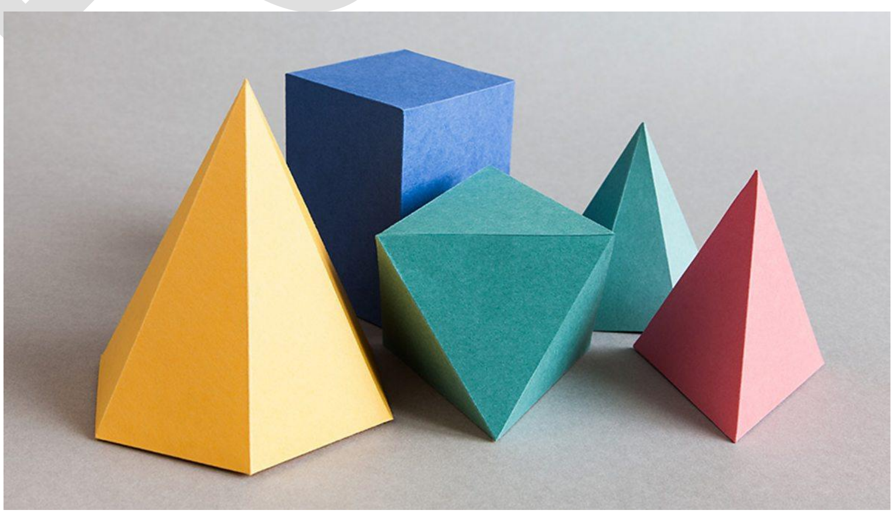
Edexcel OL Mathematics

CODE: (4MA1)

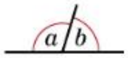
Unit 1

Shape and space



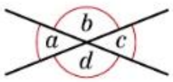
BASIC PRINCIPLES

- Angles on a straight line.



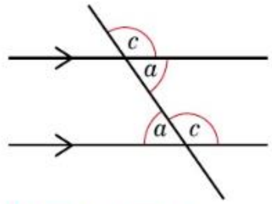
$a + b = 180°$

- Vertically opposite angles.



$a = c, b = d$

- Parallel lines.

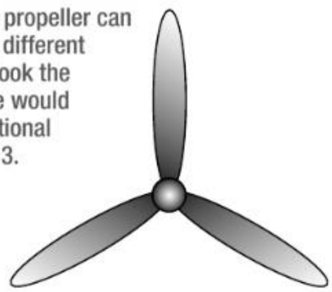


Alternate angles a are equal

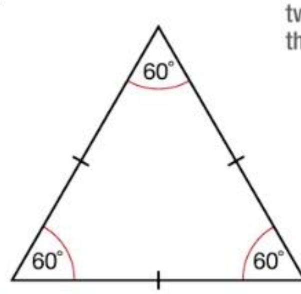
Corresponding angles c are equal

- Lines of symmetry. If a shape is folded along a line of symmetry, both halves will match exactly.

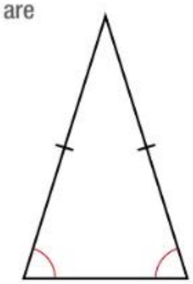
- **Rotational symmetry**. If a shape has rotational symmetry then it still looks the same after a rotation of less than one turn. This propeller can be rotated to three different positions and still look the same. In maths, we would say that it has rotational symmetry of order 3.



- Equilateral triangle. All three sides are the same length and all three angles are 60°.



- Isosceles triangle. Two sides are the same length and two angles are the same.

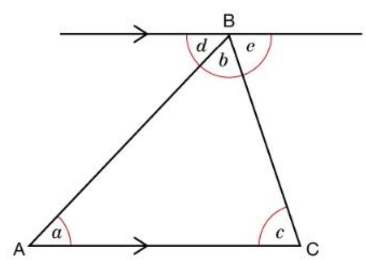


TRIANGLES

The angle **sum** of a triangle is 180°.

PROOF

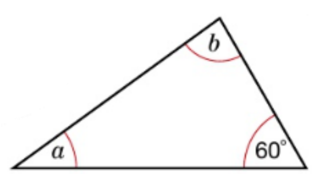
ABC is any triangle.
Through B draw a line parallel to AC.
(1)d + b + e = 180° (angles on a straight line)
(2)d = a (alternate angles)
(3) e = c (alternate angles)
Substituting (2) & (3) into (1) gives a + b + c = 180°.



EXAMPLE 1

SKILL: REASONING

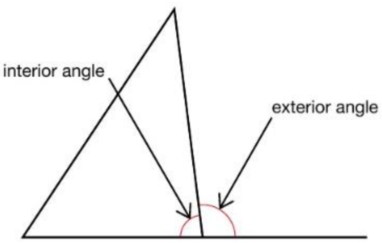
One angle of a triangle is 60°
What is the sum of the other two angles?



a+b+60 180°
⟺a + b = 120°

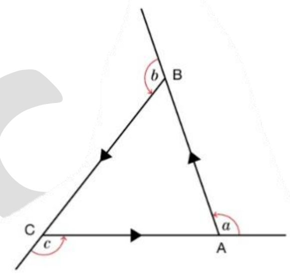
INTERIOR AND EXTERIOR ANGLES

The **exterior angle** is formed when one side of a triangle is extended.



ANGLE SUM OF THE EXTERIOR ANGLES OF A TRIANGLE

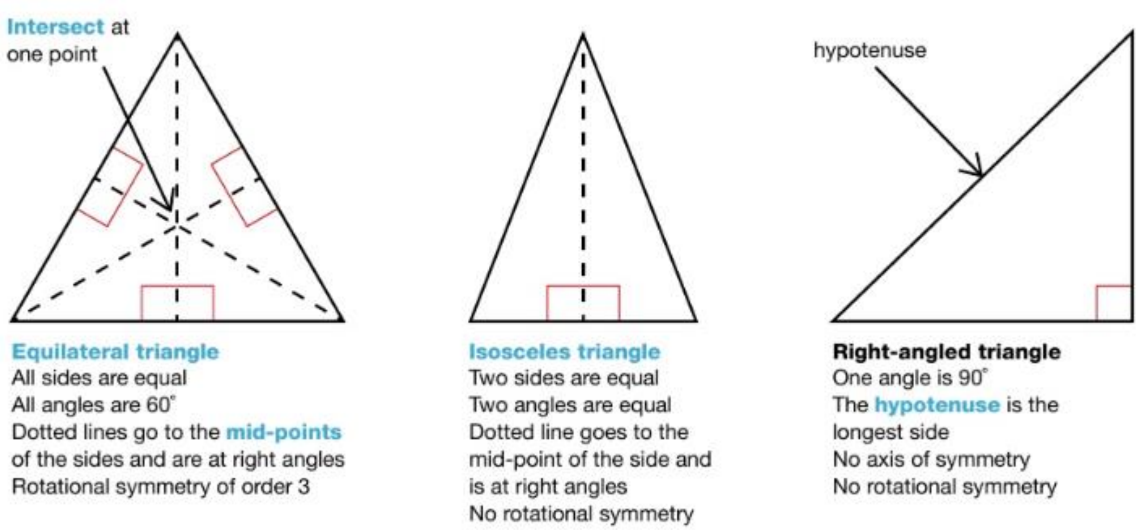
Imagine you are walking around a triangular field ABC.
Start at A, facing B.
Walk to B then turn anti-clockwise through angle b to face C.
Walk to C then turn anti-clockwise through angle c to face A. Walk to A then turn anti-clockwise through angle a to face B again.



You have turned through 360°.
You have also turned through a +b+ c.
So, a+ b+ c=360°.

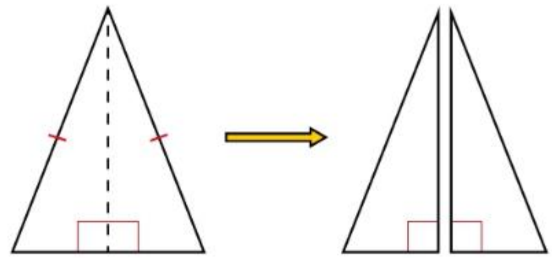
SPECIAL TRIANGLES

Some triangles have special names and properties.
A dotted line in the diagrams shows an axis of symmetry.



It is possible to have a right-angled isosceles triangle with an axis of symmetry.

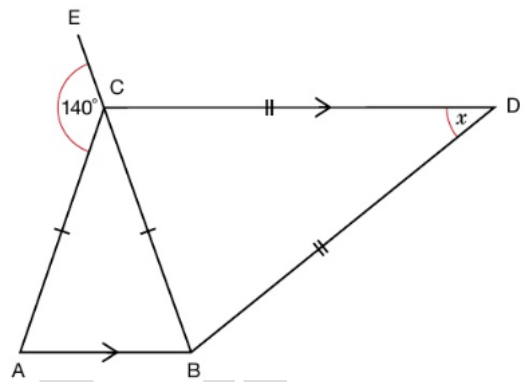
An isosceles triangle can always be split down the line of symmetry into two equal right-angled triangles. This is very important for solving problems using Pythagoras' Theorem or trigonometry later in the book.



EXAMPLE 2

SKILL: REASONING

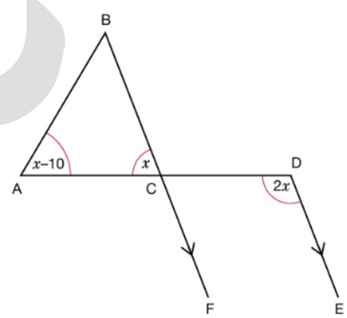
ABC and BCD are isosceles triangles.
AB is parallel to CD.
<ACE is 140°.
Find the angle marked x.
<ACE is an exterior angle of ΔABC
⇒ <CAB + <ABC = 140°
<CAB = ZABC (ΔABC is isosceles)
⇒ LABC = 70°
<ABC = <BCD (alternate angles)
<BCD =<CBD = 70° (ABCD is isosceles)
angle x = 40° (angle sum of the triangle)



EXAMPLE 3

SKILL: REASONING

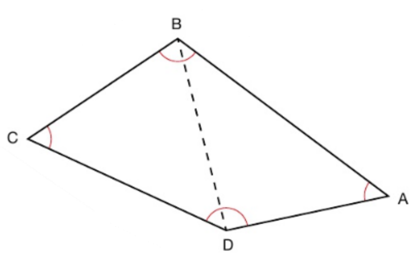
Work out the size of angle ABC.
<FCD = x (vertically opposite angles)
<FCD +<CDE = 180° (interior angles of parallel lines)
⇒3x =180⇒x = 60°
⇒<BAC 50° (<BAC = x-10)
⇒<ABC=70 (angle sum of a triangle)

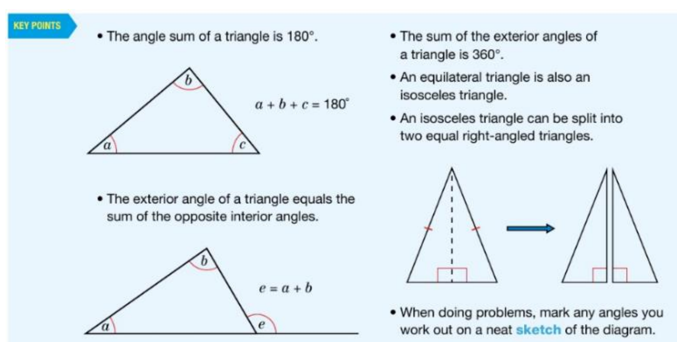


QUADRILATERALS

INTERIOR ANGLES

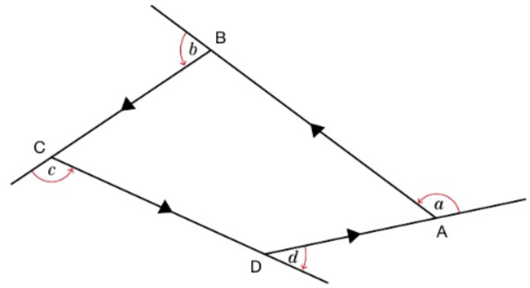
A **quadrilateral** can always be split into two triangles.
The angle sum of each triangle is 180°.
So, the angle sum of the quadrilateral is 360°.





EXTERIOR ANGLES

In the same way that you imagined walking around a triangular field, imagine walking around a quadrilateral field ABCD.



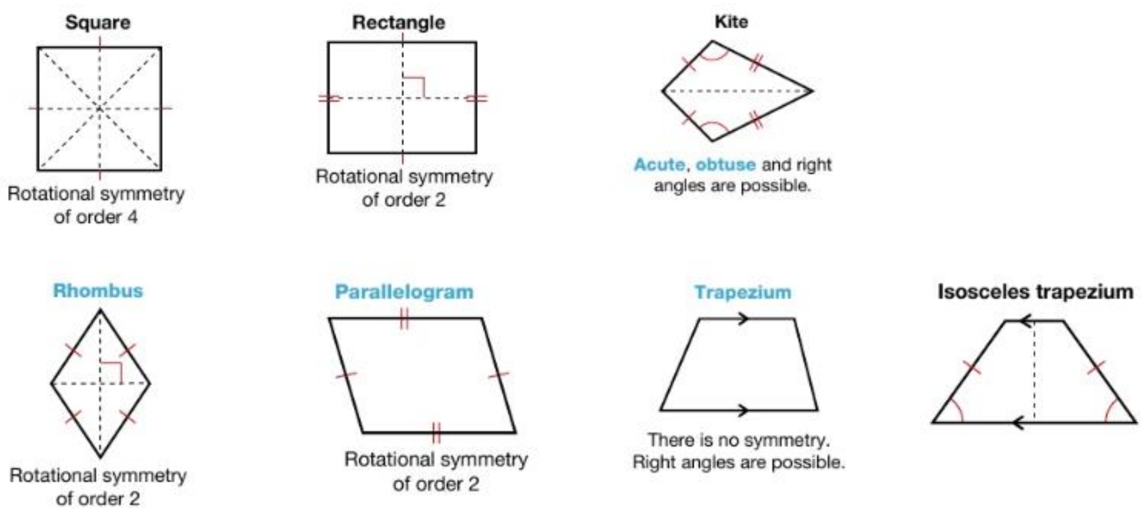
Again, you turn anti-clockwise through 360°.

So, a+b+c+d= 360°

The sum of the exterior angles of a quadrilateral is 360°.

SPECIAL QUADRILATERALS

The diagrams show quadrilaterals with special names and properties. A dotted line in the diagrams shows an axis of symmetry.

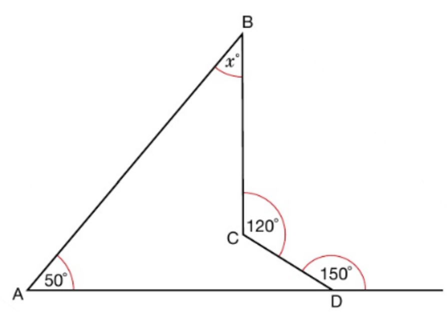


Note: the square, rhombus, and rectangle have parallel sides. This is not shown on the diagrams since it would make them too confusing.

EXAMPLE 4

SKILL: REASONING

ABCD is a quadrilateral with angles as shown.

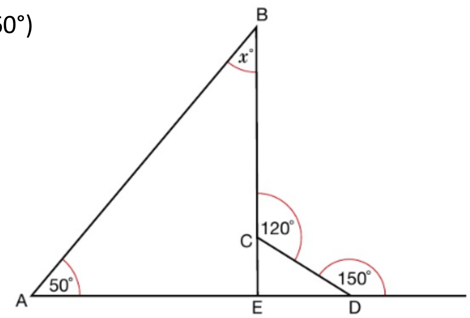


a) Find angle x.

b) Show that BC is **perpendicular** to AD.

a) <ADC = 30° (angles on a straight line add up to 180°)

<DCB = 240° (angles at a point sum to 360°)
x = 360-50-30-240 = 40° (internal angles of a quadrilateral sum to 360°)



b) Extend BC to meet AD at point E.
Then AEB = 90° (angle sum of a triangle is 180°)
⇒ BC is perpendicular to AD.

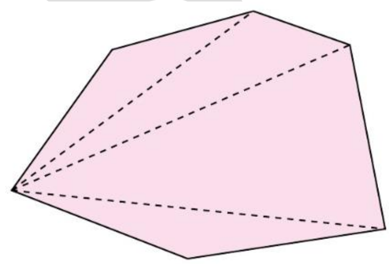
KEY POINTS

- The sum of the interior angles of a quadrilateral is 360°.
- The sum of the exterior angles of a quadrilateral is 360°.

POLYGONS

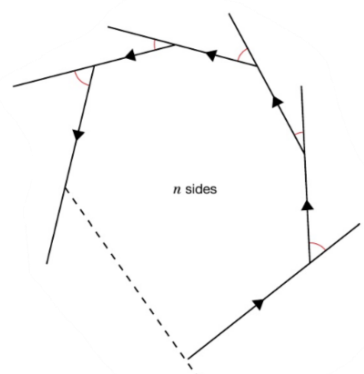
INTERIOR ANGLES

Polygons can always be divided into triangles.
The diagram shows a hexagon divided into four triangles.



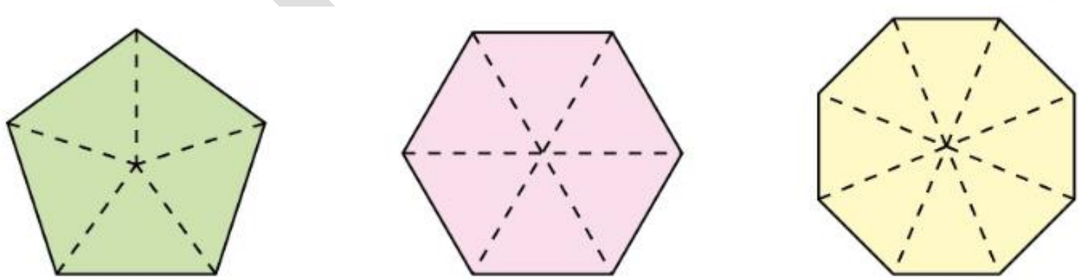
EXTERIOR ANGLES

Imagine walking around a field that is the shape of an n-sided polygon.
You will still turn through 360°.
The sum of the exterior angles of a polygon is 360° no matter how many sides it has.
If the polygon is regular, then the exterior angles are all the same and they equal 360°/n



DIVIDING REGULAR POLYGONS

All regular polygons can be divided into equal isosceles triangles.
The hexagon divides into equilateral triangles.
Dividing a regular polygon in this way can help solve many mathematical problems.



EXAMPLE 6
SKILL: REASONING
A regular polygon has twelve sides. Find the size of each exterior and interior angle.

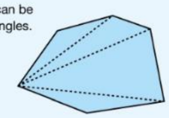
The exterior angle is $\frac{360}{n} = \frac{360}{12} = 30°$

The interior angle is $180 - \frac{360}{n} = 180 - 30 = 150°$

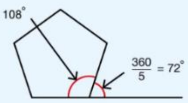
Note: the interior and exterior angles sum to 180°.

KEY POINTS

- An n-sided polygon can be divided into $n - 2$ triangles.



- The sum of the interior angles of a polygon is $(n - 2) \times 180°$.
- The sum of the exterior angles of a polygon is 360° no matter how many sides it has.
- All regular polygons can be divided into equal isosceles triangles.
- For a regular polygon



External angle $= \frac{360°}{n}$

Interior angle $= 180 - \frac{360}{n}$ degrees.

EXAMPLE 7
SKILL: REASONING
The interior angle of a regular polygon is 162°.
How many sides does it have?

The exterior angle is $180 - 162 = 18°$

The exterior angle is $18 = \frac{360}{n} \Rightarrow n = \frac{360}{18} = 20$

The polygon has 20 sides.

CONSTRUCTIONS

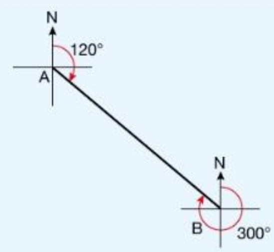
Constructions of various shapes can be done accurately with a ruler and compasses. Architects used to use this technique when producing accurate scale drawings of their building plans.

KEY POINTS

Bearings are measured

- clockwise
- from north.

A is on a bearing of 300° from B.
B is on a bearing of 120° from A.



BEARINGS AND SCALE DRAWINGS

Scale drawings show a real object with accurate sizes reduced or enlarged (scaled) by a scale factor.

EXAMPLE 8
SKILL: PROBLEM-SOLVING
A map has a scale of 1:50000.
What is the real-life distance in kilometers for 6cm on the map?

Map Real life

1 : 50 000

×6 ×6

6 : 300 000

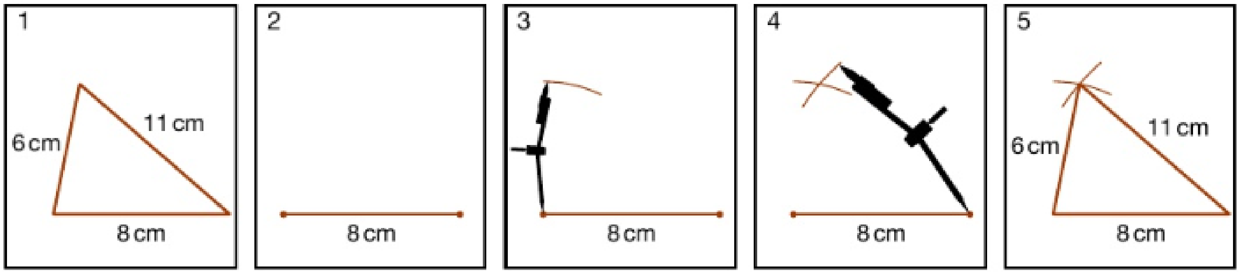
6 cm represents 6 × 50 000 = 300 000 cm

300 000 cm ÷ 100 = 3000 m — Convert cm to m.

3000 m ÷ 1000 = 3 km — Convert to km.

CONSTRUCTING TRIANGLES

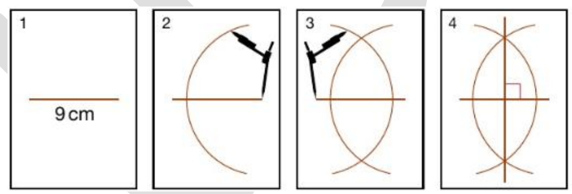
SKILL: REASONING
EXAMPLE 9
Construct a triangle with sides 11 cm, 8cm, and 6 cm.



1►Sketch the triangle first.
2►Draw the 8 cm line.
3►Open your compasses to 6 cm. Place the point at one end of the 8 cm line. Draw an arc.
4►Open your compasses to 11 cm. Draw another arc from the other end of the 8 cm line. Make sure your arcs are long enough to intersect.
5►Join the intersection of the arcs to each end of the 8 cm line. Do not erase your construction marks.

PERPENDICULAR BISECTOR

A perpendicular bisector is a line that cuts another line in half at right angles.
EXAMPLE 10
SKILL: REASONING
Draw a line 9 cm long. Construct its perpendicular bisector.

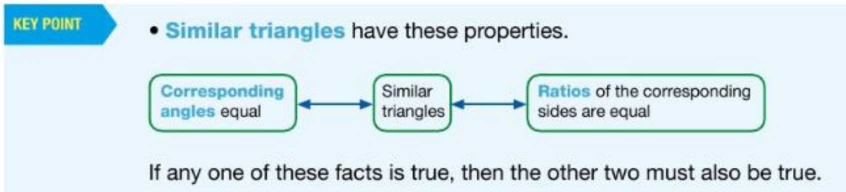


ANGLE BISECTOR

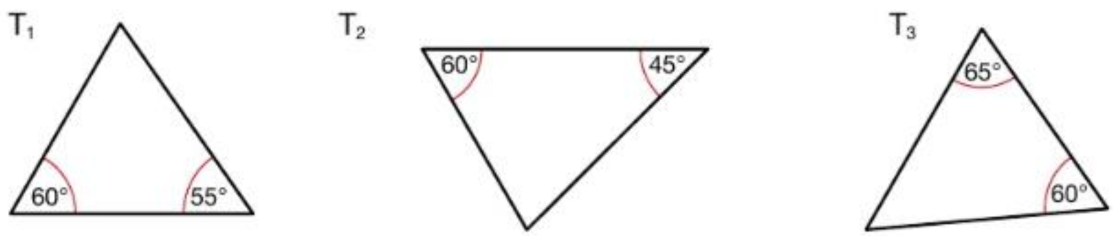
An angle bisector cuts an angle in half.

1 ►Draw an angle of 80° using a protractor.
2 ►Open your compasses and place the point at the vertex of the angle. Draw an arc that crosses both arms (lines) of the angle.
3►Keep the compasses open to the same distance.
Move them to one of the points where the arc crosses an arm.
Make an arc in the middle of the angle.
4 ► Do the same for where the arc crosses the other arm.
5►Open your compasses and place the point at the vertex of the angle. Draw an arc that crosses both arms (lines) of the angle.
Join the vertex of the angle to the point where the two small arcs intersect.
Do not erase your construction marks. This line is the angle bisector.

SIMILAR TRIANGLE

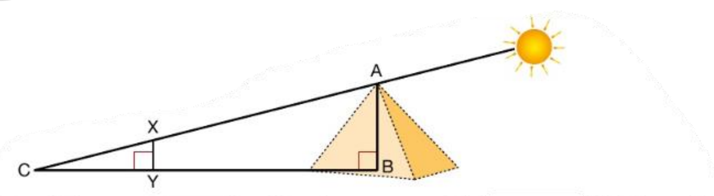


EXAMPLE 12
SKILL: REASONING
Which of these triangles are like each other?



The angle sum of a triangle is 180°.
Therefore, in $T_1$the angles are 55°, 60° and 65°, in T^2 the angles are 45°, 60° and 75°, and in T_3 the angles are 55°, 60° and 65°.
Therefore, the triangles T,$_1$and T3 are similar.

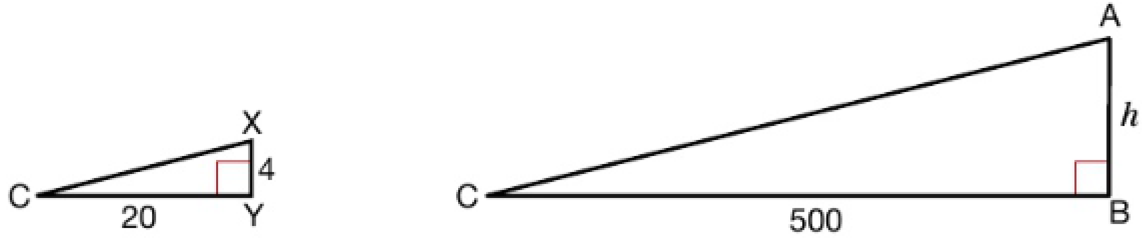
EXAMPLE 13
SKILL: PROBLEM-SOLVING
The ancient Egyptians used similar triangles to work out the heights of their pyramids. The unit they used was the cubit, a measure based on the length from a person's elbow to their fingertips.



The shadow of a pyramid reached C, which was 500 cubits from B.

An Egyptian engineer found that a pole of length 4 cubits had to be placed at Y, 20 cubits from C, for its shadow to reach C as well. What was the height of Pyramid AB?

As AB and XY are both vertical, the triangles CAB and CXY are similar in shape.

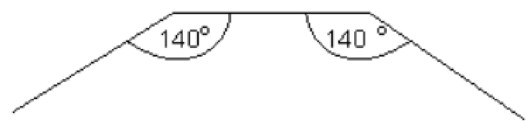


So, the ratios of their corresponding sides are equal.

$$\frac{AB}{XY} = \frac{CB}{CY} \qquad \frac{AB}{4} = \frac{500}{20} = 25 \qquad AB = 4 \times 25 = 100$$

So, the height of the pyramid is 100 cubits.

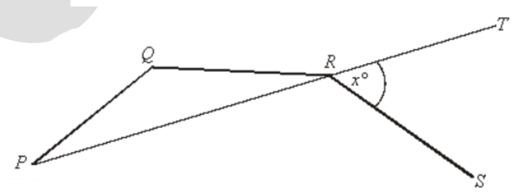
Revision questions



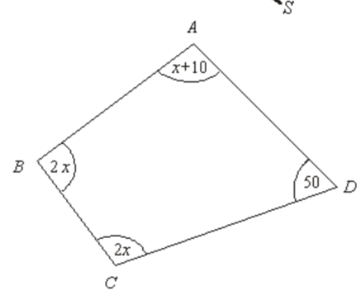
1)The diagram shows 3 sides of a regular polygon.
Each interior angle of the regular polygon is 140°.
Work out the number of sides of the regular polygon.

2)The interior angle of a regular polygon is 160°.

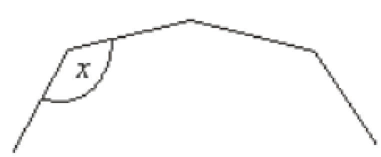
(i)Write down the size of an exterior angle of the polygon.
(ii)Work out the number of sides of the polygon.



3)PQ, QR, and RS are 3 sides of a regular decagon. PRT is a straight line. Angle TRS = x°
Work out the value of x

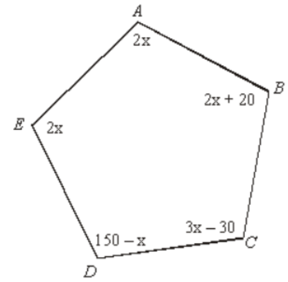


4) In this quadrilateral, the sizes of the angles, in degrees, are
x+ 10
2x
2x
50
(a)Use this information to write down an equation in terms of x.
(b)work out the value of x

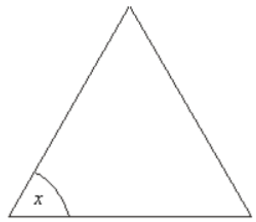


5)The diagram shows part of a regular 10-sided polygon.
Work out the size of the angle marked X.

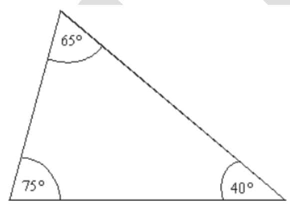
6)In the diagram all the angles are in degrees. Find the size of the angle CDE.



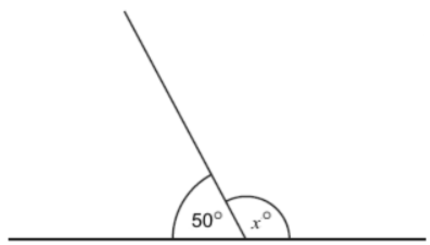
7)(a)Here is an equilateral triangle.
Write down the size of the angle marked.



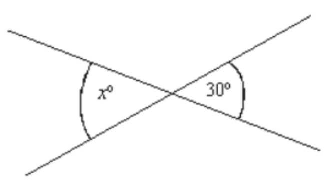
b)
Rob says this triangle is right-angled.
Rob is wrong. Explain why.



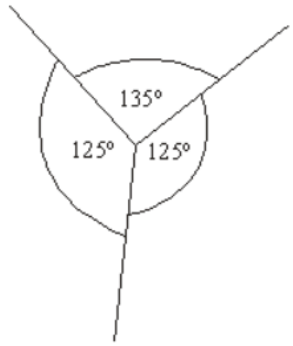
8)
(i)Work out the value of x.
(ii) Give a reason for your answer.



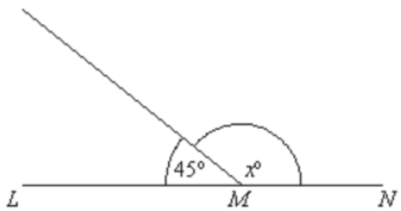
9)a)write down the value of x
b)give a reason for your answer



b)This diagram is wrong. Explain why?

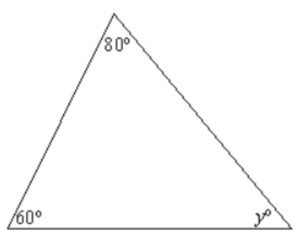


10)



a) work out the value of x
give a reason for your answer

LMN is a straight line.



b) work out the value of y