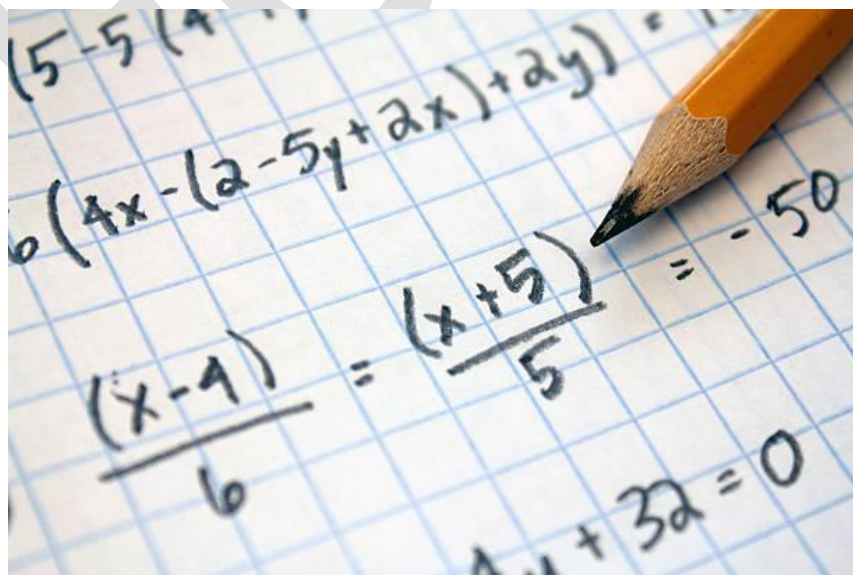# Edexcel OL Mathematics CODE: (4MA1)

Unit 1

Algebra

Chapter 04

BASIC PRINCIPLES
• Algebra uses letters, often x, to stand for numbers.
•x+3 means add three to the unknown number.
• Algebraic expressions can be treated in the same way as number expressions.

• x2 means square the unknown number.
• 3x means 3 times the unknown number.

SIMPLIFYING ALGEBRAIC EXPRESSIONS

SKILL: REASONING

EXAMPLE 1
Simplify a + 3ab-4ba a+3ab-4ba = a - ab
Note: ab = ba so 3ab and -4ba are like terms and can be simplified.

KEY POINTS
- You can only add or subtract like terms.
- $3ab + 2ab = 5ab$ but the terms in $3ab + b$ cannot be added together.
- $3a^2 + 2a^2 = 5a^2$ but the terms in $3a^2 + 2a$ cannot be added together.
- You can check your simplifications by substituting numbers.

EXAMPLE 2
Simplify $3p^3 + 2p^2 - 2p^3 + 5p^2$
$3p^3+2p^2-2p^3 + 5p^2 = 3p3 - 2p^3 + 5p^2 + 2p^2 = p^3 + 7p^2$

SIMPLIFYING ALGEBRAIC EXPRESSIONS WITH BRACKETS

EXAMPLE 3
Simplify 4r x 5t 4r x 5t=20rt

KEY POINTS
- The multiplication sign is often not included between letters, e.g. $3ab$ means $3 \times a \times b$.
- When multiplying, add like **powers** $3a^2b \times 2a^5b^4 \times a = 6a^8b^5$ (think of a as a^1).

EXAMPLE 4
Simplify $(3b)^2$ x 3b
$(3b)^2$ x 3b 3b x 36 x 3b = $27b^3$

EXPANDING BRACKETS

To simplify an expression with brackets, first multiply each term inside the bracket by the term outside the bracket, then simplify. This is called expanding the brackets.
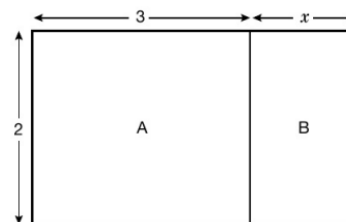
EXAMPLE 5
Simplify 2(3 + x).
2(3+x) =2x3+2 x x = 6 + 2x
The diagram helps to show that 2(3 + x) = 6 + 2x.
The area of the whole rectangle is 2(3 + x).
The area of rectangle A is 6.
The area of rectangle B is 2x

KEY POINTS
- Multiply each term inside the bracket by the term outside the bracket.
- The multiplication sign is usually left out:
  $3(x + y)$ means $3 \times (x + y) = 3 \times x + 3 \times y = 3x + 3y$
- Be very careful with negative signs outside a bracket:
  $-2(a - 3)$ means $-2 \times (a - 3) = (-2) \times (a) + (-2) \times (-3) = -2a + 6$
- When multiplying, the number 1 is usually left out:
  $-(2x + 3)$ means $-1 \times (2x + 3) = (-1) \times (2x) + (-1) \times (3) = -2x - 3$

SOLVING EQUATIONS

It is often easier to solve mathematical problems using algebra. Let the unknown quantity be x and then write down the facts in the form of an equation. There are six basics
types of the equation:

KEY POINTS
- To solve equations, do the same thing to both sides.
- Always check your answer.

$$x + 3 = 12 \qquad x - 3 = 12 \qquad 3 - x = 12$$

$$3x = 12 \qquad \frac{x}{3} = 12 \qquad \frac{3}{x} = 12$$

EXAMPLE 6

Solve x+3=12 for x.
x+3=12   (Subtract 3 from both sides)
x = 9   (Check: 9+3=12)

EXAMPLE 7
Solve x-3= 12 for x.
x-3=12   (Add 3 to both sides)
x=15   (Check: 15-312)

EXAMPLE 8
Solve 3- *x* = 12 for x
 3 - *x* = 12 (Add *x* to both sides)
3=12+*x* (Subtract 12 from both sides)
-12+3 = *x*
x = -9 (Check: 3-(-9) = 12)

EXAMPLE 9
Solve 3x = 12 for x.
3*x* =12 (Divide both sides by 3)
X=4 (Check: 3 x 4 = 12)

EXAMPLE 10
Solve x/3= 12 for x.
x/3 (Multiply both sides by 3)
x = 36 (Check: 36 ÷ 3 = 12)

EXAMPLE 11
Solve 3/x= 12 for x.
3/x= 12(Multiply both sides by x)
3 = 12x (Divide both sides by 12)
1/4 =x (Check: 3+1=12)

EQUATIONS WITH *X* ON BOTH SIDES

EXAMPLE 15
Solve 7x-3 = 3x + 5 for x
7x-3=3x+5(Subtract 3x from both sides)
7x-3x-3=5(Add 3 to both sides)
4x=5+3 (Simplify)
4x =8 (Divide both sides by 4)
X=2 (Check: 7 x 2-3=3x2+5=11)

EXAMPLE 16
Solve 5x+6=3(10-x) for x.
5x+6=3(10-x) (Multiply out the bracket)
5x+6=30-3x) Add 3x and subtract 6 from both sides)
5x+3x30-6 (Simplify)
8x = 24 (Divide both sides by 8)
x=3 (Check: 5 x 3 + 6 = 3(10-3) =21)

NEGATIVE SIGNS OUTSIDE BRACKETS

EXAMPLE 17
Solve 2(3x+1) -(2x-5) = 15 for x.
Solve 2(3x+1) -(2x-5) = 15 (remove brackets)
2(3x+1) (2x-5) = 15 (Simplify)
4x+7=15 (Subtract 7 from both sides)
4x=8 (Divide both sides by 4)
x = 2(Check: 2(3x2 + 1) -(2 x 2 -5) = 15)

KEY POINT
• $-(2x-5)$ means $-1 \times (2x-5) = (-1) \times (2x) + (-1) \times (-5) = -2x + 5$

PROBLEMS LEADING TO EQUATIONS

Let the unknown quantity be x. Write down the facts in the form of an equation and then solve it.

EXAMPLE 18

The sum of the three **consecutive** numbers is 219. What are the numbers?

Let the first number be x. Then the next two numbers are (x + 1) and (x + 2).

$x+(x+1)+(x+2)=219$

$3.x + 3 = 219$

$3x=216$

$x = 72$

So, the three numbers are 72, 73 and 74. (Check: 72 + 73+ 74 = 219)

EXAMPLE 19

SKILL: REASONING

Find the value of x and the perimeter of this isosceles triangle.
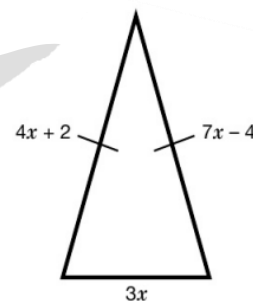
As the triangle is isosceles

$4x+2=7x-4$

$2+4=7x-4x$

$6 = 3x$

$x = 2$

Check: $4 \times 2+2=7x2-4 = 10$

The sides are 10, 10 and 6 so the perimeter is 26.

Revision equations

1)a) Multiply out and simplify $(x-8)^2$
b) (2x-4) (3x+5) is expanded and simplified.
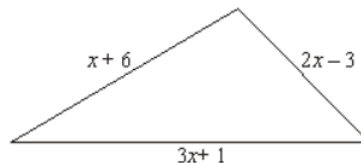c)Expand 4x2 (3x + 5)
d)Expand 4t (312)
e) Expand 7(x+5)

2)Ben is n years old.
Colin is three years younger than Ben.
(a) Write down an expression, in terms of n, for Colin's age.
(b)Daniel is twice as old as Ben.
Write down an expression, in terms of n, for Daniel's age.

3) In the diagram, all measurements are in centimetres.
The lengths of the sides of the triangle are
x + 6
2x-3
3x + 1

(a)Find an expression, in terms of x, for the perimeter of the triangle.
Give your expression in its simplest form.

The perimeter of the triangle is 37 cm.
(b) Find the value of x.

Focus College

4)David buys some stamps.

Each stamp costs 25p.

The total cost of the stamps is £3

(a) Work out the number of stamps David buys.

b) Adam, Barry and Charlie each buy some stamps.

Adam buys x stamps.

Barry buys three times as many stamps as Adam.

Write down an expression, in terms of x, for the number of stamps Barry buys.

c)Charlie buys 5 more stamps than Adam.

Write down an expression, in terms of x, for the number of stamps Charlie buys.

5)(a) Simplify 4x+3y-2x+5y

Compasses cost c pence each.

Rulers cost r pence each.

(b) Write down an expression for the total cost, in pence, of 2 compasses and 4 rulers.

6)Compasses cost c pence each.

Rulers cost r pence each.

Write down an expression for the total cost, in pence, of 2 compasses and 4 rulers.

7)a)The nth even number is 2n.

The next even number after 2n is 2n + 2 Explain why.?

(b)Write down an expression, in terms of n, for the next even number after 2n + 2

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6

8)Prove that $(3n + 1)^2 - (3n-1)^2$ is a multiple of 4, for all positive integer values of n.

9)Tarnish says,

'The sum of two prime numbers is always an even number'.

He is wrong. Explain why.

10)Prove that the difference between the squares of consecutive odd numbers is a multiple of 8