Cambridge OL

Mathematics

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Chapter 21 and Chapter 22

Algebraic fractions and Indices

Simplifying algebraic fractions

To simplify a numerical fraction, you cancel any common factors of both the numerator and the denominator to write it in its simplest terms.

In the same way, to simplify an algebraic fraction, you cancel any common factors of both the numerator and the denominator. Factors can be numbers, letters or brackets.

Example 21.1

Question

Simplify $\frac{4ab^2}{3c^2} \times \frac{9c^2}{2a^2b}$.

Solution

$$\frac{4ab^2}{3c^2} \times \frac{9c^2}{2a^2b} = \frac{\cancel{4}^{2}\cancel{a}\cancel{b^2}^{b} \times \cancel{9}^{3}\cancel{c^2}}{\cancel{3}\cancel{c^2} \times \cancel{2}\cancel{a^2}_{a}\cancel{b}} = \frac{6b}{a}$$

2, 3, a, b and c^2 are all factors of both the numerator and the denominator, so these cancel.

CHECK YOU CAN:

- simplify algebraic expressions
- expand brackets
- factorise algebraic expressions.

Example 21.2

Question

Simplify $\frac{x^2+x}{x^2-2x-3}$.

Solution

As it stands, this fraction cannot be cancelled.

First, both the numerator and denominator must be factorised.

$$\frac{x^2+x}{x^2-2x-3} = \frac{x\cancel{(x+1)}}{(x-3)\cancel{(x+1)}}$$

$$= \frac{x}{(x-3)}$$

$(x + 1)$ is a factor of both the numerator and the denominator so it cancels.

Note

Errors often occur by cancelling individual terms. Only factors, which can be individual numbers, letters or brackets, can be cancelled.

Adding and subtracting algebraic fractions

To add and subtract numerical fractions, you first write all the fractions with a common denominator. In the same way, to add and subtract algebraic fractions, you first write all the fractions with a common denominator.

Example 21.3

Question

Simplify $\frac{x+3}{2} - \frac{x-3}{3}$.

Solution

The common denominator is 6.

So $(x + 3)$ is multiplied by $6 \div 2 = 3$ and $(x - 3)$ is multiplied by $6 \div 3 = 2$.

$$\frac{x+3}{2} - \frac{x-3}{3} = \frac{3(x+3)-2(x-3)}{6}$$

$$= \frac{3x+9-2x+6}{6}$$

$$= \frac{x+15}{6}$$

Note

Do not miss out the first step. Most errors occur because of expanding the brackets wrongly.

The procedure is still the same when the denominators involve algebraic expressions as shown in the next example.

Example 21.4

Question

Simplify $\frac{3}{x+1} - \frac{2}{x}$.

Solution

The common denominator is $x(x + 1)$.

So 3 is multiplied by $x(x + 1) \div (x + 1) = x$ and 2 is multiplied by $x(x + 1) \div x = (x + 1)$.

$$\frac{3}{x+1} - \frac{2}{x} = \frac{3x-2(x+1)}{x(x+1)}$$

$$= \frac{3x-2x-2}{x(x+1)}$$

$$= \frac{x-2}{x(x+1)}$$

Chapter 22 – Indices

Simplifying algebraic expressions using indices

Indices can be used with algebraic expressions, as well as with numerical expressions.

You know that $a \times a \times a = a^3$ and $a \times a \times a \times a \times a = a^5$

So, $a^3 \times a^5 = (a \times a \times a) \times (a \times a \times a \times a \times a) = a^8$

The indices are added $a^3 \times a^5 = a^{3+5} = a^8$

Similarly $a^5 \div a^3 = (a \times a \times a \times a \times a) \div (a \times a \times a)$

$$= \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a}{\cancel{a} \times \cancel{a} \times \cancel{a}}$$

$$= a \times a$$

$$= a^2$$

The indices are subtracted $a^5 \div a^3 = a^{5-3} = a^2$

The rules for multiplying and dividing numbers in index form can also be applied to algebraic expressions.

These are the general laws for indices.

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

Using the first law, you can see that $(a^2)^3 = a^2 \times a^2 \times a^2 = a^{2+2+2} = a^6$

This is the same as $(a^2)^3 = a^{2\times 3} = a^6$

This gives another general law.

$(a^n)^m = a^{m\times n}$

Using the second law, you can see that $a^3 \div a^3 = a^{3-3} = a^0$

But $a^3 \div a^3 = 1$

This gives a fourth general law.

$a^0 = 1$

These four laws can be used to simplify algebraic expressions involving indices.

Example 22.1

Question

Simplify these expressions where possible.

a $3a^2 \times 4a^3$ **b** $\frac{6a^5}{2a^3}$ **c** $(2x^3)^4$ **d** $\frac{12ab^3 \times 3a^2b}{2a^3b^2}$ **e** $4a^2 + 3a^3$

Solution

a $3a^2 \times 4a^3 = 12a^5$ The numbers are multiplied and the indices are added.

b $\frac{6a^5}{2a^3} = 3a^2$ The numbers are divided and the indices are subtracted.

c $(2x^3)^4 = 16x^{12}$ The number is raised to the power 4 and the indices are multiplied.

d $\frac{12ab^3 \times 3a^2b}{2a^3b^2} = \frac{36a^3b^4}{2a^3b^2}$ First, simplify the numerator by multiplying the numbers and adding the indices for the a terms and the b terms separately.

$= 18a^0b^2$ Then divide the numbers and subtract the indices for the a terms and the b terms separately.

$= 18b^2$ Remember that $a^0 = 1$.

e $4a^2 + 3a^3$ This expression cannot be simplified.

The terms are not like terms so they cannot be added.

Using the laws of indices with numerical and algebraic expressions

You have met these laws of indices.

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^n)^m = a^{n\times m}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

These laws can be used together to simplify expressions involving numbers or letters.

Example 22.2

Question

Write each of these as a single power of 2, where possible.

a $2\sqrt{2}$

b $(\sqrt[3]{2})^2$

c $2^3 \div 2^{\frac{1}{2}}$

d $2^3 + 2^4$

e $8^{\frac{3}{4}}$

f $2^3 \times 4^{\frac{3}{2}}$

g $2^n \times 4^3$

Solution

a $2\sqrt{2} = 2^1 \times 2^{\frac{1}{2}} = 2^{\frac{3}{2}}$

b $(\sqrt[3]{2})^2 = \left(2^{\frac{1}{3}}\right)^2 = 2^{\frac{2}{3}}$

c $2^3 \div 2^{\frac{1}{2}} = 2^{3-\frac{1}{2}} = 2^{\frac{5}{2}}$

d $2^3 + 2^4$ These powers cannot be added.

e $8^{\frac{3}{4}} = (2^3)^{\frac{3}{4}} = 2^{3\times\frac{3}{4}} = 2^{\frac{9}{4}}$

f $2^3 \times 4^{\frac{3}{2}} = 2^3 \times (2^2)^{\frac{3}{2}} = 2^3 \times 2^3 = 2^6$

g $2^n \times 4^3 = 2^n \times (2^2)^3 = 2^n \times 2^6 = 2^{n+6}$

Key points

The laws of indices:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^n)^m = a^{n\times m}$
- $a^0 = 1$
- $a^{-n} = \frac{1}{a^n}$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$

Revision questions

1.

Simplify $\dfrac{p}{2q} \times \dfrac{4pq}{t}$.

2.

Write as a single fraction in its simplest form.

$$\frac{1}{x+2} - \frac{2}{3x-1}$$

3.

Simplify $\dfrac{ab-b^2}{a^2-b^2}$.

4.

Write as a single fraction in its simplest form.

$$\frac{1}{x} - \frac{1}{x+1}$$

Simplify.

5.

$$\frac{3+x}{9-x^2}$$

6.

Write as a single fraction in its simplest form.

$$\frac{1}{y-1} - \frac{1}{y}$$

7.

$2^{12} \div 2^{\frac{k}{2}} = 32$

Find the value of k.

8.

Simplify $\left(\dfrac{27x^{12}}{64y^3}\right)^{-\frac{1}{3}}$.

9.

Simplify fully $\left(\frac{64x^6}{25y^2}\right)^{-\frac{1}{2}}$

10.

Given that $4^{k+3} = 16 \times 2^k$

find the value of k.

Show your working clearly.

11. $25 = 125^k$

Find the value of k.

12.

Find the value of $(64x^4)^{0.5} \times 4x^{-2}$.