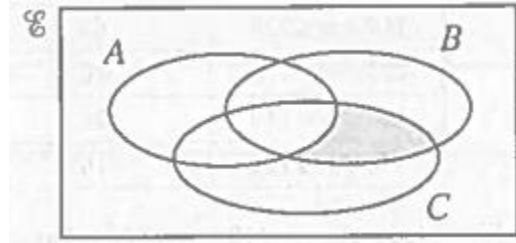Q= {x: x ≤3}.
(i) Find the value of n(PUQ).
(ii) List the elements of P'∩Q'

4) (a) Express, in set notation, as simply as possible, the subset shaded in the Venn diagram.

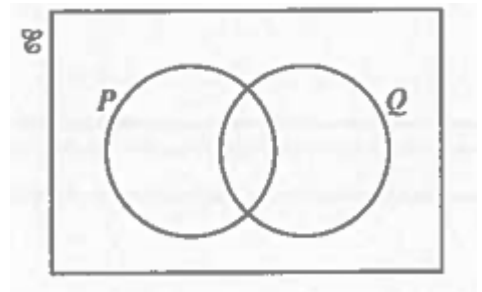(b) It is given that n (ξ) 40. n(P) = 18.
n(Q) 20 and n(P∩Q) =7.
Find
(i) n(PUQ),
(ii) n(P'∩Q').

5) (a) On the Venn diagram shown in the answer space, shade the set PUQ'.

(b) There are 27 children in a class.
Of these children, 19 own a bicycle, 15 own a scooter and 3 own neither a bicycle nor a scooter.
Using a Venn diagram, or otherwise, find the number of children who own a bicycle but not a scooter.

6)(a) ξ= {1,2,3, 4, 5.}
A= {1,2,3).
B= (5).
C= (3,4).

List the elements of
(i) AUC
(ii) BUC'
(b) A group of 60 children attend an after-school club. Of these, 35 children play football and 29 play hockey. 3 children do not play either football or hockey.
By drawing a Venn diagram, or otherwise, find the number of children who play only hockey.

6) (a) ={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)

L= {x:x is an odd number}

M = {x:x is a multiple of 3} (i) Write down

(a) L∩M.

(b) L'UM.

(ii) A number n is chosen at random from.

Find the probability that n € L U M.

(b) In a survey, a number of people were asked "Do you own a car?" and "Do you own a bicycle?". The Venn diagram shows the set C of car owners and the set B of bicycle owners.

The letters p, q and x are the numbers of people in each subset.

11 people owned neither a car nor a bicycle.
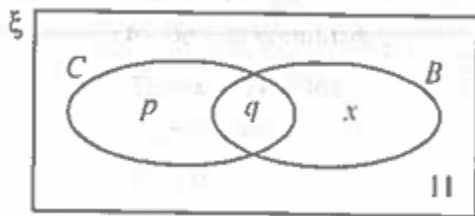
A total of 66 people owned a car.

4 times as many people owned a car only as owned a bicycle only.

(i) Write down expressions, in terms of x, for

(a) p,

(b) q.

(ii) A total of 27 people owned a bicycle.

Calculate

(a) x,

(b) the total number of people who were in the survey.

7) (1,2,3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]

A= (x: x is a multiple of 3}
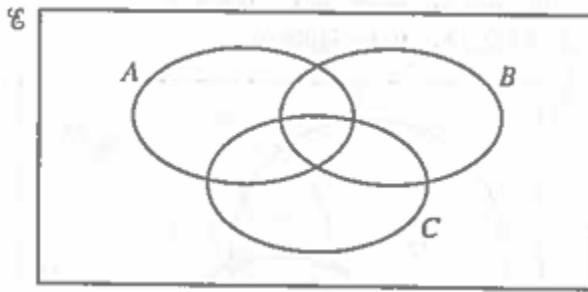
B= {x: x is a factor of 24)

C= (x: x is an odd number)

(i) Find

(a) n(B).

(b) (AUBUC)'

(ii) A number, k, is chosen at random from ξ

Find the probability that k € A ∩ B.

9) On the venn diagram, shade the set A∩B∩C'

(b) = (2, 3, 4, 5, 6, 7, 8, 9, 10)
P = {x:x is a prime number}
Q= {x:x25}
(i) Find the value of n(P∩Q).
(ii) List the elements of PUQ')

10) In a survey, 60 students are asked which of the subjects Biology (B),
History (H) and Spanish (S) they are studying.
The Venn diagram shows the results. 27 students study History.

(a) Find the values of p and q.
(b) Find n(H').
(c) Find n((BUH)∩S').