

*Edexcel*  
*IGCSE*  
*Mathematics*  
*CODE: (4MA1)*  
*Unit 1*  
*Sets*



## Basic principles

- Recognize different types of numbers (odd, even, prime, square, ...).
- Know the properties of triangles and quadrilaterals.
- Understand multiples and factors of numbers.

### Set notation.

A **set** is a collection of objects which are called the elements or members of the set.

#### EXAMPLE 1

A set described by the list.

{Anne, Nikos, Bob}

is the set consisting of the three people called Anne, Nikos, and Bob.

#### EXAMPLE 2

A set described by the rule (even numbers between 1 and 11) is the set consisting of the five numbers 2, 4, 6, 8, 10.

Sets are often labelled by a single capital letter.

$A = \{\text{odd numbers between 2 and 10}\}$  means  $A$  is the set consisting of the four numbers 3, 5, 7, 9.

The number of elements in the set  $A$  is  $n(A)$ , so  $n(A) = 4$ .

Sets can be infinite in size, for example the set of **prime numbers**.

Membership of a set is indicated by the symbol  $\in$  and non-membership by the symbol  $\notin$ .

#### EXAMPLE 3

If  $E = \{2, 8, 4, 6, 10\}$  and  $F = \{\text{even numbers between 1 and 11}\}$ , then  $(E) = 5$ ,  $(F) = 5$ ; in other words, both  $E$  and  $F$  have the same number of elements.

$3 \notin E$  means 3 is not a member of the set  $E$ .

$6 \in F$  means 6 is a member of the set  $F$ .

$E = F$  because both  $E$  and  $F$  have the same members.

The order of listing the members does not matter.

The empty set,  $\emptyset$  or  $\{\}$ , is the set with no members.

#### EXAMPLE 4

Give two examples of the empty set.

- The set of people you know over 4 m tall.
- The set of odd integers divisible by two.

#### KEY POINTS

- A set is a collection of objects, described by a list or a rule.
- Each object is an element or member of the set.
- Sets are equal if they have exactly the same elements.
- The number of elements of set  $A$  is given by  $n(A)$ .
- The empty set is the set with no members.

$$A = \{1, 3, 5\}$$

$$1 \in A, 2 \notin A$$

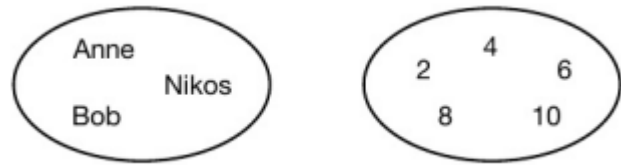
$$B = \{5, 3, 1\}, B = A$$

$$n(A) = 3$$

$$\{\} \text{ or } \emptyset$$

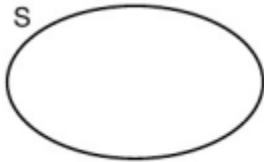
## VENN DIAGRAMS

Sets can be shown in a diagram called a Venn diagram after the English mathematician John Venn (1834-1923). The members of the set are shown within a closed curve.

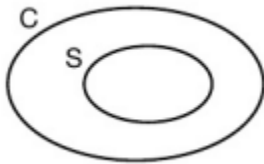


When the number of elements is large, a closed curve is drawn and labelled to indicate the set.

If  $S = \{\text{striped cats}\}$  then the Venn diagram is



If  $C = \{\text{cats in the world}\}$ , S and C can be shown on a Venn diagram as



Set S is shown inside set C because every member of S is also a member of C. S is called a subset of C. This is written as  $S \subset C$ .

### EXAMPLE 5

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

a List the subset  $O = \{\text{odd numbers}\}$

b List the subset  $P = \{\text{prime numbers}\}$

c Is  $Q = \{8, 4, 6\}$  a subset of A?

d Is  $R = \{0, 1, 2, 3\}$  a subset of A?

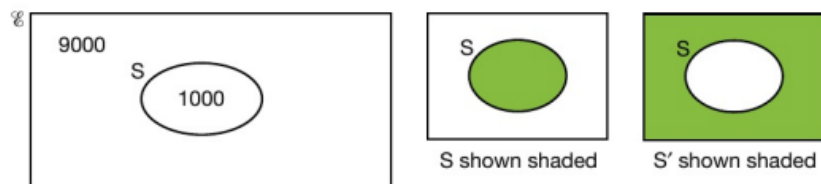
a  $O = \{1, 3, 5, 7, 9\}$

b  $P = \{2, 3, 5, 7\}$

c  $Q \subset A$  because every member of Q is also a member of A.

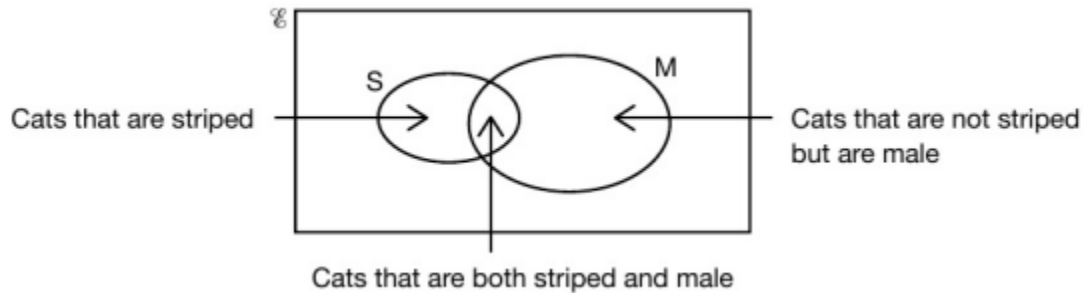
d  $R \not\subset A$  because the element 0 is a member of R but it is not a member of A.

The diagram shows that there are 9000 non-striped cats outside S. This set is denoted by  $S'$  and is called the **complement** of S.



## INTERSECTION OF SETS

Sets can overlap. Let  $M = \{\text{male cats}\}$ .  $S$  and  $M$  overlap because some cats are both striped and male.  $S$  and  $M$  are shown on the Venn diagram.



### EXAMPLE 6

SKILL: REASONING

$\mathcal{U} = \{\text{all positive integers less than 10}\}$ ,  $P = \{\text{prime numbers less than 10}\}$  and  $O = \{\text{odd numbers less than 10}\}$ .

a Show this information on a Venn diagram.

b Find the set  $P \cap O$  and  $n(P \cap O)$ .

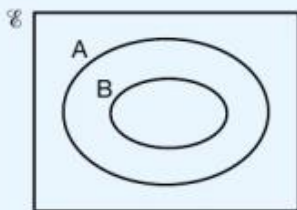
c List  $P'$ .

#### KEY POINTS

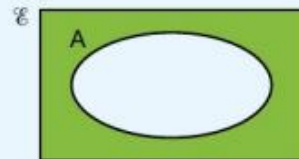
- The universal set,  $\mathcal{U}$ , contains all the elements being considered in a particular problem.



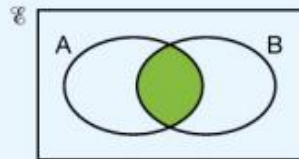
- $B$  is a subset of  $A$ ,  $B \subset A$ , if every member of  $B$  is a member of  $A$ .



- The complement of set  $A$ ,  $A'$ , is the set of all elements not in  $A$ .



- The intersection of  $A$  and  $B$ ,  $A \cap B$ , is the set of elements which are in both  $A$  and  $B$ .



## UNION OF SETS

The union of two sets  $A$  and  $B$  is the set of elements that belong to  $A$  or to  $B$  or to both  $A$  and  $B$ , and is written  $A \cup B$ .

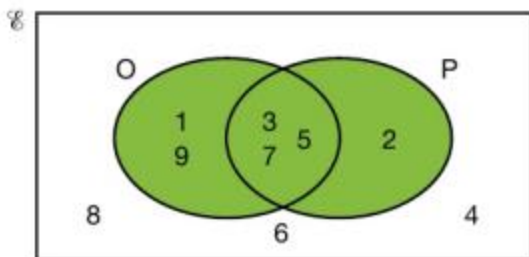
### EXAMPLE 7

$\mathcal{U} = \{\text{all positive integers less than 10}\}$ ,  $P = \{\text{prime numbers less than 10}\}$  and  $O = \{\text{odd numbers less than 10}\}$ .

a Show this information on a Venn diagram.

b Find the set  $P \cup O$  and  $n(P \cup O)$ .

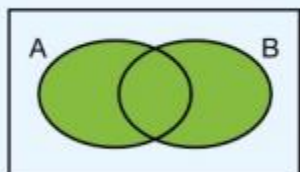
a The set  $P \cup O$  is shown shaded in the Venn diagram.



b From the Venn diagram  $P \cup O = \{1, 2, 3, 5, 7, 9\}$  and  $n(P \cup O) = 6$

**KEY POINT**

• The union of A and B,  $A \cup B$ , is the set of elements which are in A or B or both.



## Revision questions

1) A and B are two sets.

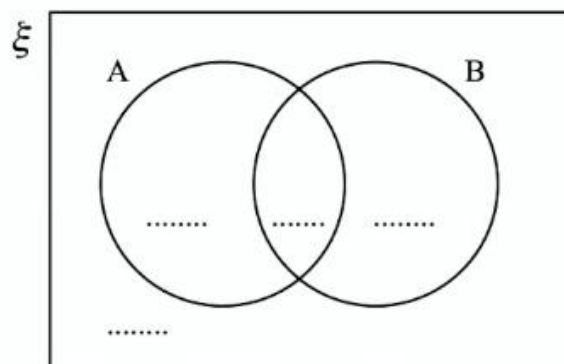
$$n(U) = 37$$

$$n(A) = 22$$

$$n(A \cap B) = 12$$

$$n(A \cup B) = 30$$

Complete the Venn diagram to show the number of elements in each region.



b) Find

i)  $n(A \cap B')$

ii)  $n(A' \cup B')$

2) There are 31 students in a class.

The only languages available for the class to study are French and Spanish.

17 students study French.

15 students study Spanish.

6 students study neither French nor Spanish.

Using a Venn diagram, or otherwise, work out how many students study only one language.

3) A garage tests cars for faults.

There are three types of fault - braking, steering and lighting.

A car fails the test if it has one or more of these three types of fault.

Last week, 11 cars had braking faults,

9 cars had steering faults,

7 cars had lighting faults,

no car had both steering faults and lighting faults,

2 cars had both braking faults and steering faults,

3 cars had both braking faults and lighting faults.

By drawing a Venn diagram, or otherwise, find the number of cars which failed the test last week.

4) Each student in a group of 32 students was asked the following question.

"Do you have a desktop computer (D), a laptop (L) or a tablet (T)?"

Their answers showed that,

19 students have a desktop computer,

17 students have a laptop,

16 students have a tablet,

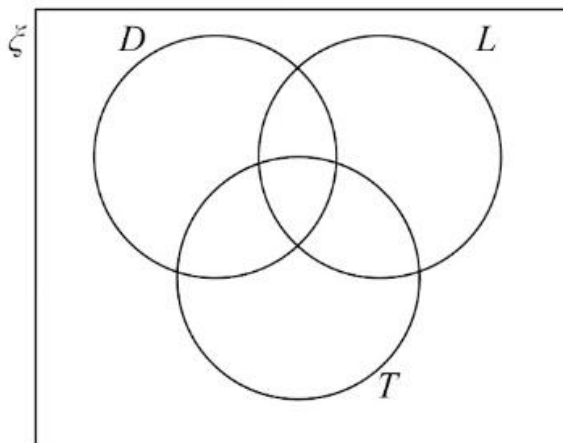
9 students have both a desktop computer and a laptop,

11 students have both a desktop computer and a tablet,

7 students have both a laptop and a tablet,

5 students have all three.

Using this information, complete the Venn diagram to show the number of students in each appropriate subset.



5) Some students were asked the following question.

"Which of the subjects Russian (R), French (F) and German (G) do you study?"

Of these students

4 study all three of Russian, French and German

10 study Russian and French

13 study French and German

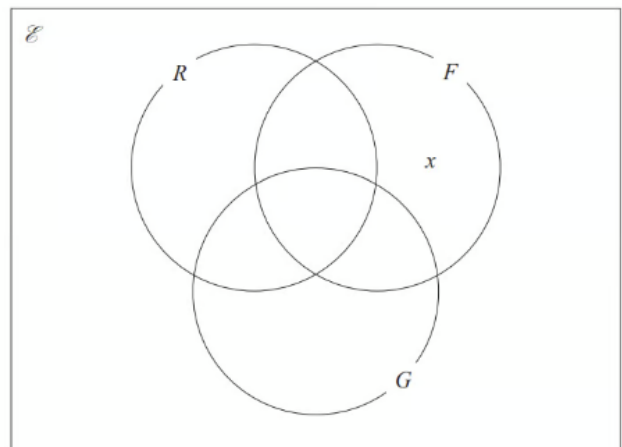
6 study Russian and German 24 study German

11 study none of the three subjects

the number who study Russian only is twice the number who study French only.

Let  $x$  be the number of students who study French only.

Show all this information on the Venn diagram, giving the number of students in each appropriate subset, in terms of  $x$  where necessary.



6) Some students in a school were asked the following question.

"Do you have a dog (D), a cat (C) or a rabbit (R)?"

Of these students

28 have a dog.

18 have a cat.

20 have a rabbit.

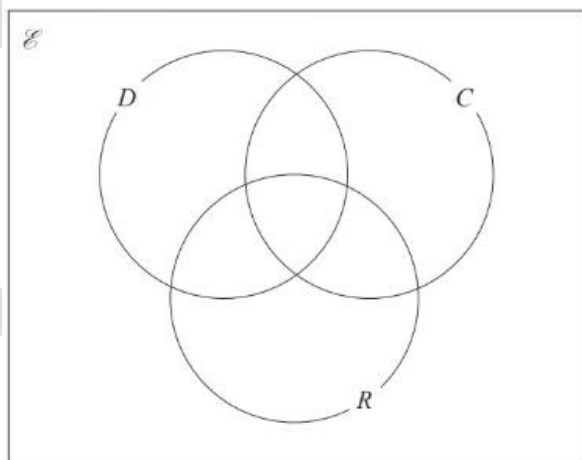
8 have both a cat and a rabbit 9 have both a dog and a rabbit  $x$  have both a dog and a cat.

6 have a dog, a cat, and a rabbit.

5 have not got a dog or a cat or a rabbit.

Using this information, complete the Venn diagram to show the number of students in each appropriate subset.

Give the numbers in terms of  $x$  where necessary.



b) given that a total of 50 students answered the question work out the  $x$ .

7) a)  $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{\text{odd numbers}\}$

$P = \{\text{prime number}\}$

List the members of the set  $A \cap P$ .

b)  $\mathcal{E} = \{\text{whole numbers}\}$

$A = \{\text{factors of 100}\}$

$B = \{\text{multiples of 5}\}$

List the members of the set  $A \cap B$ .

8) a)  $\mathcal{U} = \{\text{positive whole numbers less than 19}\}$

$A = \{\text{odd numbers}\}$

$B = \{\text{multiples of 5}\}$

$C = \{\text{multiples of 4}\}$

List the members of the set,

i)  $A \cap B'$ ,

ii)  $B \cup C$ .

b)  $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{\text{even numbers}\}$

$B = \{\text{multiples of 3}\}$

List the members of set  $B'$ .