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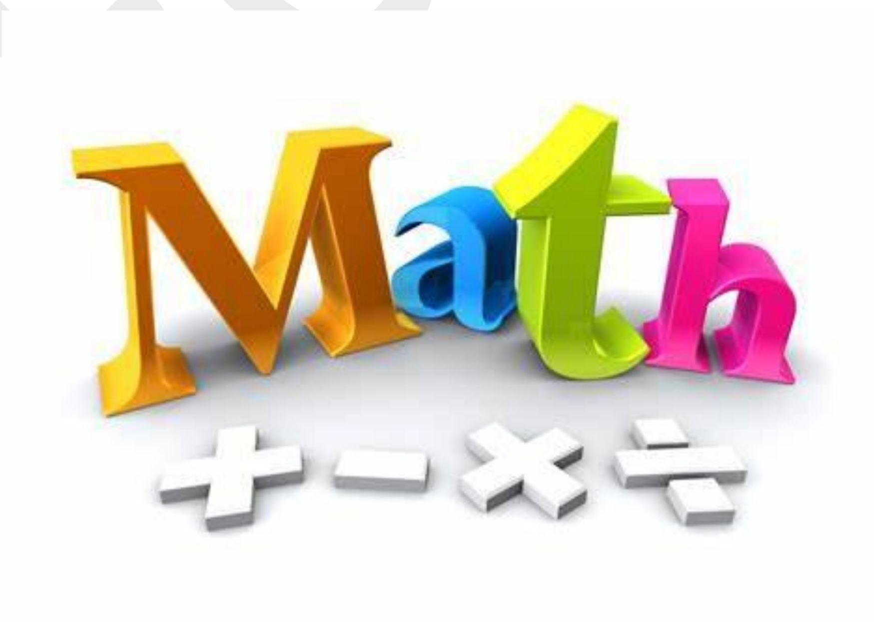
OL IGCSE

Mathematics

CODE: (4CP0)

Unit 07

4MB1



Number 07

BASIC PRINCIPLES

- **Simplify** fractions.
- Use a scientific calculator to work out arithmetic calculations (including use of the memory, sign change and power keys).
- All fractions can be written as either **terminating decimals** or decimals with a set of recurring digits.
- Fractions that produce terminating decimals have, in their simplest form, **denominators** with only 2 or 5 as factors. This is because 2 and 5 are the only factors of 10 (decimal system).
- The dot notation is used to indicate which digits recur. For example,
  - $0.\dot{3} = 0.333\ldots$
  - $0.\dot{2}\dot{3} = 0.232\,323\ldots$
  - $0.\dot{0}5\dot{6} = 0.056\,056\,056\ldots$
  - $1.2\dot{3}\dot{4} = 1.234\,343\,4\ldots$

RECURRING DECIMALS

Fractions that have an exact decimal equivalent are called terminating decimals.
Fractions that have a decimal equivalent that repeats itself are called recurring decimals.

EXAMPLE 1

SKILLS ANALYSIS

Change $0.\dot{5}$ to a fraction.

Let $x = 0.555\,555\ldots$ (Multiply both sides by 10 as one digit recurs)

$10x = 5.555\,555\ldots$ (Subtract the value of x from the value of $10x$)

$9x = 5$ (Divide both sides by 9)

$x = \frac{5}{9}$

EXAMPLE 2

SKILLS ANALYSIS

Change $0.\dot{7}\dot{9}$ to a fraction.

Let $x = 0.797\,979\ldots$ (Multiply both sides by 100 as two digits recur)

$100x = 79.797\,979\ldots$ (Subtract the value of x from the value of $100x$)

$99x = 79$ (Divide both sides by 99)

$x = \frac{79}{99}$

KEY POINT

- To change a recurring decimal to a fraction, first form an equation by putting x equal to the recurring decimal. Then multiply both sides of the equation by 10 if one digit recurs, by 100 if two digits recur, and by 1000 if 3 digits recur etc.

NO. OF REPEATING DIGITS	MULTIPLY BY
1	10
2	100
3	1000

EXAMPLE 3

SKILLS ANALYSIS

Change $0.\dot{1}2\dot{3}$ to a fraction.

Let $x = 0.123\,123\ldots$	(Multiply both sides by 1000 as three digits recur)
$1000x = 123.123\,123\ldots$	(Subtract the value of x from the value of $1000x$)
$999x = 123$	(Divide both sides by 999)
$x = \frac{123}{999}$	
$x = \frac{41}{333}$	(In its simplest form)

EXAMPLE 4

SKILLS ANALYSIS

Change $0.3\dot{4}\dot{5}$ to a fraction.

Let $x = 0.3\dot{4}\dot{5}$	(Multiply by 10 to create recurring digits after the decimal point)
$10x = 3.454\,545\ldots$	(Multiply by 100 as there are two recurring digits after the decimal point)
$1000x = 345.454\,545\ldots$	(Subtract $10x$ from $1000x$)
$990x = 342$	
$x = \frac{342}{990} = \frac{19}{55}$	(Write answer in its simplest form)

ADVANCED CALCULATOR PROBLEMS

The efficient use of a calculator is key to obtaining accurate solutions to more complex calculations. Scientific calculators automatically apply the operations in the correct order. However, the use of extra brackets may be needed in some calculations. The calculator's memory may also be a help with more complicated numbers and calculations.

The instruction manual should be kept and studied with care.

EXAMPLE 5

SKILLS ANALYSIS

Calculate the following, giving answers **correct to** 3 **significant figures**.

a $\left(\dfrac{1.76 \times 10^{3}}{\sqrt{5.3 \times 10^{-2}}}\right)^{4}$ **b** $\sqrt[5]{\dfrac{5.4 \times 10^{5}}{\pi^{3}}}$

a [( ] [$\frac{\square}{\square}$] 1.76 [×10^x] 3 [▼] [√] 5.3 [×10^x] −2 [ )] [$x^\square$] 4 [=]

$3.41585\ldots \times 10^{15} = 3.42 \times 10^{15}$ (3 s.f.)

b [$\sqrt[\square]{\square}$] 5 [$\frac{\square}{\square}$] 5.4 [×10^x] 5 [▼] π [$x^\square$] 3 [=]

$7.04999\ldots = 7.05$ (3 s.f.)

Become familiar with the function keys on your own calculator as they may be different from those shown here.

SKILLS
ADAPTIVE LEARNING

ACTIVITY 1

Calculations such as $3 \times 2 \times 1$ can be represented by the symbol 3!

Similarly $4! = 4 \times 3 \times 2 \times 1$

So $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$ and so on…

The $x!$ symbol is called the **factorial** of x.

Thus $x! = x \times (x - 1) \times (x - 2) \times \ldots \times 3 \times 2 \times 1$

The factorial function is very useful in mathematics when considering patterns.

Scientific calculators have this function represented by the button $\boxed{x!}$

Note that $\frac{4!}{3!} = \frac{4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 4$

1 ▶ Find the values of the following expressions without the use of a calculator.

a $5!$ **b** $6!$ **c** $7!$ **d** $\frac{5!}{4!}$ **e** $\frac{10!}{8!}$ **f** $\frac{100!}{98!}$

2 ▶ Find the values of the following with the use of a calculator.

a $\frac{10!}{7!5!}$

b $\left(\frac{12!}{9!} - \frac{6!}{3!}\right)^{\frac{3!}{2!}}$

c $\left(\frac{\sin(10 \times 3!)}{\cos(10 \times 3!)}\right)^{3!}$

d $\sqrt[5!]{\pi^{5!}}$

e $1\frac{1}{1!} + 1\frac{1}{2!} + 1\frac{1}{3!} + 1\frac{1}{4!} + 1\frac{1}{5!}$

f $\left(1\frac{1}{1!} - 1\frac{1}{2!} + 1\frac{1}{3!} - 1\frac{1}{4!} + 1\frac{1}{5!}\right)^{\frac{10!}{5 \times 9!}}$

3 ▶ Solve these equations.

a $x! = 3\,628\,800$

b $x^2 - x + 4! = 12 + 3!x$

c $2^{x!} = 16\,777\,216$

d $(x!)^2 = 1\,625\,702\,400$

4 ▶ Show that $\frac{n!}{(n-2)!} = n^2 - n$

Algebra 07

BASIC PRINCIPLES

- **Expand** brackets.
- Expand the **product** of two linear expressions.
- Substitute into formulae.
- **Factorise** quadratic expressions of the form $x^2 + bx + c$
- Solve linear equations.
- Solve **quadratic equations** of the form $x^2 + bx + c = 0$ by factorising
- Solve problems by setting up and solving quadratic equations of the form $x^2 + bx + c = 0$

SOLVING QUADRATIC EQUATIONS BY FACTORISING

KEY POINTS

- There are three types of quadratic equations with $a = 1$
  - If $b = 0$ | $x^2 - c = 0$ (Rearrange)
    $\Rightarrow$ $x^2 = c$ (Square root both sides)
    $\Rightarrow$ $x = \pm\sqrt{c}$
  - If $c = 0$ | $x^2 + bx = 0$ (Factorise)
    $\Rightarrow$ $x(x + b) = 0$ (Solve)
    $\Rightarrow$ $x = 0$ or $x = -b$
  - If $b \neq 0$ and $c \neq 0$ | $x^2 + bx + c = 0$ (Factorise)
    $\Rightarrow$ $(x + p)(x + q) = 0$ (Solve)
    $\Rightarrow$ $x = -p$ or $x = -q$
    where $p \times q = c$ and $p + q = b$
- If c is negative then p and q have opposite signs to each other.
- If c is positive then p and q have the same sign as b.

EXAMPLE 1

Solve these quadratic equations.

a $x^2 - 64 = 0$ **b** $x^2 - 81 = 0$ **c** $x^2 - 7x = 0$ **d** $x^2 - 10x + 21 = 0$

a $x^2 = 64$
$x = 8$ or $x = -8$

b $(x - 9)(x + 9) = 0$
$x = 9$ or $x = -9$

c $x(x - 7) = 0$
$x = 0$ or $x = 7$

d $(x - 7)(x - 3) = 0$
$x = 7$ or $x = 3$

Part **b** shows that if c is a square number then using the difference of two squares gives the same answers.

FURTHER FACTORISATION

When a ≠ 1 the factorisation may take longer

KEY POINT

- Always take out any **common factor** first.

EXAMPLE 2

Solve these quadratic equations.

a $9x^2 - 25 = 0$ **b** $3x^2 - 12x = 0$ **c** $12x^2 - 24x - 96 = 0$

a $9x^2 - 25 = 0$

$\Rightarrow 9x^2 = 25$

$\Rightarrow x^2 = \frac{25}{9}$

$\Rightarrow x = \pm\frac{5}{3}$

b $3x^2 - 12x = 0$

$\Rightarrow 3x(x - 4) = 0$

$\Rightarrow x = 0$ or $x = 4$

c $12x^2 - 24x - 96 = 0$

$\Rightarrow 12(x^2 - 2x - 8) = 0$

$\Rightarrow 12(x + 2)(x - 4) = 0$

$\Rightarrow x = -2$ or $x = 4$

If there is no simple number factor, then the factorisation takes more time.

EXAMPLE 3

Solve $x(3x - 13) = 10$

Expand the brackets and rearrange into the form $ax^2 + bx + c = 0$

$3x^2 - 13x - 10 = 0$

Factorise $3x^2 - 13x - 10$

The first terms in each bracket multiply together to give $3x^2$ so the factorisation starts as

$(3x \ldots\ldots)(x \ldots\ldots)$

The last terms in each bracket multiply together to give −10. The possible pairs are −1 and 10, 1 and −10, 2 and −5, −2 and 5.

The outside and inside terms **multiply out** and **sum** to $-13x$

So the factorisation is $(3x + 2)(x - 5)$

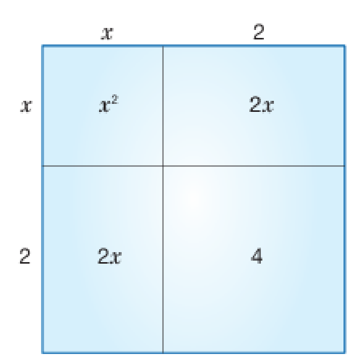
$\Rightarrow (3x + 2)(x - 5) = 0$

$\Rightarrow 3x + 2 = 0$ or $x - 5 = 0$

$\Rightarrow x = -\frac{2}{3}$ or $x = 5$

SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

If a quadratic equation cannot be factorised, then the method of completing the square can be used to solve it. This involves writing one side of the equation as a perfect square and a constant. A perfect square is an expression like (x + 2)², which is equal to x² + 4x +4



EXAMPLE 4 Solve $x^2 + 4x - 3 = 0$

$$x^2 + 4x - 3 = 0 \Rightarrow x^2 + 4x + 4 = 7 \quad \text{(Add 7 to both sides to make LHS a perfect square)}$$
$$\Rightarrow (x + 2)^2 = 7 \quad \text{(Square root both sides)}$$
$$\Rightarrow x + 2 = \pm\sqrt{7}$$
$$\Rightarrow x = -2 + \sqrt{7} \text{ or } -2 + \sqrt{7}$$

We want to write $x^2 + bx + c$ in the form $(x + p)^2 + q$ to have a perfect square.

$$x^2 + bx + c = (x + p)^2 + q \qquad (1)$$
$$x^2 + bx + c = x^2 + 2px + p^2 + q \quad \text{(Multiplying out)}$$

So if we take $b = 2p$ and $c = p^2 + q$ then the expressions will be equal.

This means $p = \frac{b}{2}$

Also as $c = p^2 + q$ then $q = -p^2 + c$ or $q = -\left(\frac{b}{2}\right)^2 + c$

So substituting for p and q in (1) gives

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

EXAMPLE 5 Write these quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 5$ **b** $x^2 + 5x - 1$

a $\underset{\substack{\uparrow\\ b}}{x^2 + 4x} \underset{\substack{\uparrow\\ c}}{+ 5} = (x + \underset{\substack{\uparrow\\ \frac{b}{2}}}{2})^2 - (\underset{\substack{\uparrow\\ \frac{b}{2}}}{2})^2 + \underset{\substack{\uparrow\\ c}}{5} = (x + 2)^2 + 1$

b $\underset{\substack{\uparrow\\ b}}{x^2 + 5x} \underset{\substack{\uparrow\\ c}}{- 1} = \left(x + \underset{\substack{\uparrow\\ \frac{b}{2}}}{\frac{5}{2}}\right)^2 - \left(\underset{\substack{\uparrow\\ \frac{b}{2}}}{\frac{5}{2}}\right)^2 \underset{\substack{\uparrow\\ c}}{- 1} = \left(x + \frac{5}{2}\right)^2 - \frac{29}{4}$

If b is negative then care is needed with the signs.

EXAMPLE 6 Write $x^2 - 8x + 7$ in the form $(x + p)^2 + q$

Treat $x^2 - 8x + 7$ as $x^2 + (-8)x + 7$ so $\frac{b}{2} = -4$

Then $x^2 + (-8)x + 7$
$$= (x + (-4))^2 - (-4)^2 + 7$$
$$= (x - 4)^2 - 16 + 7$$
$$= (x - 4)^2 - 9$$

EXAMPLE 7 Write $4x^2 + 12x - 5$ in the form $a(x + p)^2 + q$

$$4x^2 + 12x - 5 = 4(x^2 + 3x) - 5$$

Now $x^2 + 3x = \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$

So $4(x^2 + 3x) - 5 = 4\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right] - 5$
$$= 4\left(x + \frac{3}{2}\right)^2 - 9 - 5$$
$$= 4\left(x + \frac{3}{2}\right)^2 - 14$$

EXAMPLE 8

Write $-x^2 + 4x + 1$ in the form $a(x + p)^2 + q$

$$-x^2 + 4x + 1 = -1[x^2 - 4x] + 1$$
$$= -1[(x - 2)^2 - 2^2] + 1$$
$$= -(x - 2)^2 + 5$$

Any quadratic equation can be written in the form $p(x + q)^2 + r = 0$ by completing the square. It can then be solved.

EXAMPLE 9

By completing the square solve

a $x^2 + 2x - 2 = 0$ **b** $2x^2 - 6x = 4$

a $x^2 + 2x - 2 = 0 \Rightarrow (x + 1)^2 - 1 - 2 = 0$ (Rearrange)

$\Rightarrow (x + 1)^2 = 3$ (Square root both sides)

$\Rightarrow x + 1 = \pm\sqrt{3}$

$\Rightarrow x = -1 + \sqrt{3}$ or $x = -1 - \sqrt{3}$

b $2x^2 - 6x = 4 \Rightarrow x^2 - 3x = 2$ (Divide both sides by 2)

$$\Rightarrow \left(x - \frac{3}{2}\right)^2 - \left(\frac{-3}{2}\right)^2 = 2$$
$$\Rightarrow \left(x - \frac{3}{2}\right)^2 = \frac{17}{4}$$
$$\Rightarrow x - \frac{3}{2} = \frac{\pm\sqrt{17}}{2}$$
$$\Rightarrow x = \frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

KEY POINTS

- $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$
- Take care with the signs if b is negative.
- $ax^2 + bx + c$ can be written as $a\left(x^2 + \frac{b}{a}x\right) + c$ before completing the square for $x^2 + \frac{b}{a}x$
- Give your answers in **surd** form (unless told otherwise) as these answers are exact.

SOLVING QUADRATIC EQUATIONS USING THE QUADRATIC FORMULA

The quadratic formula is used to solve quadratic equations that may be difficult to solve using other methods. Proof of the formula follows later in the chapter.

KEY POINT

- The quadratic formula

  If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The quadratic formula is very important and is given on the formula sheet. However, it is used so often that you should memorise it.

EXAMPLE 10

Solve $3x^2 - 8x + 2 = 0$ giving answers **correct to** 3 s.f.

Here $a = 3$, $b = -8$, $c = 2$

Substituting into the formula gives $x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 3 \times 2}}{2 \times 3} = \dfrac{8 \pm \sqrt{64 - 24}}{6}$

so $x = \dfrac{8 + \sqrt{40}}{6} = 2.39$ or $x = \dfrac{8 - \sqrt{40}}{6} = 0.279$ to 3 s.f.

Note how the negative number, −8, is handled in Example 10. Be very careful when substituting negative numbers into the formula; this is usually the reason for any mistakes.

EXAMPLE 11

Solve $3.5x + 2.3x^2 = 4.8$ giving answers correct to 3 s.f.

$2.3x^2 + 3.5x - 4.8 = 0$ (Rearrange the equation into the form $ax^2 + bx + c = 0$)

$a = 2.3$, $b = 3.5$, $c = -4.8$ (Write down the numbers to substitute in the formula)

$x = \dfrac{-3.5 \pm \sqrt{3.5^2 - 4 \times 2.3 \times (-4.8)}}{2 \times 2.3} = \dfrac{-3.5 \pm \sqrt{56.41}}{4.6}$ (Substitute the numbers)

so $x = \dfrac{-3.5 + \sqrt{56.41}}{4.6} = 0.872$ or $x = \dfrac{-3.5 - \sqrt{56.41}}{4.6} = -2.39$ to 3 s.f.

KEY POINTS

- a is the coefficient of x^2, it is not necessarily the first number in the equation. Always rearrange the equation into the form $ax^2 + bx + c = 0$ before using the formula.
- If the question asks for answers to a number of s.f. or d.p. then it is fairly certain that the quadratic formula is required.

PROOF OF THE QUADRATIC FORMULA

The following proof is for the case when $a = 1$, i.e. for the equation $x^2 + bx + c = 0$

$x^2 + bx + c = 0 \Rightarrow \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$ (completing the square)

$\Rightarrow \left(x + \frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 - c$ (rearranging)

$\Rightarrow \left(x + \frac{b}{2}\right)^2 = \frac{b^2}{4} - c$ (squaring the fraction)

$\Rightarrow \left(x + \frac{b}{2}\right)^2 = \frac{b^2 - 4c}{4}$ (**simplifying**)

$\Rightarrow x + \frac{b}{2} = \pm\sqrt{\frac{b^2 - 4c}{4}}$ (square rooting both sides)

$\Rightarrow x + \frac{b}{2} = \frac{\pm\sqrt{b^2 - 4c}}{2}$ (square rooting fraction)

$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$ (rearranging and simplifying)

PROBLEMS LEADING TO QUADRATIC EQUATIONS

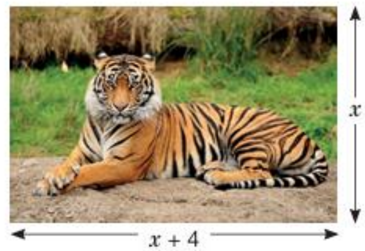
Example 12 revises the process of setting up and solving a quadratic equation to solve a problem. The steps to be followed are:

- Where relevant, draw a clear diagram and put all the information on it.
- Let x stand for what you are trying to find.
- Form a quadratic equation in x and simplify it.
- Solve the equation by factorising, completing the square or by using the formula.
- Check that the answers make sense.

EXAMPLE 12

The width of a rectangular photograph is 4 cm more than the height.

The area is 77 cm².
Find the height of the photograph.



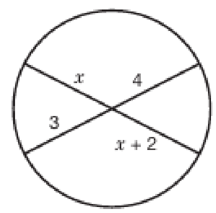
Let x be the height in cm.
Then the width is $x + 4$ cm.
As the area is 77 cm²,

$x(x + 4) = 77$ (Multiply out the brackets)
$x^2 + 4x = 77$ (Rearrange into the form $ax^2 + bx + c = 0$)
$x^2 + 4x - 77 = 0$ (Factorise)
$(x - 7)(x + 11) = 0$

So, $x = 7$ or -11 cm
The height cannot be negative, so the height is 7 cm.

EXAMPLE 13

The **chords** of a circle **intersect** as shown.
Find the value of x.



Using the intersecting chords theorem:

$x(x + 2) = 3 \times 4$ (Multiply out the brackets)
$x^2 + 2x = 12$ (Rearrange into the form $ax^2 + bx + c = 0$)
$x^2 + 2x - 12 = 0$

This expression does not factorise.

Using the quadratic formula with $a = 1$, $b = 2$ and $c = -12$ gives

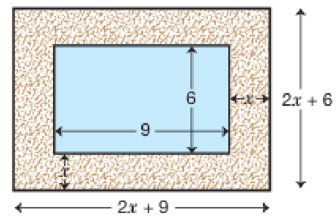
$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times (-12)}}{2}$$

$x = 2.61$ or -4.61 (3 s.f.)

As x cannot be negative, $x = 2.61$ to 3 s.f.

EXAMPLE 14

A rectangular fish pond is 6 m by 9 m. The pond is surrounded by a concrete path of constant width. The area of the pond is the same as the area of the path. Find the width of the path.



Let x be the width of the path.

The area of the path is $(2x + 9)(2x + 6) - 9 \times 6$
$= 4x^2 + 30x + 54 - 54$
$= 4x^2 + 30x$

The area of the pond is $9 \times 6 = 54\,\text{m}^2$.

As the area of the path equals the area of the pond,

$4x^2 + 30x = 54$ (Rearrange into the form $ax^2 + bx + c = 0$)
$4x^2 + 30x - 54 = 0$ (Divide both sides of the equation by 2)
$2x^2 + 15x - 27 = 0$ (Factorise)
$(2x - 3)(x + 9) = 0$

So, $x = 1.5$ or -9 m
As x cannot be negative, the width of the path is 1.5 m.

SOLVING QUADRATIC INEQUALITIES

Squares and square roots involving **inequalities** need care.

EXAMPLE 15

Solve $x^2 - 4 < 0$

First **sketch** $y = x^2 - 4$

To do this, find where the graph intersects the x-axis by solving the equation $x^2 - 4 = 0$.
$x^2 - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow x = -2$ or $x = 2$.

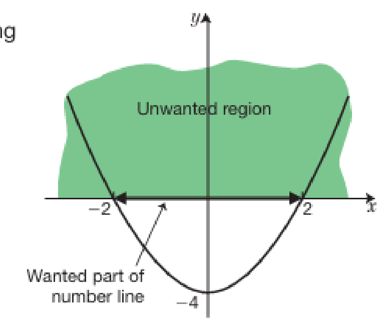
So the graph intersects the x-axis at $x = -2$ and $x = 2$.

Also, when $x = 0$, $y = -4$, so the graph cuts the y-axis at -4.

The graph is a **parabola**, which is U-shaped.

The required region is below the x-axis. As this is one region, the answer is one inequality.

The solution is $-2 < x < 2$.



EXAMPLE 16

Solve $x^2 - x - 2 \leq 0$, showing the solution set on a number line.

First sketch $y = x^2 - x - 2$

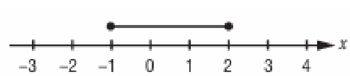
To do this, find where the graph intersects the x-axis by solving the equation $x^2 - x - 2 = 0$

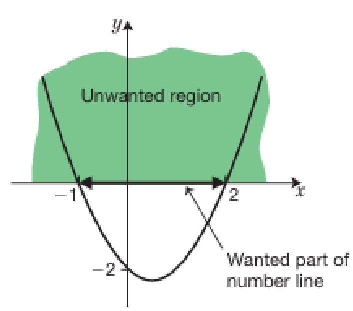
$x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = 2$ or $x = -1$
so the graph intersects the x-axis at $x = 2$ and $x = -1$

When $x = 0$, $y = -2$

The required region is below the x-axis. As this is one region, the answer is one inequality.

The solution is $-1 \leq x \leq 2$.





EXAMPLE 17

Solve $x^2 - 5x + 6 \geq 0$

First sketch $y = x^2 - 5x + 6$

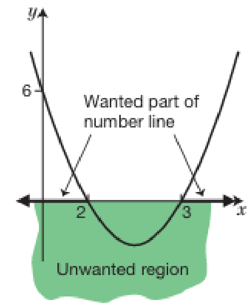
To do this, find where the graph intersects the x-axis by solving the equation $x^2 - 5x + 6 = 0$

$x^2 - 5x + 6 = 0 \Rightarrow (x - 2)(x - 3) = 0 \Rightarrow x = 2$ or $x = 3$
so the graph intersects the x-axis at $x = 2$ and $x = 3$

When $x = 0$, $y = 6$

The required region is above the x-axis. As this has two components, the answer is two inequalities.

The solution is $x \leq 2$ or $x \geq 3$



KEY POINT

- To solve a quadratic inequality, sketch the graph of the quadratic function.

GRAPHS 6

BASIC PRINCIPLES

- Substitute positive and negative numbers (including fractions) into algebraic expressions (including those involving squares and cubes) and those of the form $\frac{1}{x}$
- Recognise linear and **quadratic graphs**.
- Draw graphs (linear and quadratic) using tables of values.
- A graph of p against q implies that p is on the vertical axis and q is on the horizontal axis. These values are **variables** and can represent physical quantities such as distance, speed, time and many others. Often a **range** of values is stated where the graph is a 'good' mathematical model and the values can be trusted to produce accurate results.

CUBIC GRAPHS $y = ax^3 + bx^2 + cx + d$

A **cubic function** is one in which the highest **power** of x is x^3.

All cubic functions can be written in the form $y = ax^3 + bx^2 + cx + d$ where a, b, c and d are constants.

The graphs of cubic functions have distinctive shapes and can be used to model real-life situations.

EXAMPLE 1

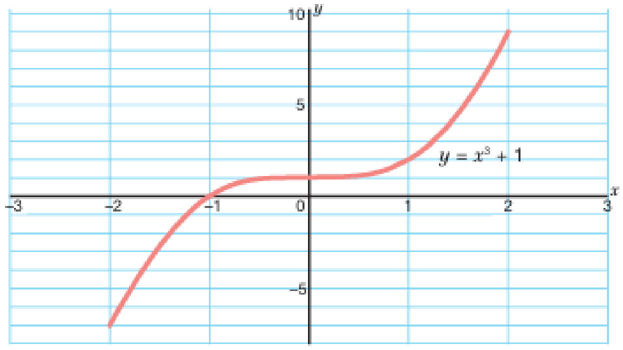
Plot the graph of $y = x^3 + 1$ for $-2 \leq x \leq 2$

Method 1 The complete table is shown with all cells filled in to find y.

x	**−2**	**−1**	**0**	**1**	**2**
x^3	−8	−1	0	1	8
+1	+1	+1	+1	+1	+1
y	**−7**	**0**	**1**	**2**	**9**

Method 2 The table is produced showing only x and y values.
This can be generated from functions in many calculators.

x	–2	–1	0	1	2
y	–7	0	1	2	9



EXAMPLE 2

SKILLS

REASONING

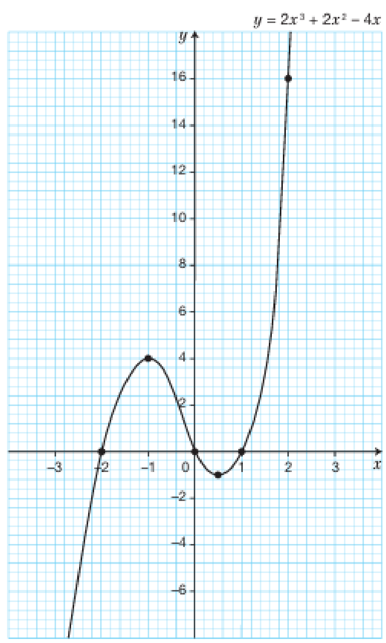
Draw the graph of $y = 2x^3 + 2x^2 - 4x$ for $-2 \le x \le 2$ by creating a suitable table of values.

x	**–2**	**–1**	**0**	$\frac{1}{2}$	**1**	**2**
$2x^3$	–16	–2	0	$\frac{1}{4}$	2	16
$2x^2$	8	2	0	$\frac{1}{2}$	2	8
$-4x$	8	4	0	–2	–4	–8
y	**0**	**4**	**0**	$-1\frac{1}{4}$	**0**	**16**

As $y = 0$ for both $x = 0$ and $x = 1$, $x = \frac{1}{2}$ is used to find the value of y between $x = 0$ and $x = 1$

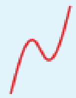
Plot the points and draw a smooth curve through all the points.

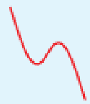
Note: Modern calculators often have a function that enables tables of graphs to be produced accurately and quickly so that only the x and y values are shown. However, you will still be expected to be able to fill in tables like the one above to plot some graphs.



KEY POINTS

- A cubic function is one whose highest power of x is x^3
  It is written in the form $y = ax^3 + bx^2 + cx + d$

  When $a > 0$ the function looks like this  or 

  When $a < 0$ the function looks like this  or 

  The graph **intersects** the y-axis at the point $y = d$

RECIPROCAL GRAPHS $y = \frac{a}{x}$

A reciprocal function is in the form $y = \frac{a}{x}$ where a is a constant.
The graph of a reciprocal function produces a curve called a hyperbola.

ACTIVITY 2

SKILLS ANALYSIS

Draw the graph of $y = \frac{1}{x}$, where $x \neq 0$, for $-3 \leq x \leq 3$

Dividing a number by 0 gives no value, so it is necessary to investigate the behaviour of y as x gets closer to zero from both sides. Use this table of values.

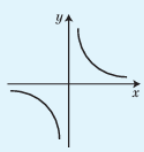
x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
y										

The graph of a **reciprocal** function has two parts, both of which are smooth curves.

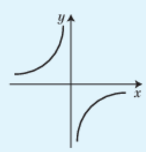
KEY POINT

- The graph of $y = \frac{a}{x}$ is a **hyperbola**.

  The curve approaches, but never touches, the axes. The axes are called **asymptotes** to the curve.



$a > 0$



$a < 0$

Other reciprocal graphs which involve division by an x term include, for example,

$y = 1 + \frac{4}{x}$, $y = 2x - \frac{1}{x}$, $y = \frac{12}{x-1}$ and $y = 3x + \frac{1}{x^2}$

EXAMPLE 3

SKILLS ANALYSIS

a Draw the graph of $y = \frac{10}{x} + x - 5$ for $1 \leq x \leq 6$ by first creating a table of values.

b Use the graph to estimate the smallest value of y, and the value of x where this occurs.

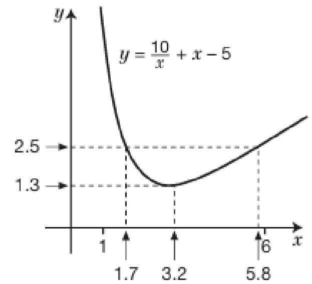
c State the values of x for which $y = 2.5$

a

x	1	2	3	4	5	6
$\frac{10}{x}$	10	5	$3\frac{1}{3}$	$2\frac{1}{2}$	2	$1\frac{2}{3}$
x	1	2	3	4	5	6
-5	-5	-5	-5	-5	-5	-5
y	6	2	$1\frac{1}{3}$	$1\frac{1}{2}$	2	$2\frac{2}{3}$

b The graph shows that the minimum value of $y \simeq 1.3$ when $x \simeq 3.2$

c When $y = 2.5$, $x \simeq 1.7$ or 5.8



SHAPE AND SPACE 7

BASIC PRINCIPLES

- The **perimeter** of a shape is the distance all the way round the shape.
  - Circle
    The perimeter of a circle is called the **circumference**.
    If C is the circumference, A the area and r the **radius**, then
    $C = 2\pi r$
    $A = \pi r^2$

  - **Semicircle**
    A semicircle is half a circle cut along a **diameter**.
    The perimeter is half the circumference of the circle plus the diameter, so $P = \pi r + 2r$
    The area is half of the area of the circle, so $A = \frac{\pi r^2}{2}$

  - **Quadrant**
    A quadrant is quarter of a circle.
    The perimeter is a quarter of the circumference of the circle plus twice the radius, so $P = \frac{\pi r}{2} + 2r$
    The area is a quarter of the area of the circle, so $A = \frac{\pi r^2}{4}$

- Find the perimeter of rectangles and triangles.
- Use Pythagoras' theorem.
- Recognise **similar** shapes.
- Use the **ratio** of corresponding sides to work out **scale factors**.
- Find missing lengths on similar shapes.

CIRCLES

EXAMPLE 1

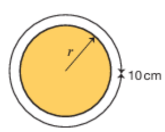
The circumference of a circle is 10 cm. Find the radius.

Using $C = 2\pi r$

$10 = 2\pi r$ (Make r the **subject** of the equation)

$r = \frac{10}{2\pi}$

$= 1.59$ cm to 3 s.f.



EXAMPLE 2

The area of a circle is 24 cm². Find the radius.

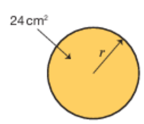
Using $A = \pi r^2$

$24 = \pi r^2$ (Make r the subject of the equation)

$r^2 = \frac{24}{\pi}$

$r = \sqrt{\frac{24}{\pi}}$

$= 2.76$ cm to 3 s.f.



EXAMPLE 3

Find the perimeter and area of the shape shown.

The radius of the quadrant BCD is 3 cm, so BC = 3 cm.

Perimeter = AB + BC + **arc** CD + DE + EA

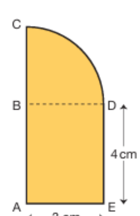
$= 4 + 3 + \frac{2 \times \pi \times 3}{4} + 4 + 3$

$= 18.7$ cm (to 3 s.f.)

Area = area of quadrant BCD + area of rectangle ABDE

$= \frac{9\pi}{4} + 12$

$= 19.1$ cm² (to 3 s.f.)

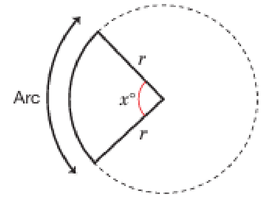


ARCS

An arc is part of the circumference of a circle.

The arc shown is the fraction $\frac{x}{360}$ of the whole circumference.

So the arc length is $\frac{x}{360} \times 2\pi r$



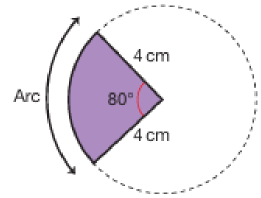
EXAMPLE 4

Find the perimeter of the shape shown.

Using Arc length $= \frac{x}{360} \times 2\pi r$

Arc length $= \frac{80}{360} \times 2\pi \times 4 = 5.585\,\text{cm}$

Perimeter $= 5.585 + 4 + 4 = 13.6\,\text{cm}$ to 3 s.f.



EXAMPLE 5

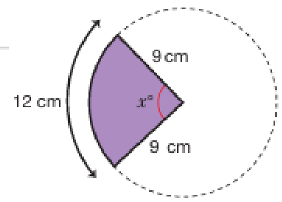
Find the angle marked x.

Using Arc length $= \frac{x}{360} \times 2\pi r$

$$12 = \frac{x}{360} \times 2\pi \times 9 \quad \text{(Make } x \text{ the subject of the equation)}$$

$$x = \frac{12 \times 360}{2\pi \times 9}$$

$$= 76.4° \text{ to 3 s.f.}$$



EXAMPLE 6

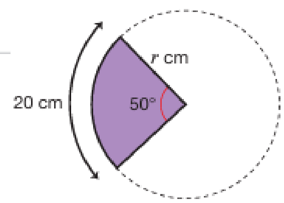
Find the radius r.

Using Arc length $= \frac{x}{360} \times 2\pi r$

$$20 = \frac{50}{360} \times 2\pi r \quad \text{(Make } r \text{ the subject of the equation)}$$

$$r = \frac{20 \times 360}{50 \times 2\pi}$$

$$= 22.9\,\text{cm to 3 s.f.}$$



KEY POINT

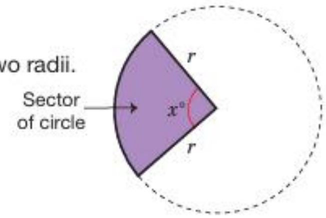
- Arc length $= \frac{x}{360} \times 2\pi r$

SECTORS

A **sector** of a circle is a region whose perimeter is an arc and two radii.

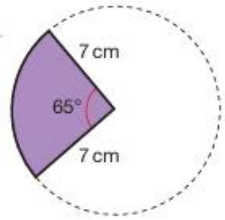
The sector shown is the fraction $\frac{x}{360}$ of the whole circle.

So the sector area is $\frac{x}{360} \times \pi r^2$



EXAMPLE 7

Find the area of the sector shown.



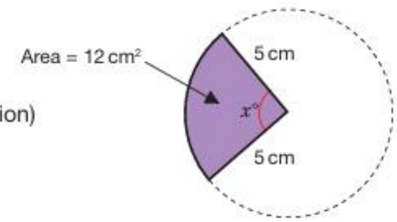
Using Sector area $= \frac{x}{360} \times \pi r^2$

$A = \frac{65}{360} \times \pi \times 7^2$

$= 27.8\,\text{cm}^2$ to 3 s.f.

EXAMPLE 8

Find the angle marked x.



Using Sector area $= \frac{x}{360} \times \pi r^2$

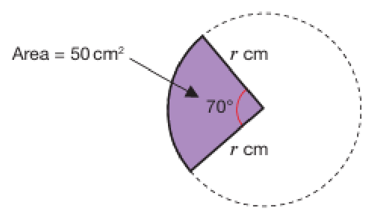
$12 = \frac{x}{360} \times \pi \times 5^2$ (Make x the subject of the equation)

$x = \frac{12 \times 360}{\pi \times 5^2}$

$= 55.0°$ to 3 s.f.

EXAMPLE 9

Find the radius of the sector shown.



Using Sector area $= \frac{x}{360} \times \pi r^2$

$50 = \frac{70}{360} \times \pi r^2$ (Make r the subject of the equation)

$r^2 = \frac{50 \times 360}{70 \times \pi}$

$r = \sqrt{\frac{50 \times 360}{70 \times \pi}}$

$= 9.05\,\text{cm}$ (to 3 s.f.)

KEY POINT

- Sector area $= \frac{x}{360} \times \pi r^2$

SOLIDS

SURFACE AREA AND VOLUME OF A PRISM

ACTIVITY 1

SKILLS REASONING

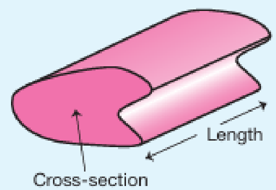
Curved surface area of cylinder

Take a rectangular piece of paper and make it into a hollow (empty) cylinder.

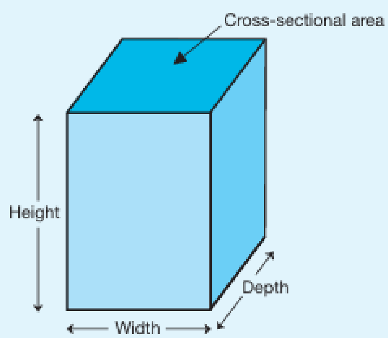
Use your paper to explain why the curved surface area of a cylinder = $2\pi rh$

KEY POINTS

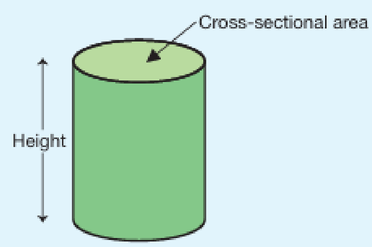
- Any solid with parallel sides that has a constant **cross-section** is called a **prism**.



- Volume of a prism = area of cross-section × length
- A **cuboid** is a prism with a rectangular cross-section.

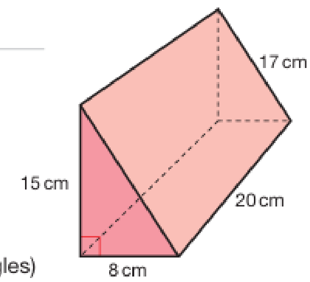


- Volume of a cuboid = width × depth × height
- A cylinder is a prism with a circular cross-section.



- If the height is h and the radius r, then volume of a cylinder = $\pi r^2 h$
- Curved surface area of a cylinder = $2\pi rh$

EXAMPLE 10 Calculate the volume and surface area of the prism shown.



The cross-section is a **right-angled triangle**.

Area of cross-section $= \frac{1}{2} \times 8 \times 15 = 60\,\text{cm}^2$

So

Volume = area of cross-section × length

Volume $= 60 \times 20 = 1200\,\text{cm}^3$

Surface area = **sum** of the areas of the 5 faces (2 triangles + 3 rectangles)

$= 2(\frac{1}{2} \times 8 \times 15) + (17 \times 20) + (8 \times 20) + (15 \times 20)$

$= 920\,\text{cm}^2$

EXAMPLE 11 Calculate the volume and total surface area of an unopened cola can that is a cylinder with diameter 6 cm and height 11 cm.

Using $V = \pi r^2 h$ with $r = 3$ and $h = 11$

$V = \pi \times 3^2 \times 11$

$= 311\,\text{cm}^3$ to 3 s.f.

Total surface area = area of 2 circles + curved surface area

$A = 2 \times \pi r^2 + 2\pi rh$

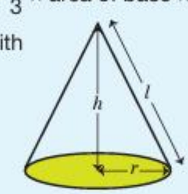
$= 2 \times \pi \times 3^2 + 2 \times \pi \times 3 \times 11$

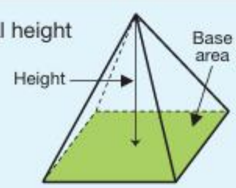
$= 264\,\text{cm}^2$ to 3 s.f.

VOLUME AND SURFACE AREA OF A PYRAMID, CONE AND SPHERE

KEY POINTS

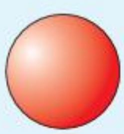
- Volume of a pyramid $= \frac{1}{3} \times$ area of base × vertical height
- A cone is a pyramid with a circular base.



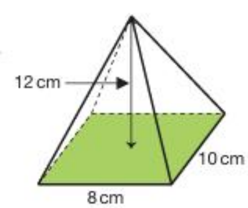


- Volume of a cone $= \frac{1}{3} \times$ area of base × vertical height

  $= \frac{1}{3} \times \pi r^2 \times h$, where r = radius and h = vertical height
- Curved surface area of a cone $= \pi rl$, where l is the slant (sloped) height
- For a sphere of radius r,
  - Volume $= \frac{4}{3}\pi r^3$
  - Surface area $= 4\pi r^2$



EXAMPLE 12 Find the volume of the rectangular-based pyramid shown.

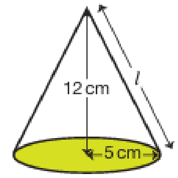


Using $V = \frac{1}{3} \times$ area of base × vertical height

$V = \frac{1}{3} \times 8 \times 10 \times 12$

$= 320\,\text{cm}^3$

EXAMPLE 13 Find the total surface area of the cone shown.



Use Pythagoras' Theorem to work out l.

$l^2 = 5^2 + 12^2$

$l^2 = 169$

$l = 13$

Curved surface area of a cone $= \pi r l$

$= \pi \times 5 \times 13$

$= 204\,\text{cm}^2$

The base is a circle with area $\pi r^2 = \pi \times 5^2$

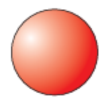
$= 78.5\,\text{cm}^2$

Total surface area = curved surface area + area of base

$= 204 + 78.5$

$= 283\,\text{cm}^2$ to 3 s.f.

EXAMPLE 14 A table tennis ball has a volume of $33\,\text{cm}^3$. Find the radius and surface area.



Using Volume of a sphere $= \frac{4}{3}\pi r^3$

$33 = \frac{4}{3}\pi r^3$ (Make r the subject of the equation)

$r^3 = \frac{33 \times 3}{4 \times \pi}$

$r = \sqrt[3]{\frac{33 \times 3}{4 \times \pi}}$

$= 1.99\,\text{cm}$

Using Surface area of a sphere $= 4\pi r^2$

$A = 4 \times \pi \times 1.99^2$

$= 49.8\,\text{cm}^2$ to 3 s.f.

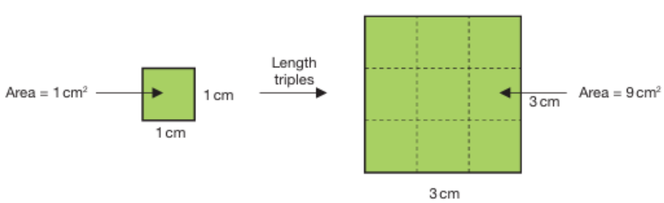
SIMILAR SHAPES

AREAS OF SIMILAR SHAPES

When a shape doubles in size, then the area does NOT double, but increases by a factor of four.

The linear scale factor is 2, and the area scale factor is 4.

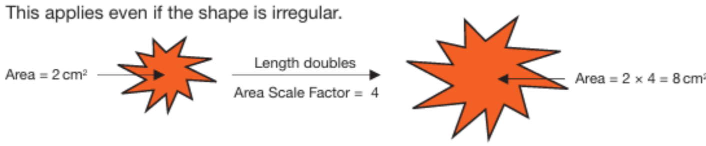
If the shape triples in size, then the area increases by a factor of nine.



Note: The two shapes must be similar. Two similar shapes that are the same size are **congruent**.

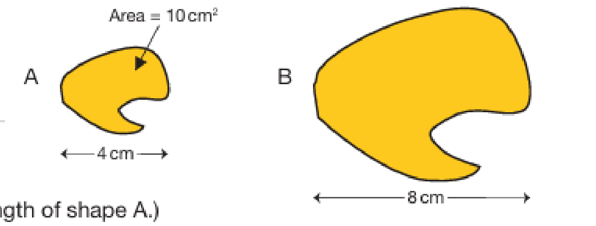
If a shape increases by a linear scale factor of k, then the area scale factor is k^2.

This applies even if the shape is irregular.



EXAMPLE 15

The two shapes shown are similar.
The area of shape A is 10 cm^2.
Find the area of shape B.



The linear scale factor $k = \frac{8}{4} = 2$

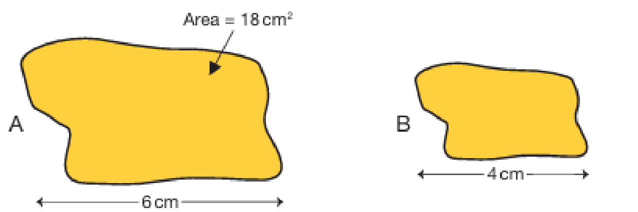
(Divide the length of shape B by the length of shape A.)

The area scale factor $k^2 = 2^2 = 4$

So the area of shape B = $10 \times 4 = 40$ cm^2

EXAMPLE 16

The two shapes shown are similar.
The area of shape A is 18 cm^2.
Find the area of shape B.



The linear scale factor $k = \frac{4}{6} = \frac{2}{3}$

(Divide the length of shape B by the length of shape A.)

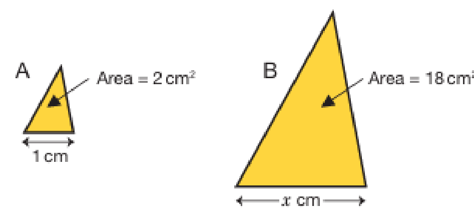
The area scale factor $k^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

So the area of shape B = $18 \times \frac{4}{9} = 8$ cm^2

EXAMPLE 17

The two triangles are similar, with dimensions and areas as shown.

What is the value of x?



The area scale factor $k^2 = \frac{18}{2} = 9$

(Divide the area of shape B by the area of shape A.)

The linear scale factor $k = \sqrt{9} = 3$

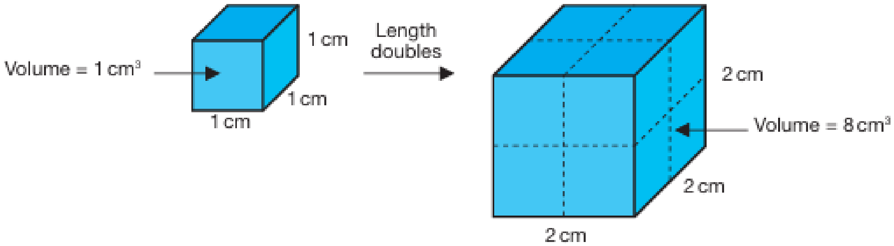
So $x = 1 \times 3 = 3$ cm

KEY POINT

- When a shape is enlarged by linear scale factor k, the area of the shape is enlarged by scale factor k^2.

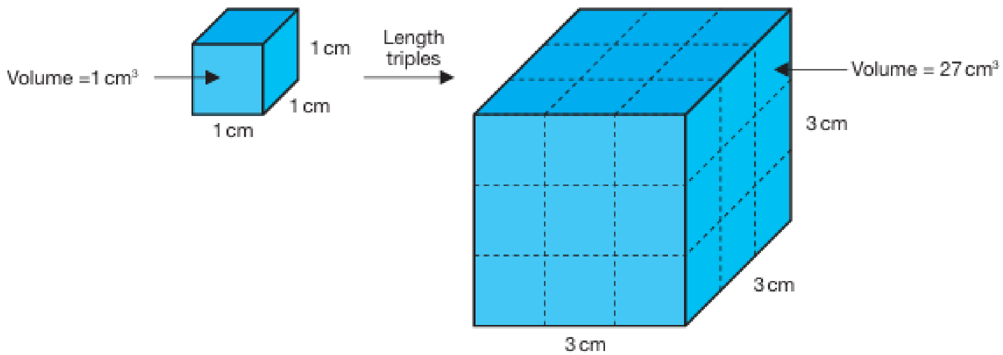
VOLUMES OF SIMILAR SHAPES

When a solid doubles in size, the volume does NOT double, but increases by a factor of eight.



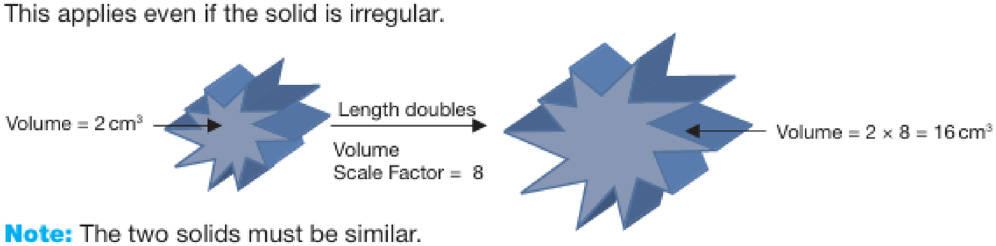
The linear scale factor is 2, and the volume scale factor is 8.

If the solid triples in size, then the volume increases by a factor of 27.



If a solid increases by a linear scale factor of k, then the volume scale factor is k^3.

This applies even if the solid is irregular.



Note: The two solids must be similar.

EXAMPLE 18

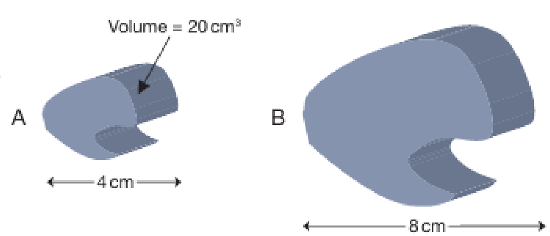
The two solids shown are similar.
The volume of solid A is $20\,\text{cm}^3$.
Find the volume of solid B.

The linear scale factor $k = \frac{8}{4} = 2$

(Divide the length of solid B by the length of solid A.)

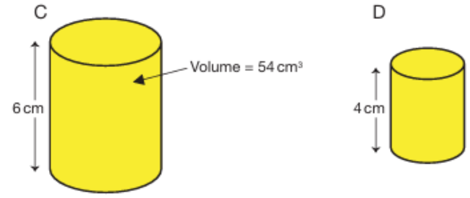
The volume scale factor $k^3 = 2^3 = 8$

So the volume of solid B $= 20 \times 8 = 160\,\text{cm}^3$



EXAMPLE 19

The two cylinders shown are similar.
The volume of cylinder C is $54\,cm^3$.
Find the volume of cylinder D.



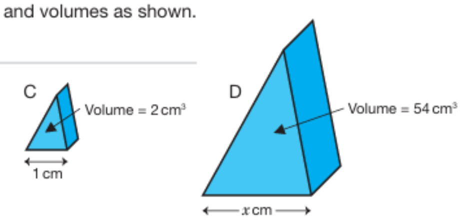
The linear scale factor $k = \frac{4}{6} = \frac{2}{3}$
(Divide the height of cylinder D by the height of cylinder C.)

The volume scale factor $k^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

So the volume of cylinder D $= 54 \times \frac{8}{27} = 16\,cm^3$

EXAMPLE 20

The two prisms are similar, with dimensions and volumes as shown.
What is the value of x?



The volume scale factor $k^3 = \frac{54}{2} = 27$
(Divide the volume of solid D by the volume of solid C.)

The linear scale factor $k = \sqrt[3]{27} = 3$

So $x = 1 \times 3 = 3\,cm$

KEY POINT

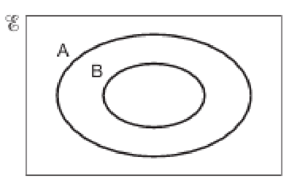
- When a shape is enlarged by linear scale factor k, the volume of the shape is enlarged by scale factor k^3.

SETS 3

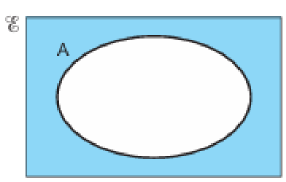
BASIC PRINCIPLES

- A **set** is a collection of objects, described by a list or a rule. A = {1, 3, 5}
- The number of elements of set A is given by n(A). n(A) = 3
- The **universal set** contains all the elements being discussed in a particular problem. $\mathscr{E}$

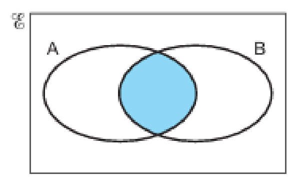
- B is a **subset** of A if every member of B is a member of A. B ⊂ A



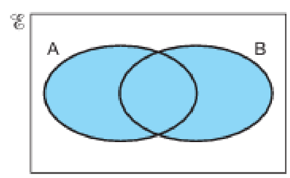
- The **complement** of set A is the set of all elements not in A. A′



- The **intersection** of A and B is the set of elements which are in both A and B. A ∩ B



- The **union** of A and B is the set of elements which are in A or B or both. A ∪ B



- Use **Venn diagrams** to represent two or three sets.
- Use algebra to solve problems involving sets.
- Calculate the probability of an event.
- Calculate the probability that something will not happen given the probability that it will happen.

PROBABILITY

Venn diagrams can be very useful in solving certain probability questions. Venn diagrams can only be used if all the outcomes are equally likely.

The probability of the event A happening is given by $\frac{n(\text{A})}{n(\mathscr{E})}$

EXAMPLE 1

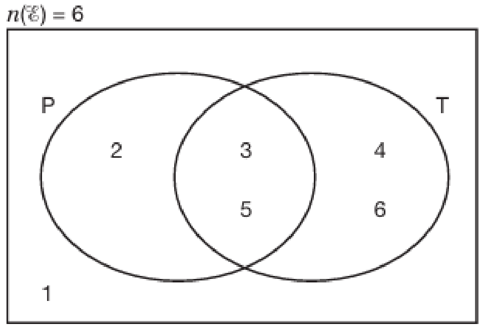
SKILLS
PROBLEM SOLVING

A fair six-sided die is thrown. What is the probability of throwing

a a **prime number**
b a number greater than 2
c a prime number or a number greater than 2
d a prime number that is greater than 2?

As all the outcomes are equally likely a Venn diagram can be used.

$n(\mathscr{E}) = 6$



The Venn diagram shows all the outcomes and the subsets
P = {prime numbers}, T = {numbers greater than 2}.

There are six possible outcomes so $n(\mathscr{E}) = 6$

a The probability of a prime number is $\frac{n(\text{P})}{n(\mathscr{E})} = \frac{3}{6} = \frac{1}{2}$

b The probability of a number greater than 2 is $\frac{n(\text{T})}{n(\mathscr{E})} = \frac{4}{6} = \frac{2}{3}$

c A prime number or a number greater than 2 is the set $\text{P} \cup \text{T}$, so the probability is $\frac{n(\text{P} \cup \text{T})}{n(\mathscr{E})} = \frac{5}{6}$

d A prime number that is greater than 2 is the set $\text{P} \cap \text{T}$, so the probability is $\frac{n(\text{P} \cap \text{T})}{n(\mathscr{E})} = \frac{2}{6} = \frac{1}{3}$

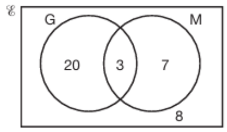
Note: You cannot add the probabilities from parts **a** and **b** to obtain the answer to part **c**. The Venn diagram shows that if you do this you count the numbers 3 and 5 twice.

EXAMPLE 2

SKILLS

PROBLEM SOLVING

The Venn diagram shows the number of students studying German (G) and Mandarin (M).



A student is picked at **random**. Work out

a P(G ∩ M)
b P(G′)
c P(G ∪ M)

a Total number of students: 20 + 3 + 7 + 8 = 38

$P(G \cap M) = \frac{\text{number of students in } G \cap M}{\text{total number of students}} = \frac{3}{38}$

b $P(G') = \frac{7+8}{38} = \frac{\text{number of students in } G'}{\text{total number of students}} = \frac{15}{38}$

c $P(G \cup M) = \frac{20+3+7}{38} = \frac{30}{38}$

KEY POINTS

- Venn diagrams can be used to work out probabilities if all the outcomes are equally likely. The probability of event A, written P(A), is $\frac{n(A)}{n(\mathscr{E})}$
- Venn diagrams are very useful in problems that ask for the probability of A and B or the probability of A or B.
  - The probability of A and B, written P(A ∩ B), is $\frac{n(A \cap B)}{n(\mathscr{E})}$
  - The probability of A or B (or both), written P(A ∪ B), is $\frac{n(A \cup B)}{n(\mathscr{E})}$

CONDITIONAL PROBABILITY USING VENN DIAGRAMS

Sometimes additional information is given which makes the calculated probabilities conditional on an event having happened

EXAMPLE 3

SKILLS

PROBLEM SOLVING

A mathematics teacher sets two tests for 50 students. 40 students pass the first test, 38 students pass the second test and 30 students pass both tests. Find the probability that a student selected at random

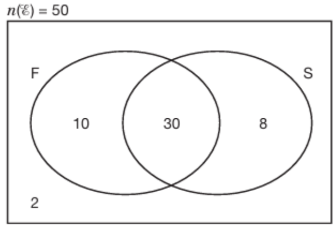
a passed both tests
b passed the second test given that the student passed the first test.

Let F = {students who passed the first test}
Let S = {students who passed the second test}
The information is shown in a Venn diagram.

The numbers must add up to 50, so 2 students failed both tests.

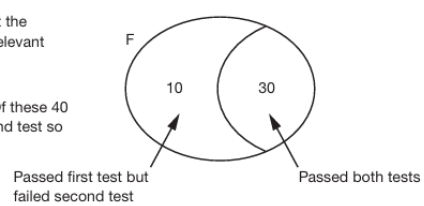
a 30 out of 50 students passed both tests so the probability is $\frac{30}{50} = 0.6$

$n(\mathscr{E}) = 50$



b As the student passed the first test the student must be in the set F. The relevant part of the Venn diagram is shown.

40 students passed the first test. Of these 40 students, 30 also passed the second test so the probability is $\frac{30}{40} = 0.75$



When you are given further information, then you are selecting from a subset rather than from the universal set. This subset becomes the new universal set for that part of the question.

The notation P(A|B) means ‘the probability of A given B has occurred’ or more simply ‘the probability of A given B’.

EXAMPLE 4

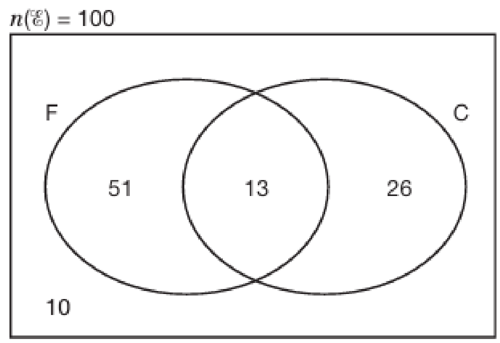
SKILLS
PROBLEM SOLVING

The Venn diagram shows the results of a survey of shopping habits.

F = {people who bought food online}

C = {people who bought clothes online}

Work out P(F|C) for this survey.



P(F|C) means the probability a person buys food online given that they buy clothes online.

The subset to select from is C

$n(C) = 13 + 26 = 39$

$\Rightarrow P(F|C) = \frac{13}{39} = \frac{1}{3}$

KEY POINTS

- Conditional probability means selecting from a subset of the Venn diagram.
- P(A|B) means 'the probability of A *given* B'.

ACTIVITY 1

SKILLS
PROBLEM SOLVING

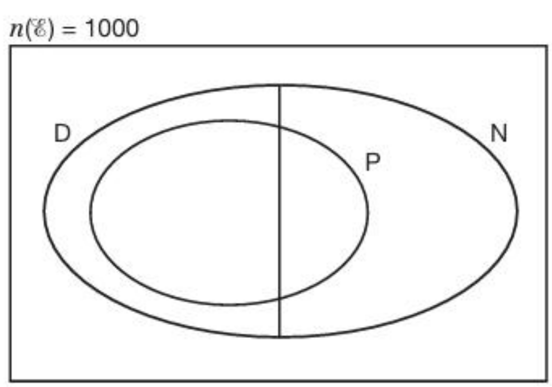
In a group of 1000 athletes it is known that 5% have taken a performance-enhancing drug. 98% of those who have taken the drug will test positive under a new test, but 2% of those who have not taken the drug will also test positive.

Let D = {athletes who have taken the drug}

N = {athletes who have not taken the drug}

P = {athletes who test positive}

Copy and complete the Venn diagram to show this data.



What is the probability that an athlete has not taken the drug given that their test result is positive? Comment on your answer.

Revision questions

1.

$$\text{Show that } \frac{1}{7} \times \frac{2}{3} \text{ is equal to } \frac{2}{21}$$

2.

$$\text{Show that } \frac{3}{5} - \frac{1}{3} \text{ is equal to } \frac{4}{15}$$

3.

$$\text{Show that } 3\frac{3}{4} \times \frac{7}{9} = 2\frac{11}{12}$$

4.

Work out £1.50 as a fraction of 60p

Circle your answer.

$\frac{2}{5}$ $\frac{1}{4}$ $\frac{4}{1}$ $\frac{5}{2}$

5.

$$\text{Show that } 2\frac{1}{4} \times 3\frac{1}{3} = 7\frac{1}{2}$$

Write the numbers 3, 4, 5 and 6 in the boxes to give the greatest possible total.

You may write each number only once.

$$\square\frac{1}{\square} + \square\frac{2}{\square}$$

6.

There are 60 children in a club.

In the club, the ratio of the number of girls to the number of boys is $3:1$

$\frac{3}{5}$ of the girls play a musical instrument.

$\frac{4}{5}$ of the boys play a musical instrument.

What fraction of the 60 children play a musical instrument?

7.

Each month Edna spends all her income on rent, on travel and on other living expenses.

She spends $\frac{1}{3}$ of her income on rent.

She spends $\frac{1}{5}$ of her income on travel.

She spends $\$420$ of her income on other living expenses.

Work out her income each month.

8.

Without using a calculator, work out $\frac{5}{6} \div 1\frac{1}{3}$.

You must show all your working and give your answer as a fraction in its simplest form.

9.

Without using a calculator, work out $2\frac{2}{3} \times 2\frac{3}{4}$.

You must show all your working and give your answer as a mixed number in its simplest form.

10.

Without using a calculator, work out $1\frac{1}{7} \times 2\frac{1}{10}$.

You must show all your working and give your answer as a mixed number in its simplest form.