Edexcel

AS Level

Physics

CODE: (WPH11)

Topic 4

Electric circuits

4A 1 Electric current

▲ **fig A** Electric current is a measure of the rate of movement of electric charge. Lightning strikes can have currents of 10 000 amperes.

ELECTRIC CHARGE

Some particles have an electric **charge**. For example, the electron has a negative charge. In SI units, electric charge is measured in **coulombs** (C) and the amount of charge on a single electron in these units is -1.6×10^{-19} C.

$$e = -1.6 \times 10^{-19}\,\text{C}$$

This means that you would have one coulomb of negative charge if you collected together 6.25×10^{18} electrons, as shown in this calculation:

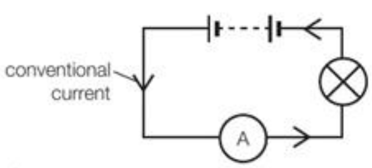
$$\text{total charge, } Q = ne = 6.25 \times 10^{18} \times -1.6 \times 10^{-19} = -1\,\text{C}$$

ELECTRIC CURRENT

If electric charge moves, this is referred to as an electric current, and the definition of current is the rate of movement of charge. As it is usually a physical movement of billions of tiny charged particles, such charge movements are often said to flow. Thinking of the flow of charge like the current in a river can be useful, and we will see later how current splits and recombines at circuit junctions in a manner that is like water flow. Also, as the total amount of water in a river at a given time does not change, even if the river splits, this again shows the conservation of charge.

▲ **fig B** We can measure electric current through a component using an ammeter connected in series.

Any source of electrical energy can create an electric force in order to produce a current. In **fig C**, the cell causes the electric force experienced by the negative conduction electrons so they move through the metal – they are attracted to the positive anode of the cell.



▲ **fig C** Conventional current flows from positive to negative. Negative electrons would move in the opposite direction.

CALCULATING CURRENT

The SI unit for electric current is the **ampere**, A. Current can be calculated from the equation:

$$\text{current (A)} = \frac{\text{charge passing a point (C)}}{\text{time for that charge to pass (s)}}$$

$$I = \frac{\Delta Q}{\Delta t}$$

Thus one ampere (1 A) is the movement of one coulomb (1 C) of charge per second (1 s). For example, if the lightning in **fig A** takes 0.1 seconds to transfer 1150 coulombs of charge, we could calculate the current in the lightning:

$$I = \frac{\Delta Q}{\Delta t} = \frac{1150}{0.1} = 11\,500\,\text{A}$$

The equation that defines electric current is often used in a rearranged form to find ΔQ, the amount of charge that has moved through a component in a given time, Δt.

$$\Delta Q = I\Delta t$$

IONIC CHARGE CARRIERS

In electrolytic processing of bauxite ore, charge carriers, such as free aluminium ions, move through the liquid as an electric current. These ions are positively charged and move towards the negative cathode due to the electric force. The charge on an electron is a fixed negative amount, making it easy to calculate the charge on any ion.

SUBJECT VOCABULARY

charge a fundamental property of some particles. It is the cause of the electromagnetic force, and it is a basic aspect of describing electrical effects

coulomb, C the unit of measurement for charge: one coulomb is the quantity of charge that passes a point in a conductor per second when one ampere of current is flowing in the conductor. The amount of charge on a single electron in these units is -1.6×10^{-19} C

electric **current** the rate of flow of charge. Current can be calculated from the equation:

$$\text{current (A)} = \frac{\text{charge passing a point (C)}}{\text{time for that charge to pass (s)}}$$

$$I = \frac{\Delta Q}{\Delta t}$$

ampere the unit of measurement for electric current: one ampere (1 A) is the movement of one coulomb (1 C) of charge per second (1 s)

4A 2 Electrical energy transfer

VOLTAGES

The electrical quantity **voltage** is a measure of the amount of energy a component transfers per unit of charge passing through it. It can be calculated from the equation:

$$\text{voltage (V)} = \frac{\text{energy transferred (J)}}{\text{charge passing (C)}}$$

$$V = \frac{E}{Q}$$

fig A Electric circuits transfer energy.

ELECTROMOTIVE FORCE

For a supply voltage – a component which is putting electrical energy into a circuit – the correct term for the voltage is **electromotive force**, or **emf**. If a cell supplies one joule (1 J) of energy per coulomb of charge (1 C) that passes through it, it has an emf of 1 volt (1 V).

$$\text{emf (V)} = \frac{\text{energy transferred (J)}}{\text{charge passing (C)}}$$

$$\varepsilon = \frac{E}{Q}$$

For example, if the bicycle lamp in **fig A** was powered by a cell with an emf of 1.5 V, then this cell would transfer 1.5 J of electrical energy to each coulomb of charge that passed through it.

POTENTIAL DIFFERENCE

For a component which is using electrical energy in a circuit and transferring this energy, the correct term for the voltage is **potential difference**, or **pd**. If a component uses one joule (1 J) of energy per coulomb of charge (1 C) that passes through it, it has a pd of 1 volt (1 V). The energy being used by the component could be referred to as work done, W.

$$\text{pd (V)} = \frac{\text{energy transferred (J)}}{\text{charge passing (C)}}$$

$$V = \frac{W}{Q}$$

THE ELECTRONVOLT

The **electronvolt**, eV, is a unit of energy that is generally used with sub-atomic particles. Its definition comes from the equation defining voltage:

$$V = \frac{E}{Q}$$

If an electron is accelerated by a potential difference of 1 V, the energy transferred to it will be:

$$E = V \times e$$

$$\therefore \quad E = 1 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-19}\,\text{J}$$

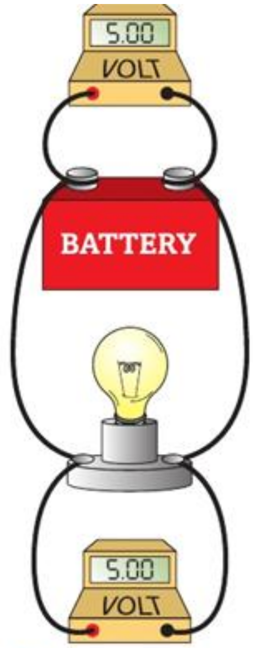
The amount of energy transferred to an electron by passing through a voltage of 1 V is an electronvolt. So 1 eV = 1.6×10^{-19} J.

ELECTRICAL MODELS

A model is a way to understand an idea or phenomenon, such as imagining an atom's structure like planets orbiting the sun. While some aspects may not accurately represent an atom, others help clarify its structure. Electricity, for example, often uses models to explain invisible aspects. Evaluating models' strengths and weaknesses is crucial.

MODELLING VOLTAGE

One model that could be used to try and understand the transfers of energy in an electric circuit could be to think of an electric circuit as a a ski area. When people go skiing, they take a lift which carries them up to the top of the hill. From there, they then slide back down to the bottom. Some skiers may slide down different routes, but they all finish back at the same lower level they started at.



▲ **fig B** We can measure voltage across a component with a voltmeter connected in parallel.

The skiers on the lift gain gravitational potential energy, which is representing electrical energy in this analogy. By the time they have all skied back down through the runs and over the obstacles, they have lost all that gpe again and are back to the same level as before they started. This illustrates the principle of conservation of energy in an electric circuit. If moving charge is given energy by a source of emf, it will transfer all that energy on its journey around the circuit, through the various pds. Around the circuit, the total of all the emfs will be the same as the total of all the pds – total energy supplied will equal total energy used.

▲ **fig C** Modelling an electric circuit as a ski area.

SUBJECT VOCABULARY

voltage a measure of the amount of energy a component transfers per unit of charge passing through it. It can be calculated by the equation:

$$\text{voltage (V)} = \frac{\text{energy transferred (J)}}{\text{charge passing (C)}}$$

$$V = \frac{E}{Q}$$

electromotive force, or **emf** a voltage as defined above, with the energy coming into the circuit

potential difference or **pd** the correct term for the voltage of a component that is using electrical energy in a circuit and transferring this energy to other stores

electronvolt the amount of energy an electron gains by passing through a voltage of 1 V

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$1 \text{ mega electronvolt} = 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$

4A 3 Current and voltage relationships

RESISTANCE CALCULATIONS

Electrical **resistance** is considered to be the opposition to the flow of current within a conductor. It can be calculated from the equation:

$$\text{resistance } (\Omega) = \frac{\text{potential difference } (V)}{\text{current } (I)}$$

$$R = \frac{V}{I}$$

OHM'S LAW

fig A Measuring the resistance of an ohmic conductor.

If the current is proportional to the voltage driving it through a component, this component is called an *ohmic conductor*, as this means it follows **Ohm's law**. The proportional relationship can be used to find the resistance of the component.

For example, what is the resistance of the resistor shown in **fig A**?

$$R = \frac{V}{I} = \frac{4.0}{0.18}$$

$$R = 22\,\Omega$$

For an ohmic conductor, the answer to the calculation of resistance would be the same for all voltages and their corresponding current values (providing the temperature remains constant – see **Section 4A.5**).

CURRENT-VOLTAGE CHARACTERISTICS

For an electric circuit design, we need to know how components will react when the pd across them changes, in order to ensure that the circuit performs its intended function under all circumstances. Part of the specification of any component is a graph of its I-V characteristics. We have already seen the graph for a simple resistor, and a metal at constant temperature would produce the same straight-line result.

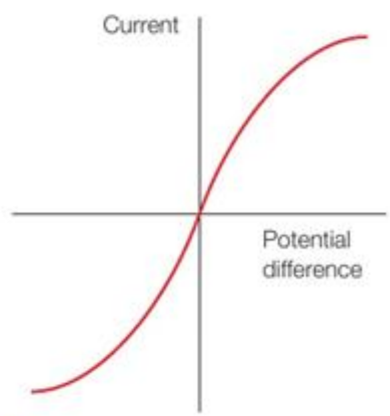
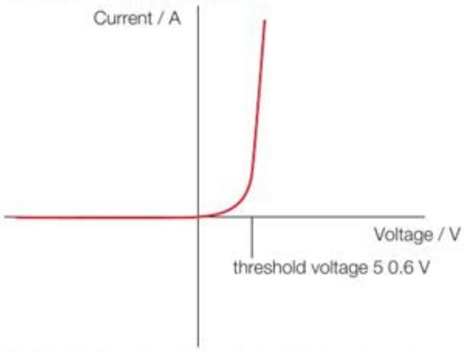


fig C A filament lamp is an example of a non-ohmic conductor. This is because the current through it affects its own temperature – higher current means a higher temperature – and controlling temperature is part of the definition of Ohm's law.

I-V GRAPH FOR A FILAMENT BULB

For the filament lamp I-V graph in fig C, we can see that for a small voltage, the current is proportional to it, as shown by the straight-line portion of the graph through the origin. At higher voltages, a larger current is driven through the lamp filament wire, and this heats it up. At hotter temperatures, metals have higher resistance. The gradient of the graph in fig B was given as the reciprocal of the resistance, and so the gradient here becomes less towards higher voltages: higher resistance means a lower gradient.

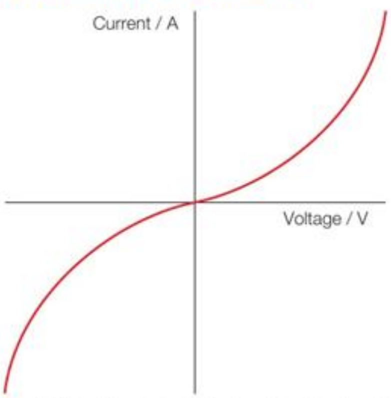
I–V GRAPH FOR A DIODE



▲ **fig D** A semiconductor diode is designed to only pass current in one direction, so the line follows the *x*-axis, $I = 0$, for negative voltages.

For the diode in **fig D**, we can see the basic idea that a diode only conducts in the forwards direction, and so there is zero current for negative voltages. It also requires a minimum voltage in the forwards direction. This *threshold voltage* is typically around 0.6 V. The curve for a diode will be looked at in detail in **Section 4A.6**, which covers the ideas of electrical conduction in different materials.

I–V GRAPH FOR A THERMISTOR



▲ **fig E** A thermistor is designed to alter its resistance with temperature; in the reverse manner to a filament bulb.

For the thermistor in **fig E**, its resistance reduces with the temperature. The gradient of the line increases with the heating effect of the increasing current. The gradient represents the reciprocal of the resistance: larger gradient value means lower resistance. This is a result of its manufacture from semiconductor materials, whose atoms release more conduction electrons as the temperature rises. This will be explained in detail in **Section 4A.6**.

SUBJECT VOCABULARY

resistance the opposition to the flow of electrical current. It can be calculated from the equation:

$$\text{resistance } (\Omega) = \frac{\text{potential difference } (V)}{\text{current } (A)}$$

$$R = \frac{V}{I}$$

Ohm's law the current through a component is directly proportional to the voltage across it, providing the temperature remains the same. The equation for this is often expressed as:

$$\text{voltage } (V) = \text{current } (A) \times \text{resistance } (\Omega)$$

$$V = I \times R$$

4A 4 Resistivity

Resistance is the result of collisions between charge carriers and atoms in the current's path. This effect will vary depending on the density of charge carriers and the density of fixed atoms, as well as the strength of the forces between them. So, pieces of different materials with identical dimensions will have differing resistances. The general property of a material to resist the flow of electric current is called **resistivity**, which has the symbol rho, ρ, and SI units ohm metres, Ω m.

RESISTIVITY INVESTIGATION EXAMPLE

An investigation like the one explained above produced the results shown in fig D. This was for an aluminium wire with a measured diameter averaging 0.22 mm (giving A = 3.8 x 10-8 m2). A single 1.5 V cell was used to drive a current, which was measured at each different length. These data were then used to calculate the resistances plotted on the graph for each length. From the graph, we can use its gradient to find the resistivity of aluminium.

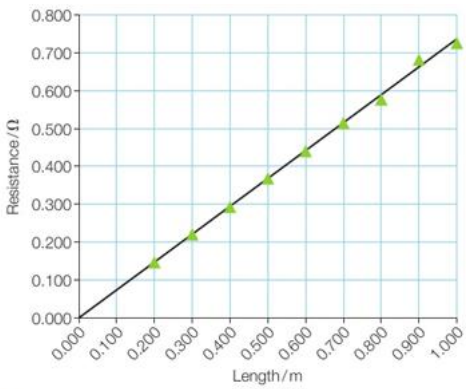


fig D From the gradient of this graph, it is possible to calculate the resistivity of the aluminium wire in the investigation.

$$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{\Delta R}{\Delta l} = \frac{0.730 - 0.000}{1.000 - 0.000} = 0.730$$

$$R = \frac{\rho l}{A}$$

As we have R on the y-axis and l on the x-axis, comparing the equation above with $y = mx + c$ shows that the gradient is equivalent to $\frac{\rho}{A}$, so we can find the resistivity by multiplying the gradient by the cross-sectional area.

$$\therefore \quad \rho = \frac{RA}{l}$$

$$\therefore \quad \rho = 0.730 \times A = 0.730 \times 3.8 \times 10^{-8} = 2.8 \times 10^{-8}\ \Omega\,\text{m}$$

Experimentally, we have found that the resistivity of aluminium is $2.8 \times 10^{-8}\ \Omega$ m.

SUBJECT VOCABULARY

resistivity for a material, the same value as the resistance between opposite faces of a cubic metre of the material

RESISTIVITY EQUATION EXAMPLE

See **fig A** and **table A**. What is the resistance of a piece of copper fuse wire if it is 0.40 mm in diameter and 2 cm long?

wire radius = 0.20 mm = 2.0×10^{-4} m

cross-sectional area, $A = \pi r^2 = \pi \times (2.0 \times 10^{-4})^2 = 1.3 \times 10^{-7}\ \text{m}^2$

$$R = \frac{\rho l}{A}$$

$$R = \frac{1.7 \times 10^{-8} \times 0.02}{1.3 \times 10^{-7}}$$

$$R = 2.6 \times 10^{-3}\ \Omega$$

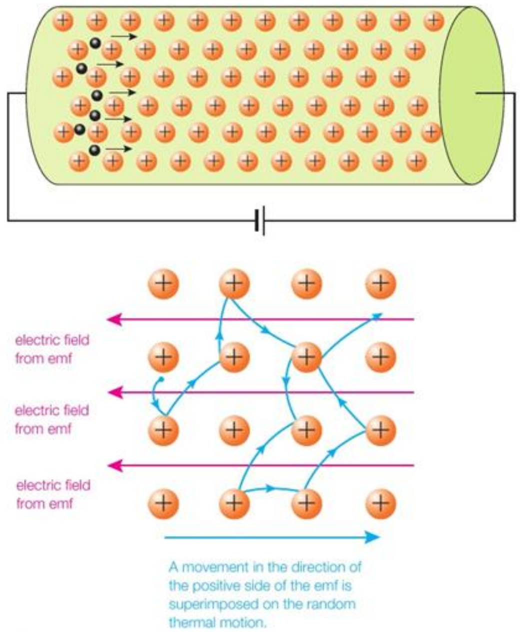
MATERIAL	RESISTIVITY, ρ / Ω m AT 20 °C	$\frac{\Delta\rho}{\Delta t}$ / % °C^{-1}
silver	1.6×10^{-8}	+0.38
copper	1.7×10^{-8}	+0.40
aluminium	2.8×10^{-8}	+0.38
constantan	4.9×10^{-7}	+0.003
germanium	4.2×10^{-1}	−5.0
silicon	2.6×10^{3}	−7.0
polyethene	2×10^{11}	
glass	$\sim 10^{12}$	
epoxy resin	$\sim 10^{15}$	

table A Resistivity varies greatly between materials, and is also dependent on temperature. Note the small change in resistivity with temperature for constantan, an alloy of copper and nickel; this information is used where accurately known resistance is important.

4A 5 Conduction and resistance

CONDUCTION IN METALS

The structure of metals has a regular lattice of metal atoms. These are bonded together through the sharing of electrons, which act as if they were associated with more than one atom. However, if a source of emf is connected across the metal, the electric field it sets up in the metal will push the negative electrons towards the positive end of the field. The slow overall movement of the electrons is called their **drift velocity.**



▲ **fig A** Conduction in metals happens as the free electrons add an overall movement along the direction of the voltage across the conductor (towards the positive anode) to their random collisions and vibrations.

EXAM HINT

In exam answers, draw the direction of the electron movement changing sharply for each collision.

The transport equation

The value of the electric current in a metal can be calculated from the fundamental movement of the electrons if we remember the definition that $I = \frac{\Delta Q}{\Delta t}$

If we consider the cylinder shaded on the diagram in **fig B** as the length of the wire that the charges move through in a time Δt, then we need to calculate how much charge flows through it in that time. There are n electrons per cubic metre of this metal, and the wire has a cross-sectional area, A. Their movement is at a drift velocity, v, and the distance this takes them along the wire in that time is Δx. So: $\Delta x = v\Delta t$.

The total charge will be the number of electrons multiplied by the charge on each, e. The number of electrons will be their density, n, multiplied by the volume of the cylinder, V, that they travel through in Δt.

$$\therefore \quad \Delta Q = n \times V \times e = n \times A\Delta x \times e = n \times Av\Delta t \times e$$

$$\therefore \quad I = \frac{\Delta Q}{\Delta t} = \frac{nAv\Delta te}{\Delta t}$$

$$\therefore \quad I = nAve$$

This is called the **transport equation**.

EXAMPLE OF A TRANSPORT EQUATION

What is the drift velocity of the electrons in a copper wire which has a diameter of 0.22 mm and carries a current of 0.50 A? The number density of conduction electrons in copper, $n = 8.5 \times 10^{28}\,\text{m}^{-3}$.

wire radius = 0.11 mm = 1.1×10^{-4} m

cross-sectional area, $A = \pi r^2 = \pi \times (1.1 \times 10^{-4})^2 = 3.8 \times 10^{-8}\,\text{m}^2$

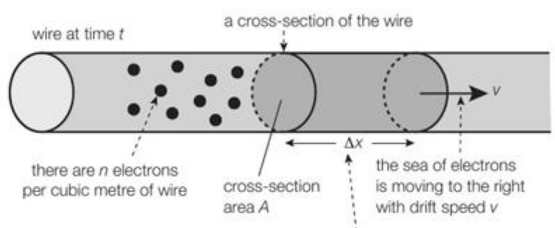
$I = nAve$

$$\therefore \quad v = \frac{I}{nAe}$$

$$= \frac{0.50}{8.5 \times 10^{28} \times 3.8 \times 10^{-8} \times 1.6 \times 10^{-19}}$$

$$\therefore \quad v = 9.67 \times 10^{-4}\,\text{ms}^{-1}$$

$v = 0.97\,\text{mms}^{-1}$



The sea of electrons has moved forward a distance $\Delta x = v\Delta t$. The shaded volume is $V = A\,\Delta x$.

▲ **fig B** A section of metal wire with dimensions to show how to make calculations using the transport equation.

RESISTANCE

Electrons in a metal's lattice have high speed but slow progress due to electrical resistance. Temperature affects the frequency of collisions, with higher temperatures causing more collisions and slower electron drift. Higher currents cause more collisions, increasing temperature and resistance. This causes the I-V graph for a filament lamp to flatten out when higher voltages drive higher currents.

RESISTIVITY

Table A in Section 4A.4 demonstrates the resistivity of materials like good conductors, semiconductors, and insulators. It reveals that resistivity varies with temperature, with semiconductors falling as temperature rises due to higher conduction electron density in the transport equation. This is because materials like silicon conduct better at higher temperatures.

MATERIAL	n / m^{-3} AT 20 °C	n / m^{-3} AT 50 °C
copper	8.42×10^{28}	8.27×10^{28}
aluminium	18.2×10^{28}	17.9×10^{28}
silicon	8.49×10^{15}	7.42×10^{17}
germanium	1.56×10^{19}	6.82×10^{20}

table A The number of charge carriers for some common materials, at room temperature and at a higher temperature.

A slight decrease in n for metals at higher temperatures is due to their thermal expansion, rather than any change in the number of available conduction electrons. It is not as significant as the increase in collisions between metal atoms and conduction electrons caused by increased thermal vibrations.

SUBJECT VOCABULARY

drift velocity the slow overall movement of the charges in a current
transport equation $I = nAvq$. This defines electric current, I, from a fundamental basis. It is the product of charge carriers, n; the charge on those carriers, q; the cross-sectional area of the conductor, A; and the drift velocity of the charge carriers in that conductor, v

4A 6 Semiconductors

CONDUCTION IN SEMICONDUCTORS

Semiconductors are solid materials with small numbers of delocalised electrons free to conduct, like silicon. Free atoms have discrete energy levels, while solid materials have many atoms close together, allowing wider energy levels and forming energy bands. The **valence band** is an energy level where electrons remain tied to atoms and do not form an electric current. Those that gain energy to move up to the **conduction band** become delocalised and can move through the semiconductor as part of a current.

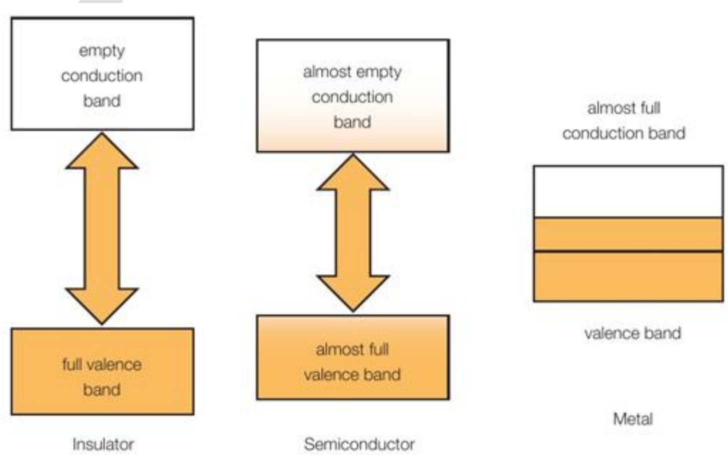


fig A The gaps between energy bands within materials explain why they are conductors or insulators. If there is a large energy gap, electrons will need to gain a lot of energy to leave their atoms and conduct a current.

Conduction 'HOLES'

Conduction holes are empty spaces left by electrons entering the conduction band, leaving atoms with positive charges. When an electron fills a hole, it moves in the opposite direction, adding to current flow in semiconductors and electrolysis. This is similar to the current flow in electrolysis.

I–V CHARACTERISTICS OF A SEMICONDUCTOR DIODE

A diode is made by joining different types of semiconductors, which normally creates an energy barrier at the junction between them. This blocks the movement of charge carriers (holes and electrons) across the barrier. This barrier can be overcome in the forward direction if a small forward voltage is applied. In the reverse direction, only very few charge carriers can pass through at low voltages. They account for a tiny 'leakage current'. Once the reverse voltage becomes large enough, it can overcome the large reverse energy barrier and force the conduction process in the opposite direction.

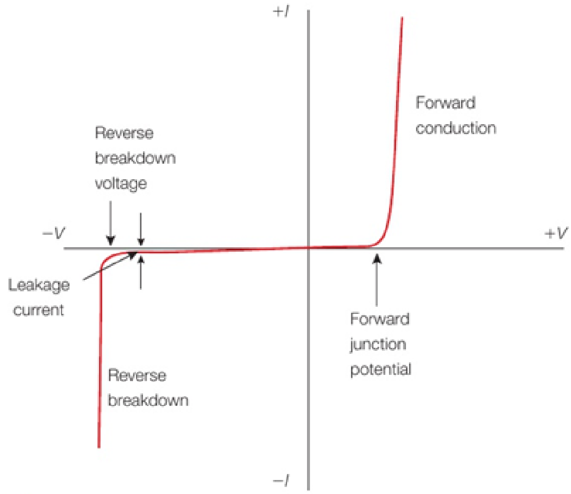


fig B The *I–V* characteristics for a semiconductor diode in detail.

LIGHT DEPENDENT RESISTORS (LDRS) AND THERMISTORS

Light dependent resistors (LDRs) and thermomers work similarly, with LDRs having lower resistance in brighter conditions due to increased conduction electrons. Thermistors, on the other hand, rely on thermal energy from the surroundings, with negative temperature coefficient thermistors being the most common type. Both work to improve conductivity.

INSULATORS

Electrical insulators can be thought of as materials in which the energy gap between the valence band and the conduction band is so large that there are virtually zero electrons available for conduction. There will therefore be no conduction holes either. A very large input of energy is required in order to make the material conduct. Often this results in melting, or other damage, before the material becomes electrically conducting.

SUPERCONDUCTIVITY

Resistance increases with higher temperatures, because the higher level of internal energy in the material causes more vibration of the fixed ions. These ions collide more often with charge carriers to reduce their speed of movement through the material. Reducing the temperature therefore reduces resistance, allowing greater current flow.

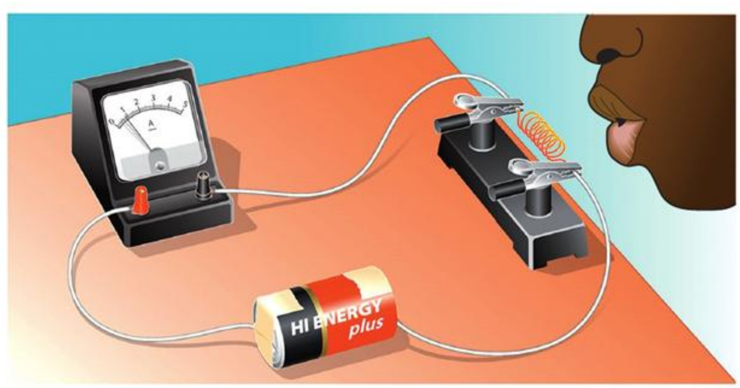


fig C If we cool a conductor, the current through it is seen to increase because its resistance drops.

Cooling to ever lower temperatures continues this trend, until something quite unexpected happens. Below a certain **critical temperature**, the resistance suddenly drops to zero. This is called **superconductivity**. The critical temperature varies with material, but for most metals it will be below -243°C.

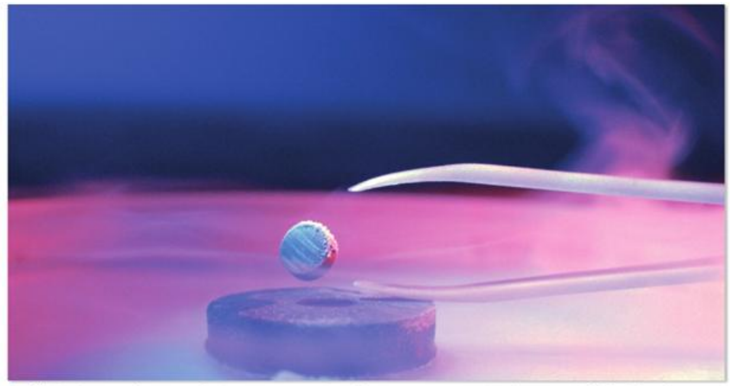


fig D Superconductors can levitate magnets, as they do not accept penetration by magnetic fields. This is the Meissner Effect.

WORKED EXAMPLE

semiconductors materials with a lower resistivity than insulators, but higher than conductors. They usually only have small numbers of delocalised electrons that are free to conduct

valence band a range of energy amounts that electrons in a solid material can have which keeps them close to one particular atom

conduction band a range of energy amounts that electrons in a solid material can have which delocalises them to move more freely through the solid

critical temperature the temperature below which a material's resistivity instantly drops to zero

superconductivity the electrical property of a material having zero resistivity

4B Complete electrical circuits

4B 1 Series and parallel circuits

CURRENT

The total amount of charge within a circuit cannot increase or decrease when the circuit is functioning.

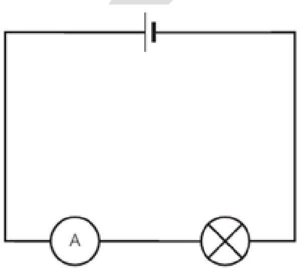


fig A Ammeters must always be in series with the component they are measuring.

CURRENTS IN SERIES CIRCUITS

Any group of components that follow in series in a circuit, with no junctions in the circuit, must have the same current through them all.

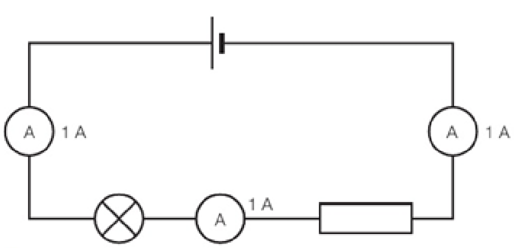
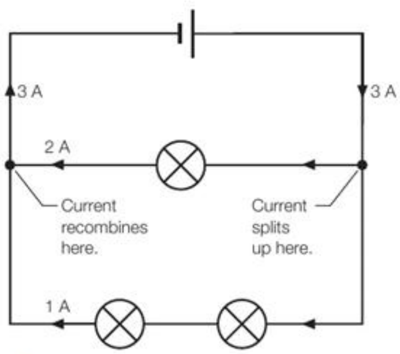


fig B The current is constant throughout a series loop in a circuit.

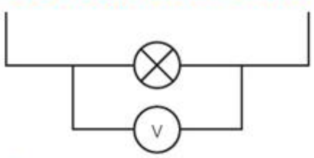
CURRENTS IN PARALLEL CIRCUITS

Whenever a current encounters a junction in a circuit, the charges can only go one way or the other, so the current must split. The proportions that travel along each possible path will be in inverse proportion to the resistance along that path. If the path has high resistance, the current is less likely to go that way. However, the total along the branches must add up to the original total current.



▲ **fig C** Current splits at circuit junctions, but the total current will remain the same, in order to conserve charge.

VOLTAGES AROUND CIRCUITS



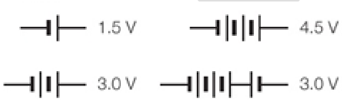
▲ **fig D** Voltmeters must always be in parallel across the component they are measuring.

LEARNING TIP

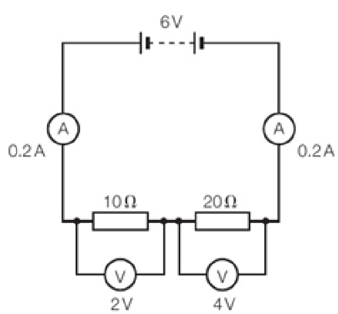
Voltages are measures of energy change as charge passes through a component. As such, they must be measured from one side to the other, i.e. the voltmeter must always be placed in parallel across the component.

VOLTAGES IN SERIES CIRCUITS

Any group of emfs that follow in series in a circuit, with no junctions in the circuit, will have a total emf that is the sum of their individual values, that accounts for the direction of their positive and negative sides. For example, two 1.5 V cells in a TV remote control will be in series, so that they supply an emf of 3.0 V. This is also true for any string of potential differences in series.



▲ **fig E** Emfs in series will add up. Take care to account for their direction, as cells in opposite directions oppose each other and cancel out the emf they would supply.



▲ **fig F** Potential differences in series will add up. What would be the total pd across the two resistors in this diagram?

VOLTAGES IN PARALLEL CIRCUITS

If the total voltage across any branch of a circuit is known, then the voltage across any other branch in parallel with it will be identical.

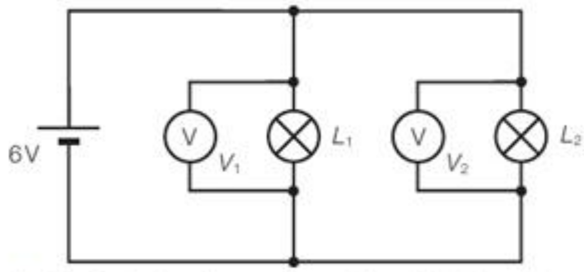


fig G Both voltmeters here will read 6 V: the voltage is the same across all parallel branches within a circuit.

RESISTANCE

Resistance can be calculated from Ohm's law as V/I. This can be done for any individual component or a whole branch of a circuit, as long as the voltage and current are known. This also means that if we follow the rules for current and voltage variations around series and parallel circuits, we come up with simple rules for the total resistance in series and parallel combinations.

RESISTORS IN SERIES

Any group of resistances that follow in series in a circuit, with no junctions in the circuit, will have a total resistance that is the sum of their individual values. In fig F, we could use the total pd across both resistors and the current through them to calculate their total resistance:

$$\text{total } R = \frac{\text{total } V}{I} = \frac{6}{0.2} = 30\,\Omega$$

You will see that this matches with our rule that

$$R_{total} = R_1 + R_2 = 10 + 20 = 30\,\Omega$$

DERIVATION

Resistors in series will have a total pd across them that is the sum of their individual pds. The current through them will be the same for each one. Take an example of three resistors in series:

potential difference: $V_{total} = V_1 + V_2 + V_3$

and $V = IR$

$\therefore \quad IR_{total} = IR_1 + IR_2 + IR_3$

$\therefore \quad R_{total} = R_1 + R_2 + R_3$

RESISTORS IN PARALLEL

The total resistance of a group of resistors in parallel can be calculated from the equation:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

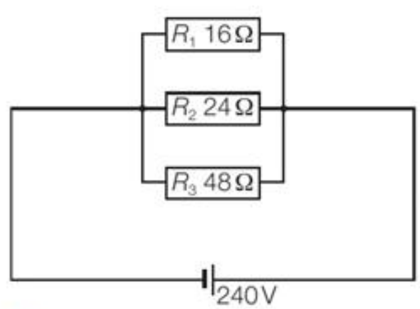


fig H Resistors in parallel follow a reciprocal sum rule to find their total resistance.

DERIVATION

Resistors in parallel will have a total current through them that is the sum of their individual currents. The pd across them will be the same for each branch. Take an example of three resistors in parallel:

current: $I_{total} = I_1 + I_2 + I_3$

and $I = \frac{V}{R}$

$$\therefore \quad \frac{V}{R_{total}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\therefore \quad \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

RESISTORS BRANCH COMBINATIONS

If a circuit has a mixture of series and parallel combinations, we must use the rules we have for each group of resistors. Then, we should use the rules again to combine these sub-totals.

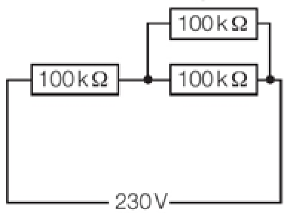


fig I Calculate the total resistance of resistor groups separately, before using the appropriate rule to combine the sub-totals together.

QUANTITY	SERIES CIRCUIT RULES	PARALLEL CIRCUIT RULES
current	current is the same throughout	$I_{total} = I_1 + I_2$
voltage	$V_{total} = V_1 + V_2$	voltage is the same on each branch
resistance	$R_{total} = R_1 + R_2$	$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$

table A Summary of the electrical rules in series and parallel circuits.

4B 2 Electrical circuits rules

ELECTRIC CURRENT RULE

Electric current rule: the algebraic sum of the currents entering a junction is equal to zero:

$\Sigma I = 0$

In order to conserve electric charge, the sum of all the currents arriving at any point (such as a junction) in a circuit is equal to the sum of all the currents leaving that point.

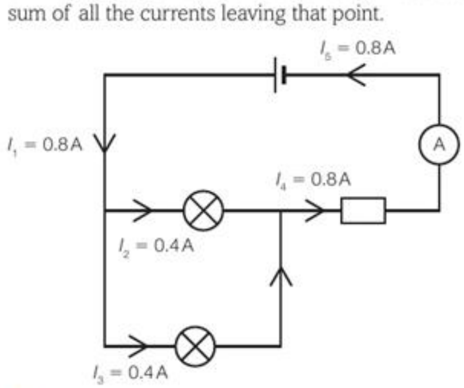


fig B Current rule example circuit.

In the circuit diagram of **fig B**, we could think about the current entering and leaving the resistor. This is 0.8 A entering and 0.8 A leaving. If we added these together, accounting for the direction:

$I_{in} + I_{out} = I_4 + -I_5 = 0.8 + -0.8 = 0$

This simple example confirms the rule at work in the resistor.

Similarly, consider the currents at the point where the circuit splits to send current through the two bulbs:

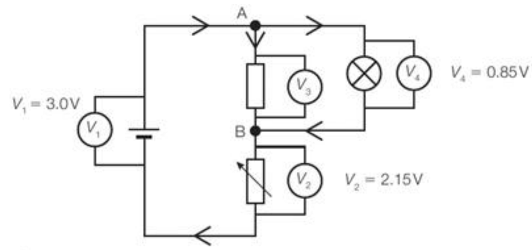
$I_1 + -I_2 + -I_3 = 0.8 + -0.4 + -0.4 = 0$

The same overall result would appear at any point in the circuit – charge must always be conserved.

ELECTRICAL VOLTAGES RULE

In order to conserve electrical energy around any closed loop, Electrical voltages rule: the sum of emfs is equal to the sum of the pds around that loop.

Note that a loop being considered might not be the whole of a circuit - the only requirement is that the loop is complete. Note also that the potential differences referred to here are defined as being the products of component currents and resistances, as per Ohm's law. So, if we isolate a particular closed loop, it may have potential differences with positive values, and with negative values when the current flows in the opposite direction.



▲ **fig C** Voltages rule example circuit.

In **fig C**, consider the voltages around the closed loop that consists of the cell, variable resistor and fixed resistor. We know V_3 must be 0.85 V, as all parallel branches must have the same voltage and it is parallel to V_4. If you move clockwise around the loop, and start at the cell, the cell is the only emf, so the sum of all emfs is 3.0 V. Following the same route to sum the potential differences, we ignore the cell as it is an emf, not a pd. This means the sum of the pds will be:

$$\Sigma V = V_3 + V_4 = 0.85 + 2.15 = 3.0\text{ V}$$

As the current is flowing in the same direction through the cell and both variable resistor and fixed resistor, these voltages are all in the same direction, and so:

$$\Sigma \varepsilon = \Sigma V$$

Next, consider just the closed loop containing the fixed resistor and the lamp. If $\Sigma \varepsilon = \Sigma V$, and there are no emfs, the pds must total zero. Starting at point A and moving clockwise around the loop, the current moves through the lamp, which is measured at V_4 = 0.85 V. Continuing to point B and then back up to complete the loop at A means going through the fixed resistor against the flow of the current. This means the pd will be negative, although also 0.85 V. So in this consideration, $V_3 = -0.85$ V.

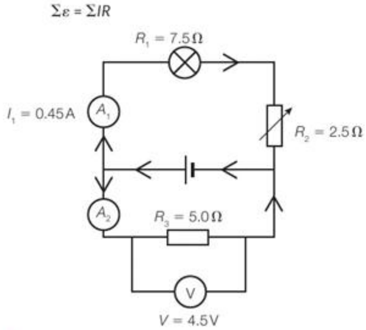
$$\Sigma V = V_3 + V_4 = -0.85 + 0.85 = 0$$

$$\therefore \quad \Sigma \varepsilon = \Sigma V$$

VOLTAGES RULE WITH CALCULATED PDS

Potential differences are the product of currents passing through resistances, thus transferring electrical energy. Each pd could be calculated from Ohm's law as V = IR. So this becomes:

$$\Sigma \varepsilon = \Sigma IR$$



▲ **fig D** Confirming $\Sigma \varepsilon = \Sigma IR$.

In **fig D**, we can confirm the voltages rule around the upper closed loop, through the cell, ammeter A_1, lamp and variable resistor. The only emf is the cell, and this will have an emf of 4.5 V, as shown on the voltmeter at the bottom of the diagram.

$$\Sigma \varepsilon = 4.5\text{ V}$$

Ammeters have zero resistance, and so A_1 contributes nothing to the sum of pds. The current must be the same through the lamp and the variable resistor, so the sum of the potential differences will be calculated from:

$$\Sigma IR = I_1R_1 + I_1R_2 = (0.45 \times 7.5) + (0.45 \times 2.5) = 3.375 + 1.125$$

$$\Sigma IR = 4.5\text{ V}$$

$$\therefore \quad \Sigma \varepsilon = \Sigma IR$$

SUBJECT VOCABULARY

electric current rule the algebraic sum of the currents entering a junction is equal to zero:

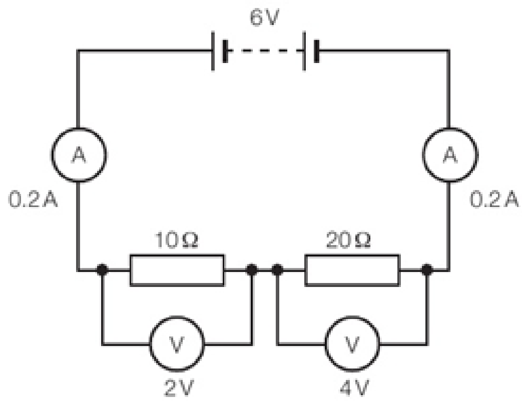
$$\Sigma I = 0$$

voltages circuit rule around a closed loop, the algebraic sum of the emfs is equal to the algebraic sum of the pds:

$$\Sigma \varepsilon = \Sigma IR$$

4B 3 Potential divers

We have seen that all the voltage supplied by emfs in a circuit loop must be used by components as potential differences at other points around the circuit loop. The way that the voltage splits up is in proportion to the resistances of the components in the circuit loop, as shown in fig B. The voltages across the two resistors must add up to 6 V. The 20 resistor has twice the resistance, so takes twice the voltage of the 10 resistor. This is a potential divider **circuit**.



▲ **fig B** All of the 6 volts supplied by the battery must be used up by the resistors. The total pd of 6 V is split into 2 V and 4 V in proportion to their resistances. Ammeters have zero resistance, so transfer zero energy and have zero pd.

Ohm's law explains why the voltage is split in proportion to the resistance. Consider the resistances and voltages in **fig B** in more general terms, as R_1 (for the 10 Ω resistor) and R_2, and their corresponding pds V_1 and V_2. This will make the calculation valid for all values of resistances, and calculating the current through each one gives:

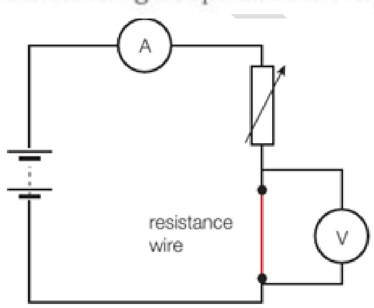
$$I_1 = \frac{V_1}{R_1} \quad \text{and} \quad I_2 = \frac{V_2}{R_2}$$

However, they both have the same current passing through them, so $I_1 = I_2$.

$$\therefore \quad \frac{V_1}{R_1} = \frac{V_2}{R_2}$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{R_1}{R_2}$$

So the voltage is split in the same proportion as the ratio of the resistances for any values of resistance.



▲ **fig C** Adjusting the variable resistor alters the proportion of the voltage it takes, and so the pd that is left for the resistance wire can be varied to any value we choose.

Section 4A.3 demonstrates the use of a potential divider circuit to analyze current against potential difference graphs. A resistance wire is used to measure current and voltage. High resistance results in a high potential difference, while decreasing resistance increases potential difference. Adjustment of resistance is necessary for I-V plot.

THE POTENTIAL DIVIDER EQUATION

As the pd in a potential divider circuit is split in proportion to the resistances of the components, we can calculate the pd across them mathematically.

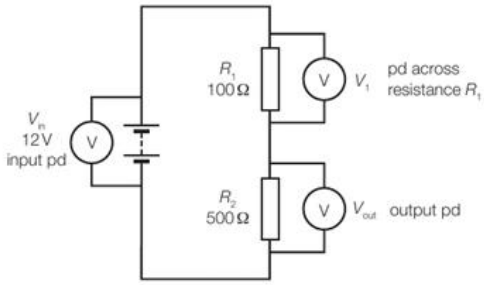


fig D Generalised potential divider circuit.

In the circuit diagram of **fig D**, we have the general terminology for the values used in calculations of a potential divider circuit: V_{out}, R_1, R_2 and V_{in}. Using these, we can calculate the pd across the component of interest, which is usually drawn as the second resistance, R_2, using the potential divider equation:

$$V_{out} = V_{in} \times \frac{R_2}{(R_1 + R_2)}$$

THE POTENTIOMETER

fig E A potentiometer allows us to supply any voltage up to the maximum provided by our emf source.

The **potentiometer** is an electrical component that uses a potential divider to supply variable voltage. It uses a moveable contact to adjust the resistance wire lengths on either side of the resistors, allowing for the adjustment of their relative resistances and voltage across a separate circuit.

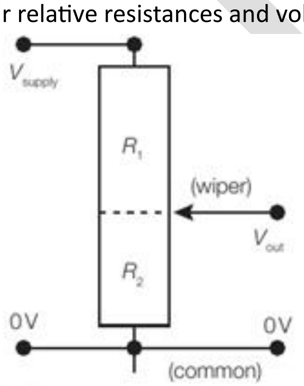
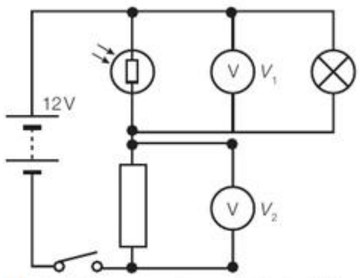


fig F Adjusting the position of the slider (wiper) on the potentiometer changes its position on the resistance wire, so that the proportions of resistance on either side change, altering the potential differences on either side of the wiper.

POTENTIAL DIVIDERS IN SENSOR CIRCUITS

If one of the resistors in a potential divider is a sensor component (such as thermistor or LDR) then we can use its changing resistance to control an external circuit.



▲ **fig G** A sense and control circuit which makes the light brighter as the light level in the room drops.

In fig G, a lamp is powered by the same voltage as the LDR, which increases resistance with ambient light levels. In bright conditions, the LDR has a low resistance, consuming a small proportion of the 12 V supplied by the battery. As the surroundings become darker, the LDR resistance increases, increasing the proportion of the 12 V dropped across the bulb, making it brighter. This circuit increases the lamp's brightness as the surroundings become darker. Many electronic components have a switch-on voltage of 5 V, which can be used with a sensor in a potential divider circuit.

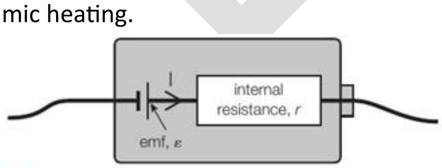
SUBJECT VOCABULARY

potential divider a circuit designed to provide specific voltage values by splitting an emf across two resistors

potentiometer a version of the potential divider in which a single resistance wire is used in two parts to form the two resistances. A sliding connection on the wire can be adjusted to alter the comparative resistances and thus alter the output pd from the potentiometer

4B 4 EMF and internal resistance

Internal resistance is a small but non-zero resistance of an electromotive force (emf) source, such as batteries. While it may not significantly affect a circuit's performance, it can cause significant power loss when large currents are used. This energy transfer by the emf source can cause a smaller voltage to the rest of the circuit, potentially affecting the circuit's performance. Additionally, a large current through the internal resistance could damage the emf source due to ohmic heating.



▲ **fig A** Any emf source could be considered to be a pure emf with a small resistive potential difference acting in series with the emf.

The effect of having an internal resistance is that an emf will never be able to fully supply its maximum voltage. There will always be a small drop in voltage over the internal resistance and this drop will be bigger with a higher current. The pd over the internal resistance, r, can be found from Ohm's law:

$$V_{internal} = Ir$$

This pd is sometimes referred to as 'lost volts', as the circuit appears to have fewer volts in use by the load than the emf should be supplying. A measurement of voltage across the terminals of a cell powering a circuit would not measure the emf; it would measure the emf minus the lost volts. This 'terminal pd' would then be:

$$V_{terminal} = \varepsilon - Ir$$

As all parallel branches have the same voltage across them, the terminal pd would also equal the pd across the load resistance.

$$V_{terminal} = V_{load} = \varepsilon - Ir$$

By considering energy conservation, we can draw up an equation for the circuit that includes the internal resistance.

$$\Sigma\varepsilon = \Sigma V$$

$$\therefore \quad \varepsilon = V_{load} + V_{internal}$$

$$\therefore \quad \varepsilon = V_{load} + Ir = IR + Ir$$

The circuit shown in **fig B** demonstrates what the mathematics means practically. The voltmeter will be measuring the voltage actually supplied by the cell, so that will be its pure emf minus the pd across the internal resistance, or lost volts. However, as the ammeter should have zero pd, the voltmeter will also be reading the value of the potential difference across the variable resistor, *R*.

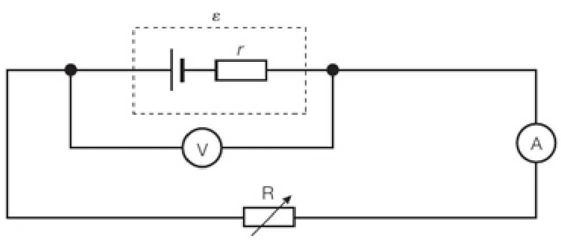
$$\varepsilon - V_r = V_R$$

Or, in terms of the ammeter reading, *I*, and the voltmeter reading, *V*:

$$\varepsilon - Ir = V$$

$$\therefore \quad \varepsilon = V + Ir$$

which is, again, a statement of energy conservation in this circuit.



▲ **fig B** Circuit for demonstrating the effect of internal resistance.

SUBJECT VOCABULARY

internal resistance the resistance of an emf source

4B 5 Power in electric circuits

ELECTRICAL WORK

Work done has the symbol *W*, and as the equation defining potential difference includes a term for the amount of energy transferred, *E*, these two will be the same.

$$W = E$$

The equation for pd, in rearranged form, gives us:

$$W = V \times Q$$

The definition of current, in rearranged form, is:

$$Q = I \times t$$

▲ **fig A** As part of this circuit, the heart will transfer some electrical energy to kinetic energy. The circuit is doing work.

Combining these will give us the work done by a component in an electric circuit:

$$W = V \times I \times t$$

▲ **fig B** Electrical work done by a light bulb.

ELECTRICAL POWER

Power, *P*, is the rate of transfer of energy, or the rate of doing work. In an electrical circuit, the energy is dissipated by a component. The mathematical definition is:

$$P = \frac{E}{t}$$

OR

$$P = \frac{W}{t}$$

Incorporating the equation for work in a circuit from above:

$$P = \frac{VIt}{t}$$

$$P = VI$$

POWER DISSIPATED BY RESISTORS

We can also write variations of the electrical power equation, substituting in terms from Ohm's law so that we can calculate the power dissipated by resistors in a circuit.

$$P = VI$$

and

$$V = IR$$

If we only know current and resistance:

$$P = IR \times I = I^2R$$

Or, if we only know pd and resistance:

$$P = V \times \frac{V}{R} = \frac{V^2}{R}$$

EFFICIENCY

▲ **fig C** Modern electrical appliances come with a label indicating their efficiency, so that consumers can choose good value for running costs when making purchases.

The work and power of electrical devices can be calculated easily, but their **efficiency** is crucial. If most energy isn't transferred to a useful store, the cost of the energy is wasted, as explained in Section 1B.2.

You will remember that efficiency is defined mathematically – and this applies to electrical energy transfers exactly as it did to mechanical energy transfers – as:

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}}$$

The answer will be a decimal between zero and one. It is common to convert this to a percentage value (multiply the decimal by 100).

SUBJECT VOCABULARY

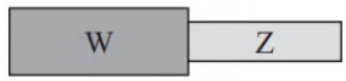
efficiency the ability of a machine to transfer energy usefully:

$$\text{efficiency} = \frac{\text{useful work done}}{\text{total energy input}}$$

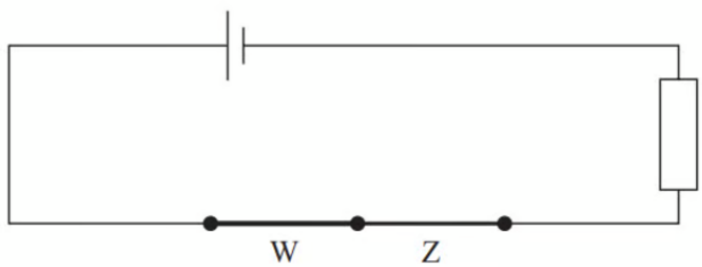
$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

Revision questions

1.a) Two copper wires, W and Z, are joined as shown. Wire W has twice the diameter of wire Z.



W and Z are connected in series with a cell and resistor, as shown.



State the purpose of the resistor in this circuit.

b)

The current I in a conductor is given by the formula

$$I = nqvA$$

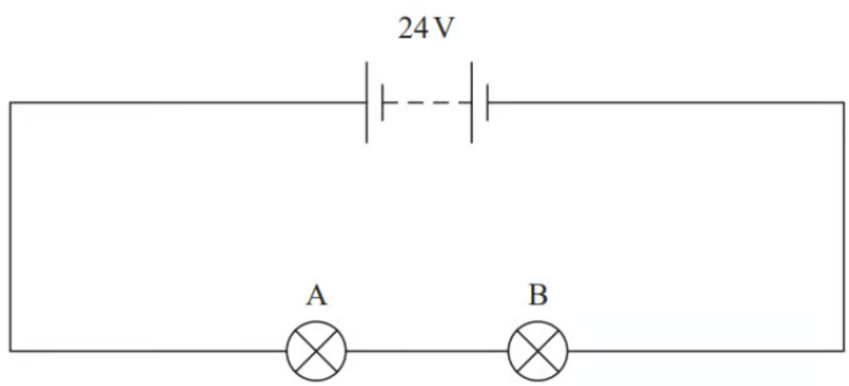
Complete the following table, which shows ratios of quantities for wires W and Z. The first row has been completed for you.

Ratio	Value	Reason
$\frac{n_w}{n_z}$	1	Both wires are made of the same material
$\frac{I_w}{I_z}$		
$\frac{v_w}{v_z}$		

2. a) A filament lamp is marked 12 V 60 W. The filament is made from a long metal wire with a diameter of 0.25 mm. The metal has a resistivity of $5.6 \times 10^{-8}\ \Omega$ m when the wire is at normal operating temperature.

Calculate the length of the wire in the filament.

b) A student has two filament lamps. Lamp A is marked 12 V 60 W and lamp B is marked 12 V 30 W. The student sets up the circuit shown.



The student states that both lamps will operate normally.

Evaluate whether the student's statement is correct.

3) a) A nichrome wire of length 0.45 m has a cross-sectional area of 2.5×10^{-7} m^2.

The resistance of the wire is 2.0 Ω.

Calculate the resistivity of nichrome.

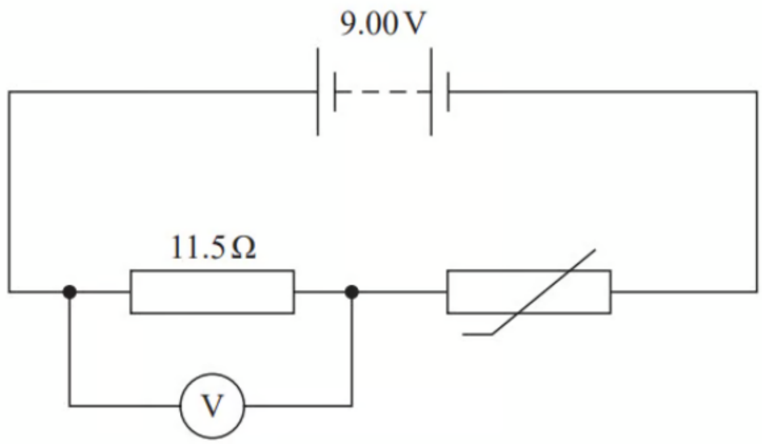
b) A potential difference of 3.0 V is applied across the nichrome wire.

Calculate the drift velocity of the conduction electrons in the nichrome wire.

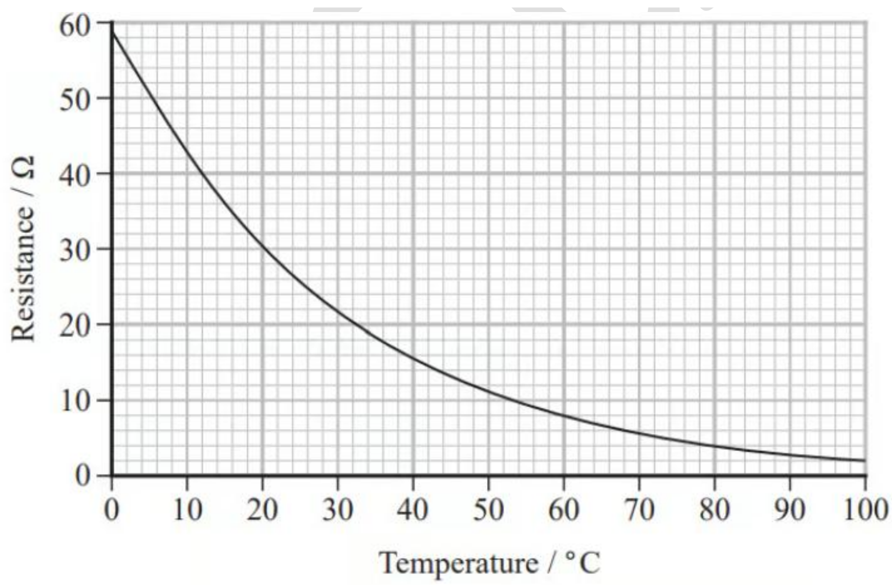
number of conduction electrons per m^3 = 9.0×10^{28} m^{-3}

4)a)

A student connected the circuit shown. The battery has negligible internal resistance.



The graph shows how the resistance of the thermistor varies with temperature.



The reading on the voltmeter is 3.42 V. Determine the temperature of the thermistor.

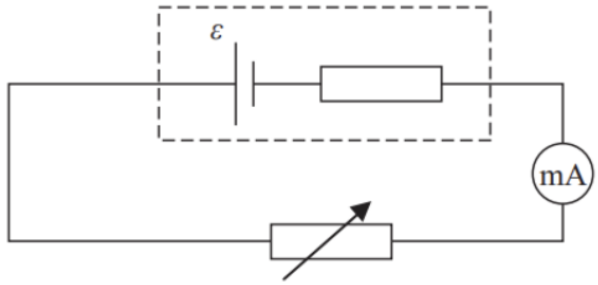
b)

The student suggests that if the e.m.f. of the battery is doubled, the reading on the voltmeter will double.

Assess whether the student's suggestion is correct.

5)a)

A cell of e.m.f. ε is connected in series with a variable resistor with resistance *R* as shown. The internal resistance of the cell is *r*.



When *R* is 12 Ω, the reading on the ammeter is 107 mA. The circuit is switched on for 300 seconds. In this time, 50 J of energy is transferred by the cell.

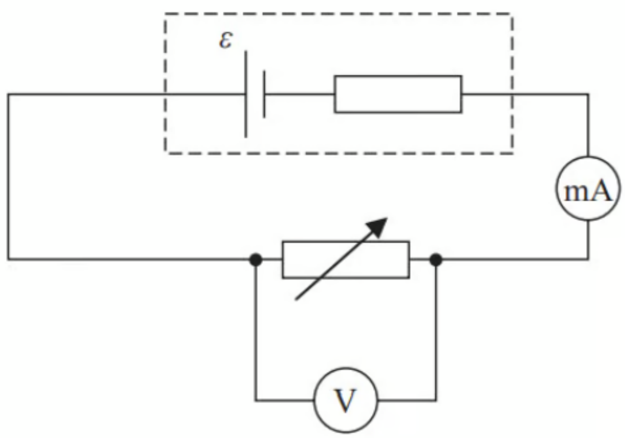
b)

Increasing *R* would make the terminal potential difference value closer to ε.

Explain why, without further calculation.

c)

A voltmeter is added to the circuit as shown.



Explain how this circuit can be used to determine a value for *r* using a graphical method.