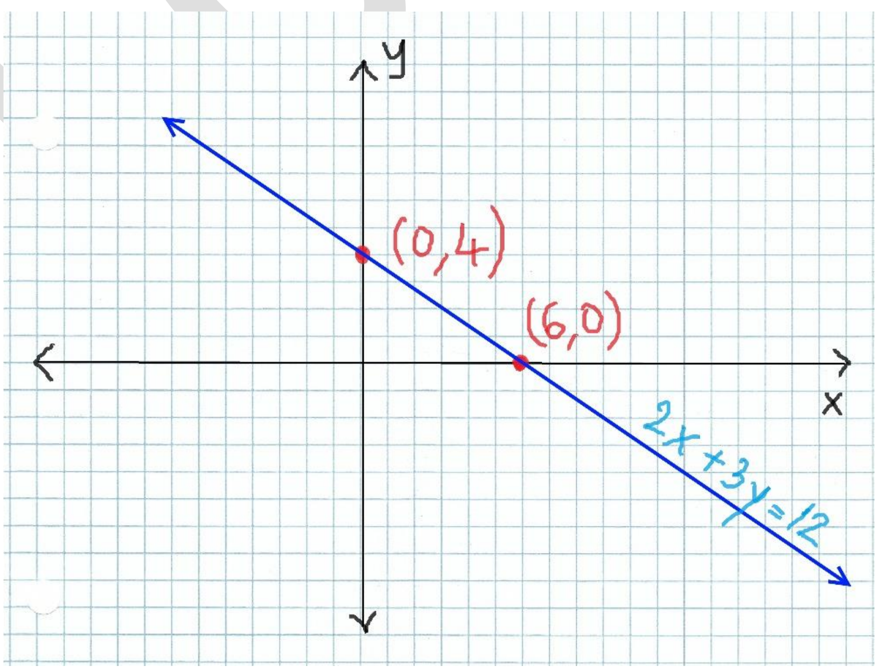
Edexcel OL Mathematics CODE: (4MA1)

Unit 2

Graphs 02



LEARNING OBJECTIVES

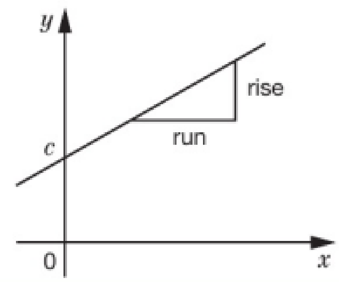
- Find the equation of a line.
- Sketch graphs using the gradient and intercepts
- Solve a pair of simultaneous equations using a graph

BASIC PRINCIPLES

- For a straight line, m is the **gradient**

$$m = \frac{\text{change in the } y \text{ coordinates}}{\text{change in the } x \text{ coordinates}} = \frac{\text{'rise'}}{\text{'run'}}$$

- The equation $y = mx + c$ represents a straight line with gradient m and y **intercept** c.
- The equation $ax + by = c$ also represents a straight line. It is most easily plotted by finding where it crosses the x and y axes.



STRAIGHT-LINE GRAPHS

EXAMPLE 1

FINDING THE EQUATION OF A STRAIGHT-LINE GRAPH

Find the equation of the straight line with gradient 2 that passes through the point (1, 3).

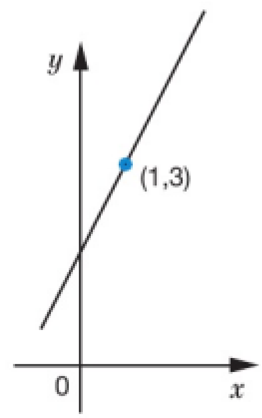
The general form is y = mx + c

The gradient, m, is 2 ⇒ y = 2x + c

(1, 3) lies on the line. Substitute x = 1 and y = 3 to find c.

3=2x1 +c ⇒ c =1

The equation is y = 2x + 1.



EXAMPLE 2

Find the equation of the straight line passing through A (2, 3) and B (6, 5).

The general form is y = mx + c

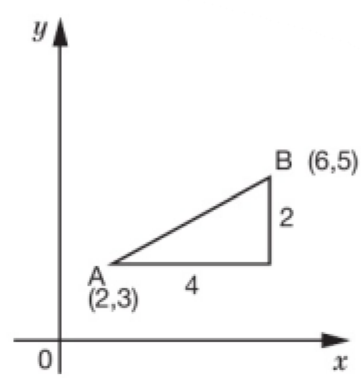
The gradient, m, is $\frac{2}{4} = \frac{1}{2} \Rightarrow y = \frac{1}{2}x + c$

(2, 3) lies on the line. Substitute x = 2 and y = 3 to find c.

This gives $3 = \frac{1}{2} \times 2 + c \Rightarrow c = 2$

The equation is $y = \frac{1}{2}x + 2$.

Check: Substituting the coordinates of B gives $5 = \frac{1}{2} \times 6 + 2$ which is correct.



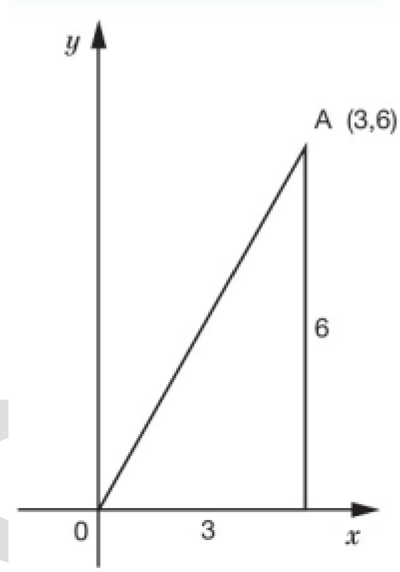
EXAMPLE 3

Find the equation of the straight line passing through A (3, 6) and the origin.

The general form is y = mx + c

The gradient, m, is $\frac{6}{3} = 2 \Rightarrow y = 2x + c$

(0, 0) lies on the line. Substitute x = 0 and y = 0 to find c.
This gives 0= 2x0 +c ⇒c=0
The equation is y = 2x.
Check: Substituting the coordinates of A gives 6 = 2 x 3 which is correct.
Note: all straight lines passing through the origin will have c = 0.



KEY POINTS

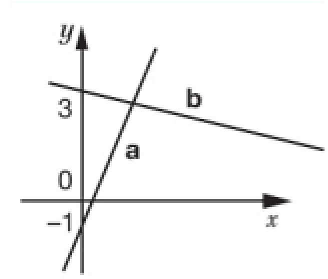
- If the gradient is m then the equation is $y = mx + c$.
- To find c, substitute a point that lies on the line.
- If the line passes through the origin, $c = 0$ so $y = mx$.

SKETCHING STRAIGHT-LINE GRAPHS

Sketching a straight-line means showing the approximate position and slope of the line without plotting any points.
A straight line in the form y = mx + c can be sketched by using the gradient and y-intercept.

EXAMPLE 04
Sketch these graphs.
a) y= 2x - 1

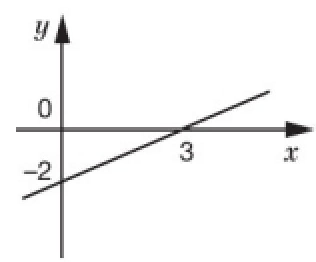
b $y = -\frac{1}{2}x + 3$



a) y = 2x-1 is a straight line with gradient 2 and y-intercept (0, -1).
b) y = -0.5.x + 3 is a straight line with gradient -0.5 and y-intercept (0, 3).

Graphs of the form ax + by = c could be sketched by rearranging the equation as y =...
It is quicker to find where the graph crosses the axes.

EXAMPLE 5
Sketch the graph 2x-3y = 6.
When y = 0, x = 3 so (3, 0) lies on the line.
When x = 0, y = -2 so (0, -2) lies on the line.



KEY POINTS

- A **sketch** is drawn roughly **to scale**, NOT plotted.
- Use the values of the gradient and intercept to sketch $y = mx + c$.
- Use where the graph crosses the axes to sketch $ax + by = c$.

SIMULTANIOUS EQUATIONS

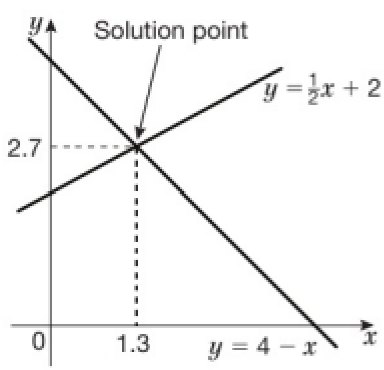
When there are two unknowns, two equations are needed to solve them. These are called simultaneous equations.
In Activity 1, the **simultaneous equations** were C = 999+ 1.1t and C = 495 + 1.8t.
The coordinates of the point of intersection of the graphs give the solution.

EXAMPLE 6
Solve the simultaneous equations y= ½ x+2 and y = 4 - x graphically.
First, make a table of values for each equation.

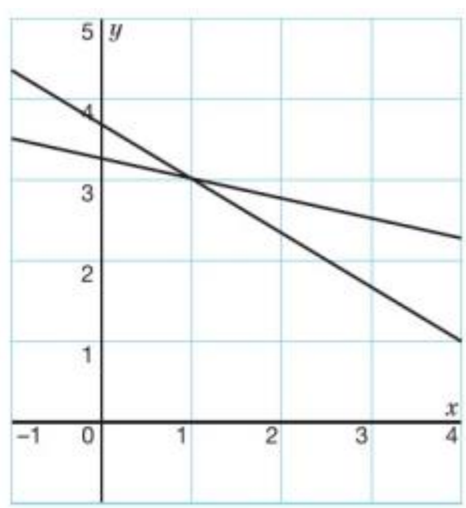
x	0	2	4
$y = \frac{1}{2}x + 2$	2	3	4

x	0	2	4
$y = 4 - x$	4	2	0

Next, draw accurate graphs for both equations on one set of axes.
The solution point is approximately x = 1.3, y = 2.7.



EXAMPLE 7
SKILL: PROBLEM SOLVING
At a craft fair stall Sarah buys two rings and three bracelets and pays $11. At the same stall Amy buys one ring and four bracelets and pays $13. How much does each item cost?



Let x be the cost of a ring and y be the cost of a bracelet.
For Sarah:2x + 3y = 11
For Amy:x + 4y = 13
The graph shows both these lines plotted.
The intersection is at x = 1 and y = 3. Each ring costs $1 and each bracelet costs $3.
Check for Sarah: 2 x 1 + 3 x 3 = 11
Check for Amy: 1x1+4x3 = 13

KEY POINTS

To solve simultaneous equations graphically:

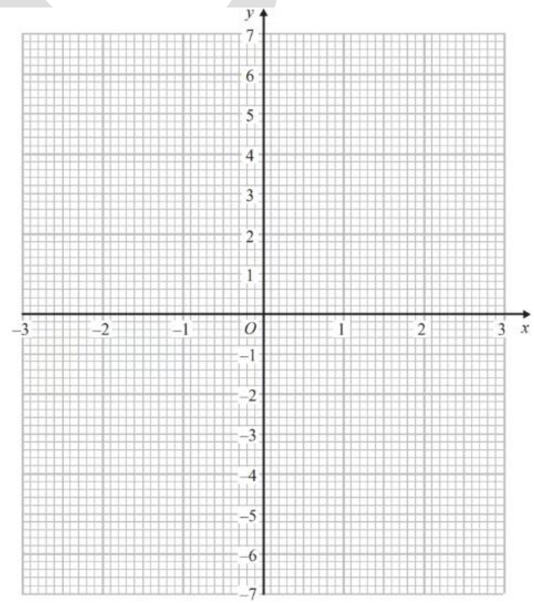
- Draw the graphs for both equations on one set of axes.
- Only plot three points for a straight-line graph.
- The solution is where the graphs **intersect**.
- If the graphs do not intersect, there is no solution.
- If the graphs are the same, there is an infinite number of solutions.

Revision questions

1)a) Complete the table of values for $y = x^2 - x - 6$

x	-3	-2	-1	0	1	2	3
y	6			-6			

b) On the grid, draw the graph of $y = x^2 - x - 6$ for values of x from 3 to 3



c)Use your graph to find estimates of the solutions to the equation x^2-x-6-2.

2)a) Solve algebraically the simultaneous equations

$x^2 - 4y^2 = 9$

$3x + 4y = 7$

b) Prove algebraically that the straight line with equation x - 2y = to the circle with equation $x^2 + y^2 = 20$

3) a) In all the following sequences, after the first two terms, the rule is to add the previous two terms to find the next term.

Write down the next two terms in this sequence.

1 1 2 3 5 8 13 ……… ……….

b) Each week Dan drives two routes, route X and route Y.

One week he drives route X three times and route Y twice. He drives a total of 134 miles that week.

Another week he drives route X twice and route Y five times. He drives a total of 203 miles that week.

Find the length of each route.

c) state an assumption that has been made in answering part (a)

4)a) The line $y = 3x + p$ and the circle $x^2 + y^2 = 53$ intersect at points A and B.
p is a positive integer.
Show that the x-coordinates of points A and B satisfy the equation.
$10x^2 + 6px + p^2 - 53 = 0$
b) The coordinates of A are (2,7) Work out the coordinates of B.
You **must** show your working.

5)The curve with equation $x^2 - x + y2 = 10$ and the straight line with equation $x - y = -4$ intersect at the points A and B.
Work out the exact length of AB.
Show your working clearly and give your answer in the form $\sqrt{a} / 2$ where a is an integer.

6)a) Solve the simultaneous equations
Show clear algebraic working.
$y = 3 - 2x$
$x^2 + y^2 = 18$

b) Solve the simultaneous equations
Show clear algebraic working.
$X - 6y = 5$
$Xy - 2y^2 = 6$

7)The straight-line L has equation $x - y = 3$
The curve C has equation $3x^2 - y^2 + xy = 9$
Land C intersect at the points P and Q.
Find the coordinates of the midpoint of PQ. Show clear algebraic working.

8)There are only r red counters and g green counters in a bag.
A counter is taken at random from the bag.
The probability that the counter is green is 3 / 7.
The counter is put back in the bag.
2 more red counters and 3 more green counters are put in the bag.
A counter is taken at random from the bag.

The probability that the counter is green is 6 / 13.
Find the number of red counters and the number of green counters that were in the bag originally.