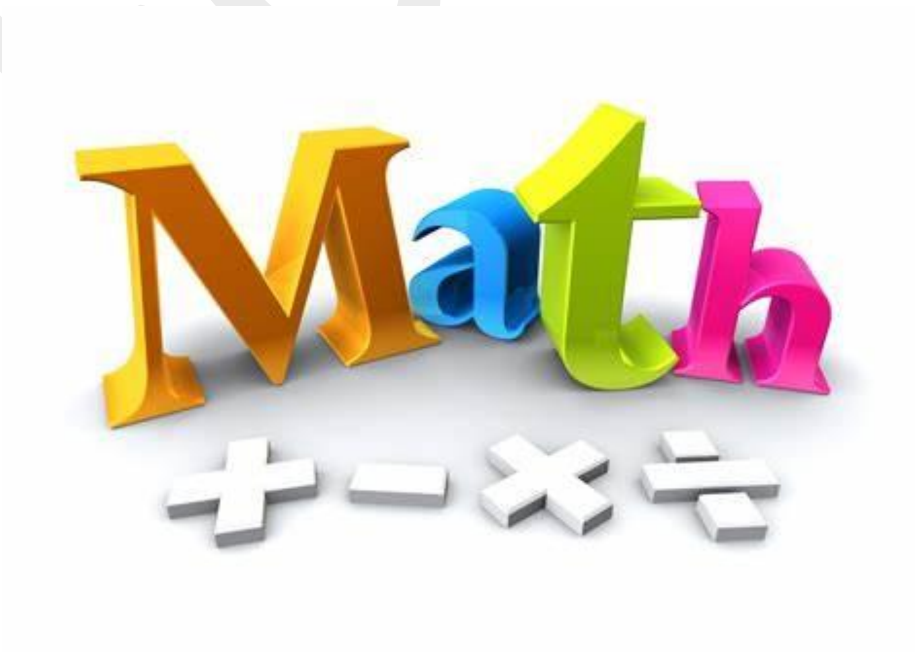
# Cambridge OL Mathematics

## CODE: (4024)

## Chapter 19 and chapter 20 Introduction to algebra and Algebraic manipulation



# Letters for unknowns

Imagine you had a job where you were paid by the hour. You would receive the same amount for each hour you worked. How could you work out how much you will earn in a week?

You would need to work it out as the number of hours you worked, multiplied by the amount you are paid for each hour. This is a formula in words. If you work 35 hours at $4.50 an hour, it is easy to work out 35 × $4.50, but what if the numbers change?

The calculation '35 × $4.50' is only correct if you work 35 hours at $4.50 an hour. Suppose you move to another job where you are paid more for each hour? You need a simple formula that always works. You can use symbols to stand for the numbers that can change

You could use ? or □, but it is less confusing to use letters.

Let the number of hours worked be $N$

the amount you are paid each hour be $$P$

the amount you earn in a week be $$W$.

Then $W = N \times P$.

### Example 19.1

**Question**

Use the formula $W = N \times P$ to find the amount earned in a week when $N = 40$ and $P = 5$.

**Solution**

$W = N \times P$

$= 40 \times 5$

$= 200$

The amount earned in a week is $200.

## Some rules of algebra

You do not need to write the × sign.

$4 \times t$ is written $4t$.

In multiplications, the number is always written in front of the letter.

$p \times 6 - 30$ is written as $6p - 30$.

You often start a formula with the single letter you are finding.

$2 \times l + 2 \times w = P$ is written as $P = 2l + 2w$.

When there is a division in a formula, it is usually written as a fraction.

$y = k \div 6$ can be written as $y = \frac{k}{6}$.

## Substituting numbers into algebraic expressions

Numbers that can be substituted into algebraic expressions can be positive, negative, decimals or fractions.

### Example 19.2

**Question**

**a** Find the value of $4x + 3$ when $x = 2$.
**b** Find the value of $3x^2 + 4$ when $x = 3$.
**c** Find the value of $2x^2 + 6$ when $x = -2$.

**Solution**

**a** $4x + 3 = 4 \times 2 + 3$ Work out each term separately and then collect together.
$= 8 + 3$
$= 11$

**b** $3x^2 + 4 = 3 \times 3^2 + 4$ Remember $3x^2$ means $3 \times x \times x$ not $3 \times x \times 3 \times x$.
$= 3 \times 9 + 4$
$= 27 + 4$
$= 31$

**c** $2x^2 + 6 = 2 \times (-2)^2 + 6$ Take special care when negative numbers are involved.
$= 2 \times 4 + 6$
$= 8 + 6$
$= 14$

## Using harder numbers when substituting

You need to be able to work with positive and negative integers, decimals or fractions, and to do so with or without a calculator.

### Example 19.3

**Question**

Find the value of

**a** $a + 5b^3$ when $a = 2.6$ and $b = 3.2$.
**b** $5cd$ when $c = 2.4$ and $d = 3.2$.

**Solution**

**a** $a + 5b^3 = 2.6 + 5 \times 3.2^3$
$= 2.6 + 5 \times 32.768$
$= 2.6 + 163.84$
$= 166.44$

**Note**
The working is shown here to remind you of the order of operations, but you should be able to do this calculation in one step on your calculator.

**b** $5cd = 5 \times 2.4 \times 3.2$
$= 12 \times 3.2$
$= 10 \times 3.2 + 2 \times 3.2$
$= 32 + 6.4$
$= 38.4$

**Note**
You could work out $2.4 \times 3.2$ first, but this would be harder – use your number facts to help you see the best way to do non-calculator calculations.

### Example 19.4

**Question**

$P = ab + 4b^2$. Find $P$ when $a = \frac{4}{5}$ and $b = \frac{3}{8}$, giving your answer as a fraction.

**Solution**

$P = \left(\frac{4}{5} \times \frac{3}{8}\right) + \left(4 \times \frac{3}{8} \times \frac{3}{8}\right) = \frac{3}{10} + \frac{9}{16} = \frac{24}{80} + \frac{45}{80} = \frac{69}{80}$

# Chapter 20 – Algebraic manipulation

## Simplifying algebraic expressions

These methods can also be used in algebra to simplify an expression.
Collect all the positive terms and all the negative terms separately.
Then add up each set and find the difference. The sign of the answer is the sign of the larger total.

### Example 20.1

**Question**

Simplify $5b - 4b - 8b + 2b$.

**Solution**

$5b + 2b - 4b - 8b$ Collect all the positive terms and all the negative terms separately.

$= 7b - 12b$ Add up each set and find the difference.

$= -5b$

You can only add or subtract **like terms**.

Like terms use only the same letter or the same combination of letters.

### Example 20.2

**Question**

Simplify $3a + 2b - 2a + 5b$.

**Solution**

$3a + 2b - 2a + 5b$

$= 3a - 2a + 2b + 5b$ Reorder with the like terms together.

$= a + 7b$

Algebraic terms can be multiplied together.

Remember that in algebra, you do not need to write the multiplication signs.

However, writing them in your working can help.

### Example 20.3

**Question**

**a** Simplify $2a \times 3b$. **b** Simplify $p \times q \times p^2 \times q$.

**Solution**

**a** $2a \times 3b$

$= 2 \times a \times 3 \times b$

$= 2 \times 3 \times a \times b$ Reorder with the numbers together.

$= 6ab$

**b** $p \times q \times p^2 \times q$

$= p \times p^2 \times q \times q$ Reorder with the like terms together.

$= p^3 \times q^2$ $p \times p^2 = p \times p \times p = p^3$.

$= p^3q^2$ You say this as '$p$ cubed $q$ squared'.

**Note**

$1a$ or $1 \times a$ is always written as just $a$.

Some people confuse $2a$ and $a^2$.

$2a = 2 \times a = a + a$, while $a^2 = a \times a$.

### Example 20.4

**Question**

A rectangle has length $3a$ and width $2b$.
Find and simplify an expression for the perimeter of the rectangle.

**Solution**

Perimeter $= 3a + 2b + 3a + 2b$

$= 6a + 4b$

## Simplifying more complex algebraic expressions

Terms such as $ab^2$, $a^2b$ and $a^3$ cannot be collected together unless they are exactly the same type.

> **Note**
> Errors are often made by trying to go too far.
> For example, $2a + 3b$ cannot be simplified any further.
> Another common error is to work out $4a^2 - a^2$ as 4. The answer is $3a^2$.

**Example 20.5**

**Question**

Simplify each of these expressions by collecting together the like terms.

**a** $2x^2 - 3xy + 2yx + 3y^2$ **b** $3a^2 + 4ab - 2a^2 - 3b^2 - 2ab$ **c** $3 + 5a - 2b + 2 + 8a - 7b$

**Solution**

**a** $2x^2 - 3xy + 2yx + 3y^2$
$= 2x^2 - xy + 3y^2$

The middle two terms are like terms, because $xy$ is the same as $yx$.

**b** $3a^2 + 4ab - 2a^2 - 3b^2 - 2ab$
$= 3a^2 - 2a^2 + 4ab - 2ab - 3b^2$
$= a^2 + 2ab - 3b^2$

Here the $a^2$ terms and the $ab$ terms can be collected, but they cannot be combined with each other or with the single $b^2$ term.

**c** $3 + 5a - 2b + 2 + 8a - 7b$
$= 3 + 2 + 5a + 8a - 2b - 7b$
$= 5 + 13a - 9b$

Here there are three different types of terms, which can be collected separately, but not combined together.

# Expanding a single bracket

> **Note**
> Remember to multiply each of the terms inside the bracket by the number or letter outside the brackets.

What is 2 × 3 + 4? Is it 14? Is it 10?

The rule is 'do the multiplication first', so the answer is 10.

If you want the answer to be 14, you need to add 3 and 4 first. You use a bracket to show this 2 × (3 + 4). Notice that this is equal to 2 × 3 + 2 × 4.

You use a bracket to show this 2 × (3 + 4).

Notice that this is equal to 2 × 3 + 2 × 4.

It is the same in algebra. a(b + c) means 'add b and c and then multiply by a' and this is the same as 'multiply b by a, multiply c by a and then add the results'.

So a(b + c) = ab + ac. This is called expanding the bracket.

**Example 20.6**

**Question**

Expand these brackets.

**a** $5(2x + 3)$ **b** $4(2x - 1)$

**Solution**

**a** $5(2x + 3) = 5 \times 2x + 5 \times 3 = 10x + 15$

**b** $4(2x - 1) = 4 \times 2x + 4 \times -1 = 8x - 4$

> **Note**
> Remember about multiplying with negative numbers. For example,
> $-3 \times x = -3x$
> $-3 \times -x = 3x$.

## Factorising algebraic expressions

Factors are numbers or letters which will divide into an expression. The factors of 6 are 1, 2, 3 and 6.

The factors of $p^2$ are 1, $p$ and $p^2$.

To factorise an expression, look for common factors.

For example, the common factors of $2a^2$ and $6a$ are 2, $a$ and $2a$.

**Example 20.7**

**Question**

Factorise these.

**a** $4p + 6$ **b** $2a^2 - 3a$ **c** $8x^2 - 12x$

**Solution**

**a** The only common factor is 2.
You divide each term by the common factor, 2.
$4p \div 2 = 2p$ and $6 \div 2 = 3$
$4p + 6 = 2(2p + 3)$

**b** The only common factor is $a$.
You divide each term by the common factor, $a$.
$2a^2 \div a = 2a$ and $3a \div a = 3$
$2a^2 - 3a = a(2a - 3)$

**c** Think about the numbers and the letters separately and then combine them.
The highest common factor of 8 and 12 is 4, and the common factor of $x^2$ and $x$ is $x$. Therefore, the common factor of $8x^2$ and $12x$ is $4 \times x = 4x$.
$4x(\ldots)$ You write this factor outside the bracket.
You then divide each term by the common factor, $4x$.
$8x^2 \div 4x = 2x$ and $12x \div 4x = 3$
$8x^2 - 12x = 4x(2x - 3)$

**Note**
*Factorise* means 'factorise fully'. Make sure that you have found all the common factors. Check that the expression in the brackets will not factorise further.

## Factorising expressions of the form *ax + bx + kay + kby*

**Example 20.8**

**Question**

Factorise $x(a + b) + 2y(a + b)$.

**Solution**

The bracket $(a + b)$ is a common factor, so this can be factorised as $(a + b)(x + 2y)$.

**Example 20.9**

**Question**

Factorise $x^2 - 3x + 2ax - 6a$.

**Solution**

There is no common factor for all four terms, but the first pair of terms have a common factor and the second pair of terms also have a common factor.

Factorising in pairs gives $x(x - 3) + 2a(x - 3)$.

The bracket $(x - 3)$ is a common factor so

$x(x - 3) + 2a(x - 3) = (x - 3)(x + 2a)$

Sometimes you have to change the order of the terms, as in the next example.

**Example 20.10**

**Question**

Factorise $5ax - 9b + 3bx - 15a$.

**Solution**

The pairs of terms in this order do not have common factors.

Changing the order gives $5ax + 3bx - 15a - 9b$.

You can then factorise in pairs.

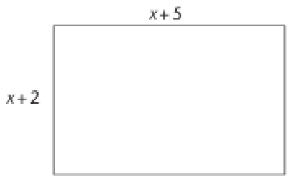
$x(5a + 3b) - 3(5a + 3b) = (5a + 3b)(x - 3)$

**Note**
Taking the factor $-3$ out means the sign of $3b$ is $+$, since $-3 \times +3b = -9b$.

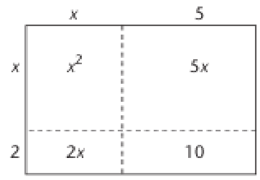
# Expanding a pair of brackets

You expand a pair of brackets by multiplying one bracket by the other bracket.

Look at this rectangle.




Now split the rectangle up.




The area of the rectangle is (x + 5)(x + 2).

So $(x + 5)(x + 2) = x^2 + 2x + 5x + 10 = x^2 + 7x + 10$

When multiplying out a pair of brackets you must multiply each term inside the second bracket by each term inside the first bracket.

The example which follows shows three methods for doing this. Choose the method you prefer and stick to it.

### Example 20.11

**Question**

Expand these.

**a** $(a + 2)(a + 5)$ **b** $(b + 4)(b - 1)$ **c** $(m - 5)(m - 4)$

**Note**
'Multiply out', 'simplify', 'expand' or 'remove' the brackets all mean the same thing.

**Solution**

**a Method 1**

Expand the first bracket and then multiply out.

$(a + 2)(a + 5) = a(a + 5) + 2(a + 5)$

$= a^2 + 5a + 2a + 10$ Notice that the middle two terms are like terms and so can be collected.

$= a^2 + 7a + 10$

**b Method 2**

| × | b | +4 |
|---|---|---|
| b | $b^2$ | +4b |
| −1 | −1b | −4 |

$(b + 4)(b - 1) = b^2 + 4b - 1b - 4$

$= b^2 + 3b - 4$

**Note**
Most errors are made in expanding brackets when negative signs are involved.

**c Method 3**

Use the word FOIL to make sure you multiply each term in the second bracket by each term in the first.

If you draw arrows to show the multiplications, you can think of a smiley face.

$(m - 5)(m - 4) = m \times m + m \times -4 - 5 \times m - 5 \times -4$

$= m^2 - 4m - 5m + 20$

$= m^2 - 9m + 20$

**Note**
F: first × first
O: outer × outer
I: inner × inner
L: last × last

There are two special types of expansion of a pair of brackets that you need to know. The first is when a bracket is squared, as in parts a and b of the next example.

The important thing with this type of expansion is to make sure that you write the brackets separately and that you end up with three terms.

The second type is shown in part **c** of the example. With this type of expansion, you get only two terms because the middle terms cancel each other out. This type is known as **the difference of two squares**, because $(A - B)(A + B) = A^2 - B^2$.

### Example 20.12

**Question**

Expand these.

**a** $(x + 3)^2$ **b** $(x - 3)^2$ **c** $(x + 3)(x - 3)$

**Solution**

**a** $(x + 3)^2 = (x + 3)(x + 3)$

The method shown here is method 1, but you can use any of the methods that were shown in Example 20.11. You will get the same answer.

$= x(x + 3) + 3(x + 3)$
$= x^2 + 3x + 3x + 9$
$= x^2 + 6x + 9$

**b** $(x - 3)^2 = (x - 3)(x - 3)$
$= x(x - 3) - 3(x - 3)$
$= x^2 - 3x - 3x + 9$
$= x^2 - 6x + 9$

**Note**
Take care with the negative signs.

**c** $(x + 3)(x - 3) = x(x - 3) + 3(x - 3)$
$= x^2 - 3x + 3x - 9$
$= x^2 - 9$

## Expanding more complex brackets

The same methods can be used for brackets containing more complex expressions. This is shown in the following example.

### Example 20.13

**Note**
Expressions of the form $ax^2 + bx + c$ are called quadratic.

**Question**

Expand and simplify these.

**a** $(3x + 2)(5x + 1)$ **b** $(5a - 2b)(3a - b)$ **c** $(y + 2)(2y - 5)(y - 1)$

**Solution**

**a** $(3x + 2)(5x + 1) = 3x(5x + 1) + 2(5x + 1)$
$= 15x^2 + 3x + 10x + 2$
$= 15x^2 + 13x + 2$

**b** $(5a - 2b)(3a - b) = 5a \times 3a + 5a \times -b - 2b \times 3a - 2b \times -b$
$= 15a^2 - 5ab - 6ab + 2b^2$
$= 15a^2 - 11ab + 2b^2$

**c** $(y + 2)(2y - 5)(y - 1) = y(2y - 5)(y - 1) + 2(2y - 5)(y - 1)$
$= y(2y^2 - 7y + 5) + 2(2y^2 - 7y + 5)$
$= 2y^3 - 7y^2 + 5y + 4y^2 - 14y + 10$
$= 2y^3 - 3y^2 - 9y + 10$

## Factorising expressions of the form $a^2x^2 - b^2y^2$

When expanding brackets earlier in this chapter, you met expressions of the form $a^2 - b^2$.

They are called **the difference of two squares**.

We have already seen that $(x - 3)(x + 3) = x^2 - 9$.

So, following this pattern, we see that factorising $x^2 - a^2$ gives $(x - a)(x + a)$.

**Note**
It is worth learning this so that you recognise it when you see it!

### Example 20.14

**Question**
Factorise $x^2 - 16$.

**Solution**
$$x^2 - 16 = x^2 - 4^2 = (x - 4)(x + 4)$$

In fact any expression which can be written as the difference of two squares can be factorised in this way.

### Example 20.15

**Question**
Factorise each of these expressions.

**a** $25x^2 - 1$

**b** $9x^2 - 4y^2$

**Solution**

**a** $25x^2 - 1 = 5^2x^2 - 1 = (5x - 1)(5x + 1)$

**b** $9x^2 - 4y^2 = 3^2x^2 - 2^2y^2 = (3x - 2y)(3x + 2y)$

When a numerical term is not a square number, check to see whether a common factor can be extracted, leaving an expression in a form that allows you to use the difference of two squares method.

### Example 20.16

**Question**
Factorise $3x^2 - 12$.

**Solution**
$3x^2 - 12 = 3(x^2 - 4)$ Take out the common factor 3.
$= 3(x^2 - 2^2)$
$= 3(x - 2)(x + 2)$

## Factorising expressions of the form $x^2 + bx + c$

In the expression $x^2 + bx + c$, $c$ is the numerical term, and $b$ is called the **coefficient** of $x$.

## Factorising quadratic expressions with a positive numerical term

The expression $(x + 6)(x + 2)$ can be multiplied out and simplified to $x^2 + 8x + 12$.

Therefore $x^2 + 8x + 12$ can be factorised as a product of two brackets, by reversing the process.

The order in which you write the brackets does not matter.

### Example 20.17

**Question**

Factorise $x^2 + 7x + 12$.

**Solution**

This will factorise into two brackets, with $x$ as the first term in each.

$x^2 + 7x + 12 = (x \quad )(x \quad )$

As both the signs are positive, both the numbers will be positive.

You need to find two numbers that multiply to give 12 and add to give 7.

These are +3 and +4.

So $x^2 + 7x + 12 = (x + 3)(x + 4)$

or $x^2 + 7x + 12 = (x + 4)(x + 3)$.

### Example 20.18

**Question**

Factorise $x^2 - 3x + 2$.

**Solution**

You need to find two negative numbers that multiply to give 2 and add to give −3. They are −2 and −1.

$x^2 - 3x + 2 = (x - 2)(x - 1)$.

**Note**

If the numerical term is positive, both the numbers in the brackets must have the same sign as the coefficient of $x$.

If the coefficient of $x$ is negative and the numerical term is positive, both the numbers in the brackets will be negative.

## Factorising quadratic expressions with a negative numerical term

### Example 20.19

**Question**

Factorise $x^2 - 3x - 10$.

**Solution**

As the numerical term is negative, you need two numbers, with opposite signs, that multiply to give −10 and add to give −3.

The numbers are −5 and +2.

$x^2 - 3x - 10 = (x - 5)(x + 2)$

### Example 20.20

**Question**

Factorise $x^2 + 4x - 12$.

**Solution**

As the numerical term is negative, you need two numbers, with opposite signs, that multiply to give −12 and add to give +4.

The numbers are +6 and −2.

$x^2 + 4x - 12 = (x + 6)(x - 2)$

# Factorising expressions of the form $a^2 + 2ab + b^2$

We have found that $(x + 3)^2 = x^2 + 6x + 9$ and $(x - 3)^2 = x^2 - 6x + 9$.

These are examples of **perfect squares**.

In general,

$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$.

The result can be used in reverse to factorise expressions of the form $a^2 + 2ab + b^2$.

So $a^2 + 2ab + b^2 = (a + b)^2$.

**Note**
Remember that if the numerical term is negative, the two numbers in the brackets have different signs and the larger number has the sign of the coefficient of $x$.

It is easy to make a mistake when factorising. Always check by multiplying out the brackets.

**Note**
The first and third terms are squares and the middle term is twice the product of the quantities that are squared.

### Example 20.21

**Question**

Factorise $x^2 + 8x + 16$.

**Solution**

The 1st and 3rd terms are $x^2$ and $4^2$ and the middle term is $8x = 2 \times 4 \times x$.

$x^2 + 8x + 16 = (x + 4)^2$

**Note**
If the middle term is negative, the sign in the bracket will be negative.

### Example 20.22

**Question**

Factorise $9x^2 - 30xy + 25y^2$.

**Solution**

The first and third terms are $(3x)^2$ and $(5y)^2$ and the middle term $30xy = 2 \times 3x \times 5y$.

The middle term is negative so the sign in the bracket will be negative.

$9x^2 - 30xy + 25y^2 = (3x - 5y)^2$

## Factorising quadratic expressions of the form $ax^2 + bx + c$

You have already learned how to factorise simple quadratic expressions such as $x^2 + bx + c$.

- If c is positive, you find two numbers with the same sign that multiply to give c and add to give b.
- If c is negative, you find two numbers with different signs that multiply to give c and add to give b.

The expression $ax^2 + bx + c$, when factorised, will be $(px + q)(rx + s) = prx^2 + (ps + qr)x + qs$.

So $pr = a$, $qs = c$ and $ps + qr = b$.

It is easiest to look at examples.

## Factorising more complex quadratic expressions with a positive numerical term

**Example 20.23**

**Question**

Factorise $3x^2 + 11x + 6$.

**Solution**

As the numerical term is positive, both numerical terms in the brackets have the same sign, and as the coefficient of $x$ is positive, they are both positive.

The only numbers that multiply to give 3 are 3 and 1.

So, as a start, $(3x + ...)(x + ...)$.

The numbers that multiply to give 6 are either 3 and 2 or 6 and 1.

So the possible answers are

$(3x + 2)(x + 3)$ or $(3x + 3)(x + 2)$, or $(3x + 6)(x + 1)$ or $(3x + 1)(x + 6)$.

By expanding the brackets, it can be seen that the first one is correct.

$3x^2 + 11x + 6 = (3x + 2)(x + 3)$

The coefficient of $x$ is $3 \times 3 + 2 \times 1 = 9 + 2 = 11$.

It is very useful to check completely by multiplying out the two brackets.

Writing out all the possible brackets can be a long process, and it is quicker to test the possibilities until you find the correct coefficient of $x$ and then multiply out the brackets to check.

**Note**

First, look for any common factor, then try the most obvious pairs first. Remember, if the sign of $c$ (the numerical term) is positive, both the numerical terms in the brackets have the same sign as $b$ (the coefficient of $x$).

**Example 20.24**

**Question**

Factorise $4x^2 - 14x + 6$.

**Solution**

First, check whether there is a common factor.

Here the common factor is 2.

$4x^2 - 14x + 6 = 2(2x^2 - 7x + 3)$

Now look at the quadratic expression.

As the numerical term is positive and the coefficient of $x$ is negative, both the numerical terms in the brackets will be negative.

The possibilities for the coefficients of the $x$-terms in the brackets are 2 and 1.

The possibilities for the numerical terms in the brackets are –1 and –3 or –3 and –1.

The coefficient of $x$ is thus $2 \times -1 + 1 \times -3 = -5$ or $2 \times -3 + 1 \times -1 = -7$.

The second is correct, so $2x^2 - 7x + 3 = (2x - 1)(x - 3)$.

So $4x^2 - 14x + 6 = 2(2x - 1)(x - 3)$.

## Factorising more complex quadratic expressions with a negative numerical term

The examples looked at so far had a positive numerical term. When c (the numerical term) is negative, the signs of the numerical terms in the brackets are different. These are, again, best shown by examples.

**Example 20.25**

**Question**

Factorise $3x^2 - 7x - 6$.

**Solution**

There is no common factor.

The signs in the numerical terms in the brackets are different.

The coefficients of the $x$-terms in the brackets are 3 and 1.

The possibilities for the numerical terms are 1 and –6 or –1 and 6, 6 and –1 or –6 and 1, 3 and –2 or –3 and 2, or 2 and –3 or –2 and 3.

Test the possibilities until you find the correct coefficient of $x$.

$3 \times 1 + 1 \times -6 = -3$

$3 \times -1 + 1 \times 6 = 3$

$3 \times 6 + 1 \times -1 = 17$

$3 \times -6 + 1 \times 1 = -17$ You don't need to test this pair as you can see that the result will be –17.

$3 \times 3 + 1 \times -2 = 7$

$3 \times -3 + 1 \times 2 = -7$ You can see from the previous calculation that this pair will give the required result.

So, $3x^2 - 7x - 6 = (3x + 2)(x - 3)$.

Multiply out to check.

# Completing the square

Another way to manipulate quadratic expressions is to arrange the x-terms in a square.
The expression $x^2 + 2x + 5$ can be written as $x^2 + 2x + 1 + 4$ = (x + 1)2 + 4. This is called completing the square. It can be used in factorising, finding maximum and minimum values and solving equations (see Chapter 23).

**Example 20.27**

**Question**

**a** Complete the square for $x^2 - 6x + 5$.
**b** Use the result to factorise $x^2 - 6x + 5$.
**c** What is the minimum value of the expression $x^2 - 6x + 5$?

**Solution**

**a** $x^2 - 6x$ are terms in the expansion of $(x - 3)^2$
(The number in the bracket is half the coefficient of $x$ in the expression.)
$x^2 - 6x + 5 = x^2 - 6x + 9 - 4 = (x - 3)^2 - 4$
**b** $(x - 3)^2 - 4$ is a difference of two squares
$(x - 3)^2 - 4 = (x - 3)^2 - 2^2 = (x - 3 - 2)(x - 3 + 2) = (x - 5)(x - 1)$
**c** $x^2 - 6x + 5 = (x - 3)^2 - 4$
Minimum value of expression when $x - 3 = 0$ (a square number cannot be negative)
Minimum = $-4$ when $x = 3$.

**Key points**

- When simplifying expressions, you can only add or subtract like terms.
- $a(b + c) = ab + ac$.
- When factorising, always take out common factors first.
- When multiplying out brackets, each term in one bracket must be multiplied by each term in the other bracket.
- Factorising $x^2 - a^2$ gives $(x - a)(x + a)$ which is a difference of two squares.
- $a^2 + 2ab + b^2 = (a + b)^2$ which is a perfect square.
- The expression $ax^2 + bx + c$ can be factorised into $(px + q)(rx + s)$ where $pr = a$, $qs = c$ and $ps + qr = b$.
- Completing the square for a quadratic expression involves arranging the $x$ terms as a square and adjusting the value of the constant.
$x^2 + 2bx + c = (x + b)^2 + c - b^2$

## Revision questions

1. Simplify $3a + 7b - 4a + b$.

2. $y = mx + c$

   Find the value of $y$ when $m = -3$, $x = -2$ and $c = -8$.

3. Complete the statement.

   When $5x = 15$, $12x$ = ....................

4. $y = x^4 - 4x^3$

   Find the value of $y$ when $x = -1$.

5. One solution of the equation $ax^2 + a = 150$ is $x = 7$.
   Find the value of $a$.

6.

$$y = p - 2qx^2$$

$p = -10$

$q = 3$

$x = -5$

Work out the value of $y$.

7.

$\frac{a}{b} = 3c$

$\frac{b}{c} = 2$

Work out the value of $a$ when $c = 8$

8. $a$ and $b$ are positive values.

Show that $\frac{7a + 2b - 3a}{8a + 6b + 2a - b}$ always simplifies to the same value.

9. Kelly is trying to work out the two values of w for which $3w - w^3 = 2$
Her values are 1 and -1

Are her values correct?
You **must** show your working.

10. Complete the statement.

When $w =$ ............... , $10w = 70$.