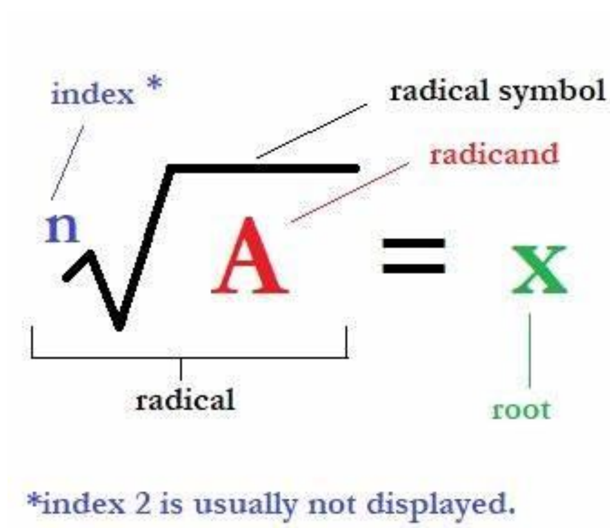
Cambridge OL

Mathematics

CODE: (4024)

Chapter 03 and Chapter 04

Power and roots and Fractional, decimals and percentages



Squares and square roots

As you can see in the diagram below, the square with side 3 has an area of 3 × 3 = 9 squares and the square with side 4 has an area of 4 × 4 = 16 squares.

The integers 1, 4, 9, 16, 25, … are the squares of the integers 1, 2, 3, 4, 5, … .

Because $16 = 4^2$, the positive square root of 16 is 4. It is written as $\sqrt{16} = 4$.

Similarly $\sqrt{36} = 6$ and $\sqrt{81} = 9$.

You can use your calculator to find squares and square roots.

Example 3.1

Question

Work out these using a calculator.

a 47^2 **b** $\sqrt{729}$

Note

Some calculators operate slightly differently. Learn which keys you need to use to do these operations on your calculator.

Solution

a $47^2 = 2209$ On your calculator, you need to press 4 7 $x^{\blacksquare}$ 2 =

b $\sqrt{729} = 27$ On your calculator, you need to press √ 7 2 9 =

You also need to be able to find squares and square roots without a calculator. Here is a list of the squares of the integers 1 to 15. You should learn these square numbers.

You will also need them for finding square roots without a calculator.

Integer	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square number	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Example 3.2

Question

Work out $8^2 - 4^2$.

Solution

Work out the squares first.

$8^2 = 8 \times 8 = 64$

$4^2 = 4 \times 4 = 16$

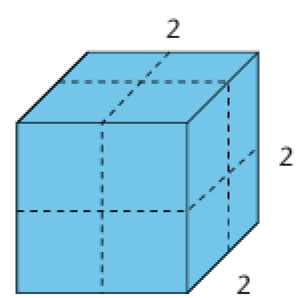
So $8^2 - 4^2 = 64 - 16$

$= 48$

Cubes and cube roots

The cube in the diagram has a volume of 2 × 2 × 2 = 8.

The cube of a number is the number multiplied by itself, and then by itself again.



The integers 1, 8, 27, 64, 125, 216, … are the cubes of the integers 1, 2, 3, 4, 5, 6, ….

Because $8 = 2^3$ the cube root of 8 is 2. It is written

Similarly $\sqrt[3]{27} = 3$ and $\sqrt[3]{64} = 4$.

You need to be able to find cubes and cube roots of some numbers without a calculator.

Here is a list of the cubes of the integers 1 to 5, and 10. You should learn these cube numbers.

Integer	1	2	3	4	5	10
Cube number	1	8	27	64	125	1000

Example 3.3

Question

Work out $\sqrt[3]{125}$ without a calculator.

Solution

$\sqrt[3]{125} = 5$ You know that $5^3 = 125$, so you also know that the cube root of 125 is 5.

You can also use your calculator to find cubes and cube roots.

Example 3.4

Question

Work out these using a calculator.

a 15^3 **b** $\sqrt[3]{4913}$

Solution

a $15^3 = 3375$ On your calculator press $\boxed{1}\boxed{5}\boxed{x^3}\boxed{=}$

b $\sqrt[3]{4913} = 17$ On your calculator press $\boxed{\sqrt[3]{\ }}\boxed{4}\boxed{9}\boxed{1}\boxed{3}\boxed{=}$

Note

Some calculators operate slightly differently.

Learn which keys you need to use to do these operations on your calculator.

Note

You also need to be able to use your calculator to find other powers and roots.

On many calculators these buttons are labelled $x^{\blacksquare}$ and $\sqrt[\blacksquare]{x}$.

Find them and learn how to use them to work out results such as $2.5^4 = 39.0625$ and $\sqrt[5]{693.43957} = 3.7$

Key points

- A square number is the result of multiplying a number by itself.
- 4^2 means 4×4. So, $4^2 = 16$.
- The square root of a number is the positive number that multiplies by itself to give that number.
- $\sqrt{\ }$ means 'the square root of'. So, $\sqrt{16} = 4$.
- A cube number is the result of multiplying a number by itself, and then by itself again. $4^3 = 4 \times 4 \times 4 = 64$.
- The fact that $4^3 = 64$ means also that the cube root of 64 is 4. This is written as $\sqrt[3]{64} = 4$.
- You should know the square numbers for 1^2 to 15^2. For example, know that $15^2 = 225$ and that $\sqrt{225} = 15$.
- You should know how to find and use the square root and cube root buttons on your calculator.
- You should know how to find and use the buttons on your calculator for other powers and roots.

Chapter 04

Fractions

- A fraction is a number written in the form a /b, where a and b are integers.
- The value on the top of the fraction is known as the **numerator**.
- The value on the bottom of the fraction is known as the **denominator**.
- The fraction 4/7 is a **proper fraction** because the numerator is smaller than the denominator.
- The fraction 7/4 is an **improper fraction** because the numerator is larger than the denominator.
- The fraction $1\frac{3}{4}$ is a **mixed number** because it is formed from an integer and a proper fraction.
- An improper fraction can be written as a mixed number

$$\frac{7}{4} = \frac{4}{4} + \frac{3}{4} = 1\frac{3}{4}$$

Example 4.1

Question

a Write $\frac{17}{6}$ as a mixed number. **b** Write $3\frac{4}{5}$ as an improper fraction.

Solution

a Divide the numerator by the denominator and write the remainder as a fraction over the denominator.

$17 \div 6 = 2$ remainder 5

so $\frac{17}{6} = 2\frac{5}{6}$

b Multiply the integer by the denominator of the fraction and add the numerator.

$3 \times 5 + 4 = 19$

so $3\frac{4}{5} = \frac{19}{5}$

Note

The denominator of the mixed number is the same as the denominator of the improper fraction.

Fraction of a quantity

- A fraction can be used to describe a share of a quantity.
- The denominator shows how many parts the quantity is divided into.
- The numerator shows how many of those parts are required.

Example 4.2

Question

a Work out $\frac{1}{8}$ of 56. **b** Work out $\frac{5}{8}$ of 56.

Solution

a Finding $\frac{1}{8}$ of 56 is the same as dividing 56 into 8 parts.

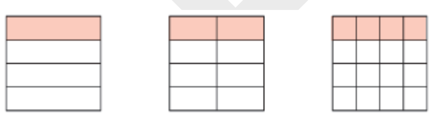
So $\frac{1}{8}$ of $56 = 56 \div 8 = 7$

b $\frac{5}{8}$ means $5 \times \frac{1}{8}$.

So $\frac{5}{8}$ of $56 = 5 \times 7 = 35$

Equivalent fractions

These squares can be divided into equal parts in different ways.



The fraction represented by the shaded parts is $\frac{1}{4}$ or $\frac{2}{8}$ or $\frac{4}{16}$.

These three fractions are equal in value and are **equivalent fractions**.

$\frac{1}{4} = \frac{2}{8} = \frac{4}{16}$ but $\frac{1}{4}$ is in its **simplest form**.

Example 4.3

Question

Write each fraction in its simplest form.

a $\frac{6}{10}$ **b** $\frac{18}{24}$

Solution

a The highest common factor of 6 and 10 is 2, so divide the numerator and denominator by 2.

$\frac{6}{10} = \frac{3}{5}$ in its simplest form.

b The highest common factor of 18 and 24 is 6, so divide the numerator and denominator by 6.

$\frac{18}{24} = \frac{3}{4}$ in its simplest form.

Note

18 and 24 also have common factors of both 2 and 3, so $\frac{9}{12}$ and $\frac{6}{8}$ are also equivalent to $\frac{18}{24}$, but $\frac{3}{4}$ is the simplest form of this fraction.

Fractions and decimals

We can use place value in a decimal to convert the decimal to a fraction.

Example 4.4

Question

Convert the decimal 0.245 to a fraction in its simplest form.

Solution

Units	.	Tenths	Hundredths	Thousandths
0	.	2	4	5

$0.245 = \frac{2}{10} + \frac{4}{100} + \frac{5}{1000}$

$= \frac{200}{1000} + \frac{40}{1000} + \frac{5}{1000}$ Convert each fraction to its equivalent with a denominator of 1000.

$= \frac{245}{1000}$ The HCF of 245 and 1000 is 5, so divide the numerator and the denominator by 5 to simplify the fraction.

$= \frac{49}{200}$

0.245 is equivalent to $\frac{49}{200}$

Note

You can use the place value of the final digit to write the decimal directly as a single fraction. The final digit here, 5, represents thousandths so 0.245 is equivalent to $\frac{245}{1000}$.

Terminating and recurring decimals

- In Chapter 1, you learnt that both terminating and recurring decimals were rational numbers.
- So, both terminating and recurring decimals can be written as fractions.
- Also, all fractions can be written as a terminating or a recurring decimal.
- You can convert the fraction 5/8 to a decimal using division.
- 5/8 = 5÷8=0.625
- This is a terminating decimal because it finishes at the digit 5.
- You can convert the fraction 1/ 6 to a decimal using division. 1/ 6 =1÷6=0.166666...
- This is a recurring decimal because the digit 6 repeats indefinitely.

Example 4.5

Question

a Convert $\frac{13}{25}$ to a decimal.

b Convert $\frac{7}{11}$ to a decimal.

c Convert $0.\dot{6}$ to a fraction using $\frac{1}{3} = 0.\dot{3}$.

d Convert $0.\dot{3}\dot{6}$ to a fraction.

Solution

a $\frac{13}{25} = 13 \div 25 = 0.52$

b $\frac{7}{11} = 7 \div 11 = 0.636\,363\ldots$ In this case the digits 6 and 3 recur so you can write the answer using dot notation.

$\frac{7}{11} = 0.\dot{6}\dot{3}$

c $0.\dot{6} = 2 \times 0.\dot{3} = 2 \times \frac{1}{3} = \frac{2}{3}$

d Let $N = 0.363\,636\ldots$

Multiply each side by 100

$100\,N = 36.363\,636\ldots$

Subtract

$99\,N = 36,\ N = \frac{36}{99} = \frac{4}{11}$

Key points

- A proper fraction has a numerator smaller than the denominator.
- An improper fraction has a numerator larger than the denominator. It can also be written as a mixed number formed from an integer and a proper fraction.
- Fractions of equal value are equivalent.
- The fraction with no common factors in its numerator and denominator is in its simplest form.
- Fractions are equivalent to terminating or recurring decimals.
- Percentages are fractions written out of 100.
- Equivalent fractions, decimals and percentages can be converted from one to another.

Dot notation for recurring decimals

Dot notation can be used when writing recurring decimals. Dots are placed over the digits that recur.

Fractions, decimals and percentages

The term per cent means 'out of 100'.

For example 75% means 75 out of every 100 or $\frac{75}{100}$.

$\frac{75}{100}$ can be written in decimal form as 0.75.

So 75% is equivalent to $\frac{75}{100}$ and 0.75.

You can find fraction and decimal equivalents of all percentages.

There is some fraction, decimal and percentage equivalents that are useful to remember.

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%

Example 4.6

Question

a Convert $\frac{3}{8}$ to a percentage.

b Convert 65% to a fraction in its simplest form.

Solution

a $\frac{3}{8} = 3 \div 8 = 0.375$ Convert to a decimal by dividing.

$0.375 \times 100 = 37.5$ Multiply by 100 for percentage.

So $\frac{3}{8} = 37.5\%$

b $65\% = \frac{65}{100}$ The HCF of 65 and 100 is 5, so divide the numerator and the denominator by 5 to simplify the fraction.

$65\% = \frac{13}{20}$

Note

You can convert directly from a fraction to a percentage using multiplication.

Revision questions

1)

(a) Express 180 as the product of its prime factors.

(b) $\sqrt{180}$ can be expressed in the form $p\sqrt{q}$, where p and q are integers. Find the smallest value of $p + q$.

2)

(a) Write 0.0040751 correct to two significant figures. [1]

(b) $\sqrt{131}$ lies between two consecutive integers. Complete the inequality below with these integers.

.... $< \sqrt{131} <$ .... [1]

(c) Add brackets to the statement below to make it correct.

$3 \times 2 + 1^2 = 49$ [1]

3) By making suitable approximations, estimate the value of $\frac{\sqrt{35.78} \times \sqrt[3]{1005}}{0.3012}$.
Show clearly the approximate values you use. [2]

4) (a) Write the value of 1234.567, correct to 2 significant figures. [1]

(b) Write down an estimate for the value of $\sqrt{\frac{28}{\pi}}$.

5) By writing each number correct to one significant figure, estimate the value of

$$\frac{29.3^2}{2.04 \times 0.874}.$$

6)

Fraction		Decimal		Percentage
$\frac{1}{2}$	=	0.5	=	50%
$\frac{3}{20}$	=	....	=	....
....	=	....	=	62.5%

7) (a) Calculate 5% of $280 000.
(b) A single carton of juice costs $4.20.
A special offer pack of 3 cartons costs $9.45. Ali bought a special offer pack instead of 3 single cartons.
Calculate his percentage saving.

8) (a) In 2005, the cost of posting a letter was 28 cents." A company posted 1200 letters and was given 4% discount on the cost.
Calculate the total discount.
(b) In 2006, the cost of posting a letter was creased from 28 cents to 35 cents.
Calculate the percentage increase in the cost of posting a letter.
(c) After the price increase to 35 cents, the cost to the company of posting 1200 letters was $399. Calculate the percentage discount that the company was given in 2006.
(d) In 2006, it cost $4.60 to post a parcel.
This was an increase of 15% on the cost of posting the parcel in 2005.
Calculate the cost of posting this parcel in 2005.

10) (a) A jar contained 370 g of jam. Usman ate 30% of the jam.
What mass of jam remained in the jar?

(b) In 2006 the population of a town was 30 000. This was 5000 more than the population in 1999. Calculate the percentage increase in population.

11) (a) Anne's digital camera stores its images on a memory card.
The memory card has 128 units of storage space. When 50 images were stored, there were 40 units of unused storage space on the memory card.

(i) Calculate the percentage of unused storage space on the memory card.

(ii) Calculate the average amount of storage space used by each image.

(b) Shop A charged 60 cents for each photograph.
Shop B charged 63 cents for each photograph and gave a discount of $1 on all purchases more than $10.

(i) Anne bought 24 photographs from Shop A and paid with a $20 note.
Calculate the change she received.

(ii) Find how much cheaper it was to buy 24 photographs from Shop B than from Shop A.

(iii) Find the smallest number of photographs for which it was cheaper to use Shop B.