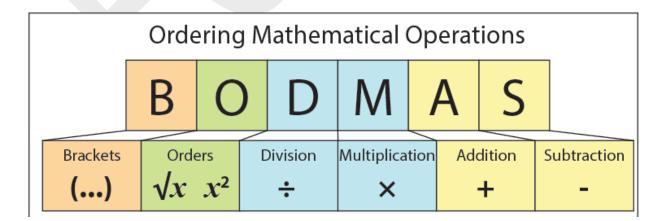


Cambridge OL Mathematics

CODE: (4024)

Chapter 05 and chapter 06

Ordering and The four operations





Ordering integers

Example 5.1

Question

Put these masses in order, smallest first.

1.2kg 1500g 175g 2kg 0.8kg

Solution

Write each value with the same units first. (It is usually easier to use the smallest unit.)

1200g 1500g 175g 2000g 800g

Then order them, writing your answer using the original units.

So order is

175 g 0.8 kg 1.2 kg 1500 g 2 kg.

Inequalities

a < b means 'a is less than b'.

 $a \le b$ means 'a is less than or equal to b'.

a > b means 'a is greater than b'.

 $a \ge b$ means 'a is greater than or equal to b'.

Ordering decimals

Example 5.2

Question

Put these decimals in order of size, smallest first.

Solution

Add zeros to make the decimals all the same length.

0.4120 0.0059 0.3250 0.0460 0.0120

Now, remove the decimal points and any zeros in front of the digits.

4120 59 3250 460 120

The order in size of these values is the order in size of the decimals.

The order is

0.0059 0.012 0.046 0.325 0.412

Note

As you get used to this, you can omit the second step.

Ordering factions

To put fractions in order, convert them to equivalent fractions all with the same denominator, and order them by the numerator.

Example 5.3

Question

Which is the bigger fraction, $\frac{3}{4}$ or $\frac{5}{6}$?

Solution

First, find a common denominator. 24 is an obvious one, as $4 \times 6 = 24$, but a smaller one is 12. $\frac{3}{4} = \frac{9}{12}$ $\frac{5}{6} = \frac{10}{12}$

 $\frac{10}{12}$ is bigger than $\frac{9}{12}$, so $\frac{5}{6}$ is bigger than $\frac{3}{4}$.

Note

Multiplying the two denominators together will always work to find a common denominator, but the lowest common multiple of the denominators is sometimes smaller.



Alternatively, you can convert each fraction to a decimal and compare the decimals as before.

Note

than 0.03.

A common error is to

write 3% as 0.3, rather

Example 5.4

Question

Put these fractions in order, smallest first.

$$\frac{3}{10}$$
 $\frac{1}{4}$ $\frac{9}{20}$ $\frac{2}{5}$ $\frac{1}{2}$

Solution

Use division to convert the fractions to decimals ($3 \div 10$, $1 \div 4$, etc.).

Now make the number of decimal places the same.

So order is
$$\frac{1}{4}$$
 $\frac{3}{10}$ $\frac{2}{5}$ $\frac{9}{20}$ $\frac{1}{2}$.

Note

Try both methods shown in the examples. The first is useful when calculators are not allowed.

Ordering fractions, decimals and percentages

Example 5.5

Question

Put these numbers in order, smallest first. 3% 0.41 0.35

Convert them all to decimals.

0.25, 0.03, 0.41, 0.275, 0.35 So the order is

0.03 0.25

0.275 0.35 Write the numbers in their original form.

0.35

Key points

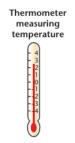
- Know what the symbols =, ≠, >, <, ≥, ≤ mean and how to use them.
- When ordering values with different metric units, change each value so they all have the same unit.
- When comparing decimals, make them all have the same number of digits after the decimal point by adding zeros to the ends.
- When comparing fractions, change each so they all have the same denominator. You can also compare fractions by changing each to a decimal and then comparing the decimals.
- The easiest way to compare a mixture of fractions, decimals and percentages is to change each to a decimal and then compare the decimals.

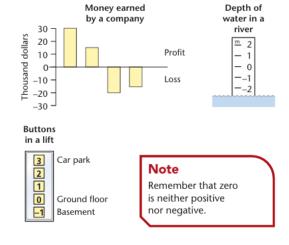
Chapter 06

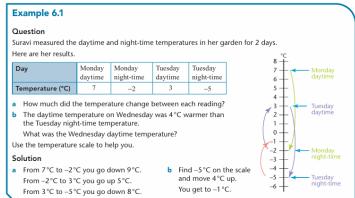
Numbers below zero

Some numbers are less than zero. These are called negative numbers. They are written as ordinary numbers with a negative sign in front.

Negative numbers are used in many situations.









Example 6.2

Question

Abal's bank account is overdrawn by \$75.

How much must he put in this account for it to be \$160 in credit?

Solution

Abal's bank account is overdrawn by \$75.

This means Abal owes the bank \$75. It can be shown as -\$75.

\$160 in credit means that there is \$160 in the account. It can be shown as +\$160.

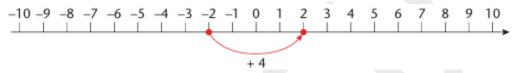
From -75 to +160 is 235, so he must put \$235 into the account.

Adding and subtracting with negative numbers

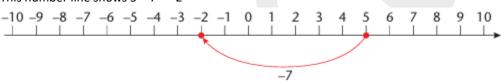
A negative number is a number less than zero.

A number line is very useful when adding or subtracting with negative numbers.

This number line shows -2 + 4 = 2



This number line shows 5 - 7 = -2



Patterns can also help.

$$-2+4=2$$
 $-2+3=1$ $-2+2=0$ $-2+1=-1$
 $-2+0=-2$ $-2-1=-3$ $-2-2=-4$
 $1-4=-3$ $1-3=-2$ $1-2=-1$ $1-1=0$
 $1-0=1$ $1--1=2$ $1--2=3$ $1--3=4$

This helps to show that

- Adding a negative number is the same as subtracting a positive
- Subtracting a negative number is the same as adding a positive number.

Example 6.3

Question

Work out these.

a
$$-2 + -6$$
 b $-2 - -5$

Solution

a -2 + -6 = -8 Adding a negative number is like subtracting a positive number.

So -2 + -6 is the same as -2 - 6.

b -2 - -5 = 3 Subtracting a negative number is like adding a positive number.

So -2 - -5 is the same as -2 + 5.

Multiplying and dividing with negative numbers

Look at these patterns.

This suggests these rules:



Example 6.4 Questions Work out these. a 6×-4 b -7×-3 c -5×8 Solution a $6 \times 4 = 24$ $(+ \times - = -)$ $50 \cdot 6 \times -4 = -24$ b $7 \times 3 = 21$ $(- \times - = +)$ $50 \cdot 7 \times -3 = 21$

From the previous page, you can see that $-3 \times 4 = -12$.

 $(-\times + = -)$

It follows that $-12 \div 4 = -3$ and $-12 \div -3 = 4$.

This suggests these rules:

 $5 \times 8 = 40$

So $-5 \times 8 = -40$

$$+ \div + = +$$
 and $- \div - = +$
 $+ \div - = -$ and $- \div + = -$

You now have a complete set of rules for multiplying and dividing positive and negative numbers.

$$+ \times + = +$$
 $- \times - = +$ $+ \div + = +$ $- \div - = +$ $+ \times - = - \div + = -$

Here is another way of thinking of these rules:

Signs different = answer is negative.

Signs the same = answer is positive.

You can extend the rules to calculations with more than two numbers.

If there is an even number of negative signs, the answer is positive.

If there is an odd number of negative signs, the answer is negative

Combining operations

As with positive numbers, multiplying and dividing is done before addition and subtraction, unless there are brackets.

Example 6.5 Question Work out these. c $5\times-4+3\times-2$ a $(-3 \times -4) + (-2 \times 3)$ **b** $6 - 2 \times 3$ Solution a $(-3 \times -4) + (-2 \times 3) = +12 + -6$ Brackets are not really needed here, as multiplication should be done first. = 12 - 6= 6 Do the multiplication first. **b** $6-2\times 3=6-6=0$ $\frac{5 \times -4 + 3 \times -2}{} = \frac{-20 + -6}{}$ Work out the numerator and the denominator separately. -6 + 4 $=\frac{-26}{-2}$ Then divide. = 13



Order of operations

When working out the answer to a calculation involving more than one operation, it is important that you use the correct order of operations.

For example, if you want to work out $2 + 3 \times 4$, should you carry out the addition or the multiplication first?

This is the correct order of operations when carrying out any calculation:

- · first, work out anything in brackets
- then, work out any powers (such as squares or square roots)
- then do any multiplication or division
- finally, do any addition or subtraction.

So, to work out $2 + 3 \times 4$, you should first do the multiplication, then the addition.

$$2 + 3 \times 4 = 2 + 12 = 14$$

If you want the addition to be done first, then brackets are needed in the calculation.

$$(2 + 3) \times 4 = 5 \times 4 = 20$$

Some calculations are written like fractions, 6 × 4

In this case, the fraction line works in the same way as brackets.

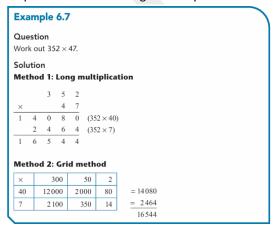
First, evaluate the numerator, then the denominator, and then do the division.

$$\frac{6 \times 4}{5 + 3} = \frac{24}{8} = 3$$

This calculation could also be written as $6 \times 4 \div (5 + 3)$.

Multiplying integers

There are several methods that can be used for multiplying three-digit by two-digit integers. These methods can be adapted for use with integers of any size.



	Example 6.6		
	Question Work out these. a $7 + 4 \div 2 - 5$ b $(6 - 2)^2 + 5 \times -3$		
	Solution		
	a 7 + 4 ÷ 2 – 5		
	= 7 + 2 - 5	First, work out the division.	
1	= 4	The addition and subtraction can be done in a single step.	
	b $(6-2)^2+5\times-3$		
	$= 4^2 + 5 \times -3$	First, work out the brackets.	
	$= 16 + 5 \times -3$	Next, work out the power.	
	= 16 + -15	Then the multiplication.	
	= 1	And finally the addition.	



Multiplying decimals

Use this method for multiplying two decimal numbers:

- · count the number of decimal places in the numbers you are multiplying
- ignore the decimal points and do the multiplication
- put a decimal point in your result so that there are the same number of decimal places in your answer as in the original calculation.

Dividing integers

There are several methods that can be used for dividing three-digit by two-digit integers. These methods can be adapted for use with integers of any size.

Example 6.9	
Question Work out 816 ÷ 34.	
Solution Method 1: Long division 816 ÷ 34 = 24	Method 2: Chunking 816 + 34 = 24
$ \begin{array}{c cccc} & 24 \\ 34 & 816 \\ & - & 680 \\ \hline & & 136 \\ & - & 136 \\ \hline & & 0 \\ \end{array} (34 \times 20) $	$ \begin{array}{r} 816 \\ -340 \\ \hline 476 \\ -340 \\ 136 \\ \hline 136 \\ 0 \end{array} $ (34×10) $ \begin{array}{r} 34 \\ 136 \\ \hline 0 \end{array} $ (34×4)
The answer is 24.	The answer is $10 + 10 + 4 = 24$.

Use this method for dividing by a decimal:

- first, write the division as a fraction
- · make the denominator an integer by multiplying the numerator and the denominator by the same power of 10
- cancel the fraction to its simplest form if possible; this gives you easier numbers to divide
- · divide to find the answer.

Example 6.8

Question

Work out these.

a 0.4 × 0.05 **b** 4.36 × 0.52

Solution

a 0.4× 0.05 There are three decimal places in the calculation, so there will be three in

 $4 \times 5 = 20$ Multiply the figures, ignoring the decimal points.

 $0.4 \times 0.05 = 0.020$ Insert a decimal point so that there are three decimal places in the answer. In this case we need to add an extra zero between the decimal point and the 2.

 $0.4 \times 0.05 = 0.02$

b 4.36 × 0.52 There are four decimal places in the calculation, so there will be four in

 $436 \times 52 = 22672$ Multiply the figures, ignoring the decimal points. Use whichever method

you prefe

 $4.36 \times 0.52 = 2.2672$ Insert a decimal point so that there are four decimal places in the answer

Dividing decimals

Example 6.10

Question

Work out these.

a 0.9 ÷ 1.5 **b** 25.7 ÷ 0.08

Solution

a $0.9 + 1.5 = \frac{0.9}{1.5}$ Write the division as a fraction

> Make the denominator an integer by multiplying both the numerator and the denominator by 10.

 $=\frac{3}{5}$ Simplify.

 $\frac{3}{5} = 5 \frac{0.6}{3.0}$ Divide, adding an extra zero after the decimal point to complete the division.

Write the division as a fraction.

Make the denominator an integer by multiplying both the numerator and the

denominator by 100.

Simplify.

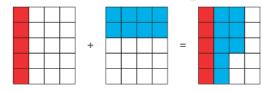
Use whichever method you prefer for the division.

 $\frac{1285}{4} = 4)1285.00$ Divide, adding extra zeros after the decimal point to complete the division.

25.7 + 0.08 = 321.25

Adding and subtracting fractions

In the diagram, each rectangle is divided into 20 small squares.



The diagram shows $\frac{1}{4} + \frac{2}{5}$.

The result of the addition has 13 squares shaded, or $\frac{13}{20}$

Use this method to add two fractions:

- change the fractions to equivalent fractions with the same denominator
- add the numerators.



$$\frac{1}{4} = \frac{5}{20} \text{ and } \frac{2}{5} = \frac{8}{20}$$
$$\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$$

The method is the same for subtracting two fractions. Change them to equivalent fractions with the same denominator, then subtract the numerators.

If the fractions are mixed numbers, change them to improper fractions first.

d $3\frac{1}{4} - 1\frac{5}{6}$

Example 6.11

Question

- Work out these. $\frac{2}{3} + \frac{5}{6}$
- Solution
- a $\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6}$

- The lowest common multiple of 3 and 6 is 6, so use 6 as the common denominator.
- The result is an improper fraction and not in its simplest form.
 - Simplify the fraction and write it as a mixed number.
 - The lowest common multiple of 4 and 3 is 12, so use 12 as the common

- First, change the mixed numbers to improper fractions.
- The lowest common multiple of 5 and 3 is 15.
 - The result is an improper fraction, so write it as a mixed number.
- First, change the mixed numbers to improper fractions.
- The lowest common multiple of 4 and 6 is 12.

In this diagram, five times as much is red, so $\frac{1}{8} \times 5 = \frac{5}{8}$.

The result is an improper fraction, so write it as a mixed number.

Note

You should always give your final answer as a fraction in its simplest form.

Multiplying fractions

Multiplying a fraction by a whole number

This shows that to multiply a fraction by an integer, you multiply the numerator by the integer. Then simplify by cancelling and changing to a mixed number if possible.



In this diagram, $\frac{1}{4}$ is red.

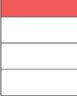
In this diagram, $\frac{1}{8}$ is red.

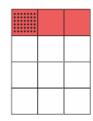
In this diagram, $\frac{1}{3}$ of the red is dotted.

This is $\frac{1}{12}$ of the original rectangle.

So
$$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$
.

To multiply fractions, multiply the numerators and multiply the denominators, then simplify if possible.







Example 6.12

Question

Work out these.

$$\frac{7}{12} \times 2$$

$$\frac{3}{4}$$
 ×

c
$$2\frac{2}{3} \times 1\frac{5}{6}$$

Note

Cancelling common factors

before multiplying makes the arithmetic simpler.

If you multiply first, you may have to cancel to give a fraction in its simplest form.

Solution

a
$$\frac{7}{12} \times 2 = \frac{14}{12}$$
 Multiply the numerator by 2.

$$=\frac{7}{6}=1\frac{1}{6}$$

Simplify the fraction and write it as a mixed number.

b
$$\frac{3}{4} \times \frac{6^3}{7} = \frac{3}{2} \times$$

Cancel the common factor of 2 in the numerator and the denominator.

$$=\frac{9}{14}$$

The result is a proper fraction in its simplest form.

c $2\frac{2}{3} \times 1\frac{5}{6} = \frac{8}{3} \times \frac{11}{6}$ First change the mixed numbers to

$$=\frac{\cancel{8}^4}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{3}}$$

 $=\frac{8^4}{3}\times\frac{11}{4^3}$ Cancel the common factor of 2 in the numerator and the denominator.

$$=\frac{4}{2} \times \frac{11}{2}$$

Multiply the numerators and multiply the denominators.

$$=\frac{44}{9}=4\frac{8}{9}$$

The result is an improper fraction, so write it as a mixed number.

Dividing fractions

When you work out 6 ÷ 3, you are finding how many 3s there are in 6.

When you work out $6 \div \frac{1}{3}$, you are finding out how many $\frac{1}{3}$ s there are in 6.

In this diagram, $\frac{1}{3}$ of the rectangle is red.



This diagram shows 6 of these rectangles.

You can see that 18 of the red squares will fit into this diagram.

So
$$6 \div \frac{1}{3} = 6 \times 3 = 18$$

You can see that dividing by $\frac{1}{3}$ is the same as multiplying by 3.

 $\frac{1}{3}$ is known as the **reciprocal** of 3.

This can be extended to division by a non-unit fraction.

For example, when you work out $4 \div \frac{2}{3}$, you are finding out how many $\frac{2}{3}$ s there are in 4.

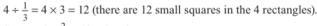
In this diagram, $\frac{2}{3}$ of the rectangle is red.



This diagram shows 4 of these rectangles.

You can see that 6 of these 2-square shapes will fit into this diagram.

You can think of the calculation in two steps.



So,
$$4 \div \frac{2}{3} = 12 \div 2 = 6$$

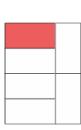
Or,
$$4 \div \frac{2}{3} = 4 \times \frac{3}{2} = \frac{12}{2} = 6$$
.

You can see that dividing by $\frac{2}{3}$ is the same as multiplying by $\frac{3}{2}$.

 $\frac{2}{3}$ is known as the reciprocal of $\frac{3}{2}$.

To find the reciprocal of any fraction, turn it upside down.

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.





Use this method for dividing by a fraction:

- find the reciprocal of the fraction you are dividing by
- multiply by the reciprocal.

Example 6.13 Question Work out these. a $\frac{8}{9} \div 2$ $2\frac{3}{4} \div 1\frac{5}{8}$ Solution $\frac{8}{9} \div 2 = \frac{8}{9} \div \frac{2}{1}$ 2 is the same as $\frac{2}{3}$ The reciprocal of $\frac{2}{1}$ is $\frac{1}{2}$, so multiply by this reciprocal. $=\frac{g'^4}{9} \times \frac{1}{Z_1}$ Cancel the common factor of 2 in the numerator and the denominator. $=\frac{4}{9}\times 1$ **b** $\frac{9}{10} \div \frac{3}{4} = \frac{9}{10} \times \frac{4}{3}$ The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$, so multiply by this reciprocal. $=\frac{g^3}{5!0}\times\frac{f^2}{1/2}$ Cancel the common factors of 2 and 3 in the numerator Remember that and the denominator. numbers, you $=\frac{6}{5}=1\frac{1}{5}$ The result is an improper fraction, so write it as a c $2\frac{3}{4} + 1\frac{5}{8} = \frac{11}{4} + \frac{13}{8}$ First, change the mixed numbers to improper fractions.

 $=\frac{11}{4}\times\frac{8}{13}$ The reciprocal of $\frac{13}{8}$ is $\frac{8}{13}$, so multiply by this reciprocal.

 $=\frac{11}{f_1}\times\frac{g^2}{13}$ Cancel the common factor of 4 in the numerator and the

 $=\frac{22}{13}=1\frac{9}{13}$ The result is an improper fraction, so write it as a

when you have a fraction calculation involving mixed should first change them to improper fractions.

Always check that your final answer is given as a mixed number in its simplest form.

Key points

- Negative numbers are less than zero.
- When multiplying and dividing numbers, an even number of negative numbers gives a positive answer. An odd number gives a negative answer.
- The correct order of operations is: brackets; powers; multiplication and division; addition and subtraction.
- When multiplying decimals, the number of decimal places in the answer is equal to the total number of decimal places in the numbers
- To divide decimals, write each as a fraction. Multiply both the numerator and denominator by a power of 10 to make the denominator an integer. Now divide.
- To add or subtract fractions, replace the fractions with equivalents with the same denominator. Then, add or subtract the numerators.
- To multiply fractions, multiply the numerators and multiply the denominators.
- To divide fractions, multiply the first fraction by the reciprocal of the fraction you are dividing by.

Revision questions

mixed number.

1)

Evaluate

(a)
$$3\frac{1}{5}-2\frac{2}{3}$$

(b)
$$4^{\frac{3}{2}}$$

2)

Express as a single fraction in its lowest terms

(a)
$$\frac{8}{9} \times \frac{3}{4}$$
,

(b)
$$\frac{3}{4} - \frac{2}{3}$$

3)

(b) Evaluate
$$\frac{1}{1.25}$$
 as a decimal.



4)

Arrange these values in order of size, starting with the smallest

- $\frac{9}{20}$ 0.39 4
- 5%
- [2]

- 5) Evaluate
- (a) 1.5-0.2×4,
- (b) 4.2+0.07.
- 6) (a) Express, correct to two significant figures, (i) 15823.769,
- (ii) 0.0030489.
- (b) Use your answers to part (a) to estimate, correct to one significant figure, the value of 15823.769 0.0030489.
- 7) (a) Evaluate 12+6+2-8.
- (b) Evaluate 2.6×0.2.
- 8) (a) Evaluate 6.3 +0.09.
- (b) Find the decimal number that is exactly halfway between 3.8 and 4.3.
- 9) (a) Express 72% as a fraction in its lowest terms.
- (b) Write down two fractions that are equivalent to 0.4.
- 10) (a) Evaluate 8+ 2 x 1.3.
- (b) Express 0.06 as a fraction, giving your answer in its lowest terms.
- 11) Evaluate
 - (a) $\frac{4}{7} \frac{2}{5}$
 - (b) $\frac{5}{8} \div \frac{2}{3}$
- 12) (a) Write these lengths in order of size, starting with the shortest.
- 500 m 5 cm 50 km 500 mm
- (b) Convert 41.6cm² to mm².