

Edexcel O/L

Mathematics

Code: 4MA1

Unit 1

Numbers



Basic Principles

- **Sign** of answer when multiplying or dividing.

$$+ \times + = + \quad + \times - = - \quad - \times + = - \quad - \times - = +$$

$$+ \div + = + \quad + \div - = - \quad - \div + = - \quad - \div - = +$$

- Finding **common factors**: Common factors of 12 and 8 are 2 and 4.
- Finding lowest **common denominator** when adding and subtracting fractions: Lowest common denominator of 6 and 4 is 12
- The value of a fraction is not changed if the top and bottom multiplied or divide same number.

$$\frac{1}{2} = \frac{3 \times 1}{3 \times 2} = \frac{3}{6} \quad \frac{4}{10} = \frac{2 \times 2}{2 \times 5} = \frac{2}{5}$$

- Converting **mixed numbers** to fraction: $1\frac{2}{3} = \frac{5}{3}$

Working with Fractions

Simplifying fractions

A fraction has been simplified when the **numerator** (the top number) and the denominator (the bottom number) are expressed as whole number with no common factors.

Examples:

Skill: Analysis

Simplify

a. $\frac{28}{42}$

b. $\frac{0.8}{1.6}$

a. $\frac{28}{42} = \frac{2 \times 14}{2 \times 21} = \frac{2 \times 7}{3 \times 7} = \frac{2}{3}$

b. $\frac{0.8}{1.6} = \frac{0.8 \times 10}{1.6 \times 10} = \frac{8}{16} = \frac{8 \times 8}{16 \times 8} = \frac{1}{2}$

How to write decimals as fractions

Change

a. 0.4

b. 0.025 to fractions.

a. $0.4 = \frac{4}{10} = \frac{2}{5}$

b. $0.025 = \frac{25}{1000} = \frac{5 \times 5}{5 \times 5 \times 40} = \frac{1}{40}$

To write a fraction as a decimal, divide the top number by the bottom number.

Change

a. $\frac{2}{5}$

b. $\frac{5}{8}$ to decimal

a. $2 \div 5 = 0.4$ (using a calculator or **long division**)

b. $5 \div 8 = 0.625$ (using a calculator or **long division**)

Key points

- Always simplify the fractions.
- When working with mixed numbers, convert to **improper fractions** first.

Multiplying fractions

If you do not know why one-half of one-third is the same as one-half multiplied by one-third, read the next example.

Examples:

Skill: Problem solving

Ella has a bar of chocolate. Her mother says she can eat one-half of one-third of the bar.

How much does Ella eat?

When Ella unwraps the bar, she finds it has six squares.



One-third of the bar is two squares



Half of this is one square.



So one-half of one-third of the bar is one square or one-sixth.

This is the same as one-half multiplied by one-third. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Key points

- The word 'of' means 'multiplied by'.
- Convert mixed numbers into improper fractions before multiplying.
- If possible, divide by common factors before multiplying.
- Treat whole numbers as fractions, e.g. $5 = \frac{5}{1}$.

Dividing fractions

To divide by a fraction, turn the fraction upside down and multiply. The next two examples explain this rule. The word 'reciprocal' is used for turning a fraction upside down.

Examples:

Skill: Problem solving

Half of Ella's chocolate bar is divided equally into three for three friends.

How does each friend receive?

Half of Ella's bar is three squares of chocolate.

When divided in three, each friend receives one square or one-sixth of the original bar.

$$\text{So } \frac{1}{2} \div 3 = \frac{1}{6}$$



By writing 3 as $\frac{3}{1}$ you can see that the rule works: $\frac{1}{2} \div \frac{3}{1} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
 $2 \div \frac{1}{3}$ means how many thirds are in two whole units.



Key points

- To divide by a fraction, turn the fraction upside down and multiply.

Adding and Subtracting fractions

This can only be done if the denominator are the same.

Examples:

Skill: Problem solving

Ella eats one-half of her bar of chocolate and then eats a further third.
 What fractions of the bar has she eaten?

Half the bar is 3 squares.



One-third of the bar is 2 squares.



One-half plus one-third equals to five-sixth or $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Key points

- To add and subtract fraction, put them over a common denominator.
- Less work is needed if the common denominator is the lowest one.

Order of Operations

The answer to $3+4 \times 2$ depends on whether the addition or multiplication is done first. So that's everybody gets the same answer to a calculation, there are rules for the order of operations. (Examples of operations: addition, subtraction, multiplication and division.)

The mnemonic BDMAS will help you remember the correct order.

Key points

- | | | |
|----------|----|--|
| • First | B | Brackets |
| • Second | I | Indices |
| • Third | DM | Division and /or Multiplication, work from left to right |
| • Fourth | AS | Addition and /or Subtraction, work from left to right |

Examples:

Skill: Interpretation

Evaluate $7-3 \div (5-2) \times 2^2 + 5$

This part of the expression being worked out at each step is highlighted in yellow.

$$7-3 \div (5-2) \times 2^2 + 5 = 7-3 \div 3 \times 2^2 + 5$$

$$7-3 \div 3 \times 2^2 + 5 = 7-3 \div 3 \times 4 + 5$$

$$7-3 \div 3 \times 4 + 5 = 7-1 \times 4 + 5$$

$$7-1 \times 4 + 5 = 7-4 + 5$$

$$7-4 + 5 = 3 + 5$$

$$3 + 5 = 8$$

Brackets

Indices

Division and /or Multiplication, work from left to right

Division and /or Multiplication, work from left to right

Addition and /or Subtraction, work from left to right

Addition and /or Subtraction, work from left to right

Significant figures and decimal places

If a piece of wood is to be cut 35.784 mm long, then this measurement is too accurate to mark out and cut, so 34.784 would be **rounded** to a suitable **degree of accuracy**. Numbers can be rounded to a certain number of **significant figures** or **decimal places**.

Significant figures (s.f.)

The first s.f. is the first non-zero digit in the number, counting from the left.

Examples:

Skill: Interpretation

Highlights the first s.f. of following numbers.

a. 27 400

b. 0.123

c. 0.000583

The first s.f. is highlighted in yellow.

a. 27 400

b. 0.123

c. 0.000583

Decimal places (d.p.)

Count after the decimal point (going from left to right).

Rounding up or down follows the same rules as for s.f.

Examples:

Skill: Interpretation

Write

a. 7.3167

b. 0.135

c. 0.0349 correct 2 d.p.

a. 3rd d.p. is $6 \geq 5$ so 3 rounds up to 4 $\Rightarrow 7.1361 = 7.14$ (2 d.p.)
(7.1361 is closer to 7.14 than 7.13)

b. 3rd d.p. is $5 \geq 5$ so 3 rounds up to 4 $\Rightarrow 0.135 = 0.14$ (2 d.p.)
(0.135 is midway between 0.14 and 0.13 but we round up in this case)

c. 3rd d.p. is $4 \geq 5$ so 3 is not rounds up $\Rightarrow 0.0349 = 0.03$ (2 d.p.)
(0.0349 is closer to 0.03 than 0.04)

This table shows $\pi = 3.141\ 592\ 654\ \dots$. Rounded to various degrees of accuracy.

DEGREE OF ACCURACY	SIGNIFICANT FIGURE	DECIMAL PLACE
5	3.1416	3.14159
3	3.14	3.142
1	3	3.1

Key points

- The first significant figure is the first non-zero digit in the number, counting from left.
- For decimal places, count after the decimal point (going from left to right).
- If the next number is greater than or equal to 5, then round up

Exercise

1. A box contains only red pencils and blue pencils. The ratio of the number of red pencils to the number of blue pencils is 2: 3. What fraction of the pencils are red?
2. Write as a decimal.
 - (a) $\frac{9}{10}$
 - (b) $\frac{1}{5}$
 - (c) $\frac{16}{10}$
 - (d) $\frac{3}{4}$
 - (e) $\frac{1}{4}$
3. Write as a fraction.
 - (a) 0.25
 - (b) 0.3
 - (c) 1.4
 - (d) 0.5
 - (e) 0.07
4. Simplify.
 - (a) $\frac{18}{24}$
 - (b) $\frac{25}{100}$
 - (c) $\frac{10}{15}$
 - (d) $\frac{25}{50}$

5. One month, Greta! spent a total of \$360 on presents. She spent $\frac{1}{5}$ of this total on presents for her parents. She spent $\frac{2}{3}$ of the remaining money on presents for her friends. She spent the rest of the money on presents for her sisters. Calculate the percentage of the \$360 that she spent on presents for her sister.
6. Collette has \$136 and spends half of her money on clothes and $\frac{1}{5}$ of her money on books. Calculate the amount she has left.
7. In a school there are 220 pupils in Year 9. 120 of these pupils are girls. What fraction of the 220 pupils are boys? Give your fraction in its simplest form.
8. Give your answers as fractions in simplest forms.
 - i. $\frac{15}{28} \div \frac{4}{7}$
 - ii. $\frac{2}{3} + \frac{1}{4} \times \frac{2}{3}$
 - iii. $\frac{12}{35} \times \frac{7}{9}$
 - iv. $\frac{1}{15} + \frac{2}{5}$
 - v. $2\frac{2}{3} + 3\frac{3}{2}$
 - vi. $\frac{5}{6} \div 1\frac{1}{3}$
 - vii. $2\frac{2}{3} \times 2\frac{3}{4}$
 - viii. $1\frac{1}{7} \times 2\frac{1}{10}$
 - ix. $\left(2\frac{1}{3} - \frac{7}{8}\right) \times \frac{6}{5}$
 - x. $1\frac{3}{4} - \frac{11}{22}$
9. Write 0.046875 correct to two significant figures.
10. To the nearest 1000, there are 18 000 people at a festival.
 - (a) Write down the minimum possible number of people at the festival.
 - (b) Write down the maximum possible number of people at the festival.
11. Write 57.3997 correct to 4 significant figures
12. $T = \frac{49.2 - 9.50}{4.085 \times 2.35}$
 By writing each number correct to 1 significant figure, work out an estimate for T. you must show all your working.

13. The mass, m kilograms, of a horse is 429kg, correct to the nearest kilogram. Complete this statement about the value of m .
 $\dots\dots\dots \leq m < \dots\dots\dots$
14. The space allowed for each tent is rectangle measuring 8m by 6m, each correct to the nearest meter. Calculate the upper bound for the area of the space allowed for each tent.
15. The length of a roll of ribbon is 30m, correct to the nearest half – meter. A piece of length 5.8m, correct to the nearest 10cm, is cut from the roll. Work out the maximum possible length of ribbon left on the roll.
16. Each month Edna spends all her income on rent, on travel and on other living expenses. She spends $\frac{1}{3}$ of her income on rent. She spends $\frac{1}{5}$ of her income on travel. She spends \$420 of her income on other living expenses. Work out her income each month.
17. There are 60 children in a club. In the club, the ratio of the number of girls to the number of boys is 3 : 1. $\frac{3}{5}$ of the girls play a musical instrument. $\frac{4}{5}$ of the boys play a musical instrument. What fraction of the 60 children play a musical instrument?
18. The average attendance across all the games at the world cup was 82.5% of capacity Round 82.5 to 2 significant figures.
19. Tommy performs the following calculation $1332.33 + 234.23 = 1566.56$
 (i) Round the answer to 1 decimal place.
 (ii) Round the answer to the nearest integer.
 (iii) Round the answer to the nearest ten.
 (iv) Round the answer to the nearest thousand.
20. In the first round of a gymnastics competition, James is given a score of $2\frac{2}{11}$. His score is later revised to account for the difficulty in that round. His new score is $2\frac{2}{11} \times 1\frac{1}{12}$
 (a) What is his new score? Give your answer as a mixed fraction.
 (b) In the second round of the competition, James scores $7\frac{1}{2}$. His score is again revised. His score is now $7\frac{1}{2} \div \frac{2}{3}$. What is his new score? Give your answer as a mixed fraction.