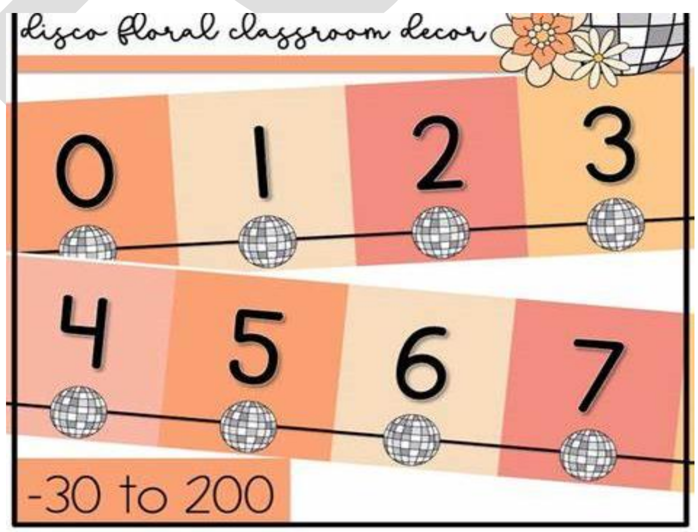
Edexcel

IGCSE

Mathematics

CODE: (4MA1)

Unit 3



Number 03

LEARNING OBJECTIVES

- ■Write a number as a product of its prime factors
- ■Find the HCF and LCM of two (or more) numbers
- ■Solve problems involving HCF and LCM
- ■Compare ratios
- ■Find quantities using ratios
- ■ Solve problems involving ratio

BASIC PRINCIPLES

Multiples

- The multiples of 4 are 4, 8, 12, 16, …. i.e. the numbers that 4 divides into exactly.

Factors

- The factors of 10 are 1, 2, 5 and 10. These are the only numbers that divide into 10 exactly.
- If the only factors of a number are 1 and itself, the number is a prime number.

Prime numbers

- These are only divisible by 1 and themselves.
- They are 2, 3, 5, 7, 11, 13, 17, 19, 23, …
- The number of prime numbers is infinite.
- 1 is not a prime number.

PRIME FACTORS

Any factor of a number that is a prime number is a prime factor.
Any number can be written uniquely as the product of its prime factors.

Example 01
Express 72 as a product of prime factors

SKILLS ANALYSIS

Keep dividing repeatedly by prime numbers or use your knowledge of the multiplication tables.

$$72 = 2 \times 36 = 2 \times 2 \times 18 = 2 \times 2 \times 2 \times 9 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

2	72
2	36
2	18
3	9
3	3
	1

$\Rightarrow 72 = 2^3 \times 3^2$

$$72 = 8 \times 9 = 2^3 \times 3^2$$

KEY POINTS

- Prime factors are factors that are prime numbers.
- Divide a number repeatedly by prime numbers to find the prime factors.
- The product of the prime factors is written in index form.
- There is only one way of expressing a number as a product of prime factors.

HCF AND LCM

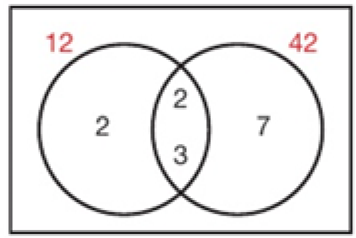
HCF is the Highest **Common Factor.**
It is the highest (largest) factor common to a set of numbers.
LCM is the Lowest Common Multiple.
It is the lowest (smallest) multiple common to a set of numbers.
To find the HCF or LCM of a set of numbers, first express the numbers as products of prime factors.

Example 02

Find the HCF and LCM of 12 and 42.

$12 = 4 \times 3 = 2^2 \times 3$

$42 = 6 \times 7 = 2 \times 3 \times 7$



Draw a Venn diagram.

The common prime factors are in the intersection set, i.e. 2 and 3.

The highest common factor (HCF) is the intersection set numbers all multiplied together:

2 x 3 = 6.

The lowest common multiple (LCM) is the union set numbers all multiplied together:

2 x 2 x 3 x 784.

Notes:

3 is a common factor, but it is not the highest common factor.

12 × 42 = 504 is a common multiple, but it is not the lowest common multiple.

Example 03

Skills; problem solving

A rope of length 672 cm and a rope of length 616cm will be cut into pieces. All the pieces must be the same length.

Find the greatest possible length of each piece.

The length must be a factor of both 672 and 616.

The greatest possible length must be the HCF of 672 and 616.

$672 = 2^5 \times 3 \times 7$, 616 = 23 x 7 x 11, HCF = $2^3 \times 7 = 56$

The greatest possible length is 56 cm.

Example 04

Skills; problem solving

Daisy and Max both walk a whole number of steps from one side of their garden to the other. Daisy's step length is 75 cm while Max's is 80 cm. What is the minimum length of their garden? The length must be a multiple of 75 and 80. The minimum length must be the LCM of 75 and 80.

$75 = 3 \times 5^2$ $80 = 2^4 \times 5$ LCM = $2^{4x}\ 3 \times 5^2 = 1200$

The minimum length of the garden is 1200 cm or 12m.

KEY POINTS

- The HCF is the highest (largest) factor that is common to a set of numbers.
- The LCM is the lowest (smallest) multiple that is common to a set of numbers.
- To find the HCF or LCM, express the numbers in prime factor form in a Venn diagram.

RATIO

Ratios are used to compare quantities (or parts). If the ratio of the quantities is given and one quantity is known, the other quantities can be found. Also, if the total quantity is known, the individual quantities can be found.

Example 05
A marinade in a recipe contains rice vinegar and soy sauce in the ratio 2:3.
How much of each ingredient is needed to make 100 ml of the marinade?

SKILLS
PROBLEM SOLVING

Add the numbers in the ratio together: $2 + 3 = 5$

Then $\frac{2}{5}$ of the marinade is rice vinegar and $\frac{3}{5}$ is soy sauce.

Amount of rice vinegar $= \frac{2}{5} \times 100 = 40$ ml

Amount of soy sauce $= \frac{3}{5} \times 100 = 60$ ml

Check: 40 ml + 60 ml = 100 ml

SKILLS
PROBLEM SOLVING

Add the numbers in the ratio together: $2 + 3 + 4 = 9$

Then the first part $= \frac{2}{9} \times £1170 = £260$

The second part $= \frac{3}{9} \times £1170 = £390$

The third part $= \frac{4}{9} \times £1170 = £520$

Check: £260 + £390 + £520 = £1170

Ratios stay the same if both sides are multiplied or divided by the same number.

To compare ratios write them as unit ratios, that is $1 : n$ or $n : 1$.

EXAMPLE 7
SKILLS
REASONING

Which is larger, 9 : 4 or 23 : 10?

Divide both sides of 9 : 4 by 4 to give 2.25 : 1

Divide both sides of 23 : 10 by 10 to give 2.3 : 1

So 23 : 10 is larger the 9 : 4

KEY POINTS

- Add the numbers in ratios together to find each proportion.
- Ratios stay the same if both sides are multiplied or divided by the same number.
- Compare ratios by writing them as unit ratios.

Algebra 03

LEARNING OBJECTIVES

- ■ Factorise algebraic expressions
- ■ Simplify algebraic fractions
- ■ Solve equations involving fractions
- ■ Solve simultaneous equations
- ■ Solve simultaneous equations for real-life applications

BASIC PRINCIPLES

- Prime factors: $600 = 2^3 \times 3 \times 5^2$
- The lowest common denominator of 6 and 4 is 12.
- The solution of simultaneous equations is given by the intersection of their graphs.

SIMPLE FACTORISING

Expanding $2a^2b(7a-3b)$ gives $14a^3b - 6a^2b^2$. The reverse of this process is called **factorising.** If the common factors are not obvious, first write out the expression to be factorised in full, writing numbers in prime factor form. Identify each term that is common to all parts and use these terms as **common factors** to be placed outside the bracket.

EXAMPLE 1 Factorise $x^2 + 4x$.

$x^2 + 4x = x \times x + 4 \times x$

$= x(x + 4)$

red terms ↑ ↑ black terms

EXAMPLE 2 Factorise $6x^2 + 2x$.

$6x^2 + 2x = 2 \times 3 \times x \times x + 2 \times x$

$= 2x(3x + 1)$

red terms ↑ ↑ black terms

EXAMPLE 3 Factorise $14a^3b - 6a^2b^2$.

$14a^3b - 6a^2b^2 = 2 \times 7 \times a \times a \times a \times b - 2 \times 3 \times a \times a \times b \times b$

$= 2a^2b(7a - 3b)$

red terms ↑ ↑ black terms

KEY POINT

- Always check your factorising by multiplying out.

SIMPLIFYING FRACTIONS

To simplify 234 / 195 it is easiest to factorise first.

EXAMPLE 4 Simplify $\frac{x^2 + 5x}{x}$

$$\frac{x^2 + 5x}{x} = \frac{{}^1\cancel{x}(x+5)}{{}_1\cancel{x}} = x + 5$$

EXAMPLE 5 Simplify $\frac{2a^3 - 4a^2b}{2ab - 4b^2}$

$$\frac{2a^3 - 4a^2b}{2ab - 4b^2} = \frac{{}^1\cancel{2}a^2\cancel{(a-2b)}}{{}_1\cancel{2}b\cancel{(a-2b)}} = \frac{a^2}{b}$$

Equations with fractions

Equations with fractions are easier to manage than algebraic expressions, because both sides of the equation can be multiplied by the lowest common denominator to **clear** the fractions.

EQUATIONS WITH NUMBERS IN THE DENOMINATOR

EXAMPLE 6 Solve $\frac{2x}{3} - 1 = \frac{x}{2}$

$\frac{2x}{3} - 1 = \frac{x}{2}$ (Multiply both sides by 6)

$4x - 6 = 3x$

$x = 6$

Check: 4 − 1 = 3

EXAMPLE 7 Solve $\frac{3}{4}(x - 1) = \frac{1}{3}(2x - 1)$

$\frac{3}{4}(x - 1) = \frac{1}{3}(2x - 1)$ (Multiply both sides by 12)

$9x - 9 = 8x - 4$

$x = 5$

Check: $\frac{3}{4}(5 - 1) = \frac{1}{3}(10 - 1) = 3$

KEY POINT

- Clear the fractions by multiplying both sides by the lowest common denominator.

EQUATIONS WITHIN THE DENOMINATOR

When the denominator contains x, the same principle of clearing fractions still applies.

EXAMPLE 8 Solve $\frac{3}{x} = \frac{1}{2}$

$\frac{3}{x} = \frac{1}{2}$ (Multiply both sides by $2x$)

$\frac{3}{x} \times 2x = \frac{1}{2} \times 2x$

$6 = x$

Check: $\frac{3}{6} = \frac{1}{2}$

EXAMPLE 9 Solve $\frac{4}{x} - x = 0$.

$\frac{4}{x} - x = 0$ (Multiply both sides by x)

$\frac{4}{x} \times x - x \times x = 0 \times x$ (Remember to multiply everything by x)

$4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$

Check: $\frac{4}{2} - 2 = 0$ and $\frac{4}{-2} - (-2) = 0$

SIMULTANEOUS EQUATIONS

If we try to solve simultaneous equations using a graph, this can take a long time and also the solutions can be inaccurate. Using algebra can be better, since this gives exact solutions, though it is impossible to solve some simultaneous equations algebraically.

There are two common ways of solving simultaneous equations using algebra: by substitution and by elimination.

SUBSTITUTION METHOD

EXAMPLE 10

A bottle and a cap together cost £1. The bottle costs 90p more than the cap. Find the cost of the bottle.

Let b be the cost of the bottle in pence, and c be the cost of the cap in pence. The total cost is 100p, and so

b + c = 100 (1)

The bottle costs 90p more than the cap, and so

b = c + 90 (2)

Substituting (2) into (1) gives

(c+90) + c = 100

2c = 10 ⟹ c=5

Substituting in (1) gives b = 95.

Therefore, the bottle costs 95p and the cap costs 5p.

Check: Equation (2) gives 95 = 5 + 90.

ELIMINATION METHOD

EXAMPLE 11 Solve the simultaneous equations $2x - y = 35$, $x + y = 118$.

$2x - y = 35$ (1)

$x + y = 118$ (2)

$3x = 153$ (Adding equations (1) and (2))

$x = 51$

Substituting x = 51 into (1) gives 102 - y = 35⇒ y = 67
The solution is x = 51, y = 67.
Check: Substituting x = 51, y = 67 into (2) gives 51 + 67 = 118.
This method only works if the numbers before either x or y are of opposite **sign** and equal in value.
The equations may have to be multiplied by suitable numbers to achieve this.

EXAMPLE 12

Solve the simultaneous equations $x + y = 5$, $6x - 3y = 3$.

$$x + y = 5 \quad (1)$$
$$6x - 3y = 3 \quad (2)$$

Multiply both sides of equation (1) by 3.

$$3x + 3y = 15 \quad (3)$$
$$6x - 3y = 3 \quad (2)$$
$$9x = 18 \quad \text{(Adding equations (3) and (2))}$$
$$x = 2$$

Substituting $x = 2$ into (1) gives $2 + y = 5 \Rightarrow y = 3$

The solution is $x = 2$, $y = 3$.

Check: Substituting $x = 2$ and $y = 3$ into (2) gives $12 - 9 = 3$.

If the numbers in front of x or y are not of opposite sign, multiply by a negative number as shown in Example 13.

EXAMPLE 13

Solve the simultaneous equations $x + 2y = 8$, $2x + y = 7$.

$$x + 2y = 8 \quad (1)$$
$$2x + y = 7 \quad (2)$$

Multiply *both* sides of equation (2) by −2.

$$x + 2y = 8 \quad (3)$$
$$-4x - 2y = -14 \quad (2)$$
$$-3x = -6 \quad \text{(Adding equations (3) and (2))}$$
$$x = 2$$

Substituting $x = 2$ into (1) gives $2 + 2y = 8 \Rightarrow y = 3$

The solution is $x = 2$, $y = 3$.

Check: Substituting $x = 2$ and $y = 3$ into equation (2) gives $4 + 3 = 7$.

Sometimes both equations have to be multiplied by suitable numbers, as in Example 14.
EXAMPLE 14
Solve the simultaneous equations 2x + 3y = 5, 5x-2y= -16.

$$2x + 3y = 5 \quad (1)$$
$$5x - 2y = -16 \quad (2)$$

Multiply (1) by 2. $\quad 4x + 6y = 10 \quad (3)$

Multiply (2) by 3. $\quad 15x - 6y = -48 \quad (4)$

$$19x = -38 \quad \text{(Adding equations (3) and (4))}$$
$$x = -2$$

Substituting $x = -2$ into (1) gives $-4 + 3y = 5 \Rightarrow y = 3$

The solution is $x = -2$, $y = 3$.

Check: Substituting $x = -2$ and $y = 3$ into equation (2) gives $-10 - 6 = -16$.

KEY POINTS

To solve two simultaneous equations by elimination:

- Label the equations (1) and (2).
- Choose which **variable** to eliminate.
- Multiply one or both equations by suitable numbers so that the numbers in front of the terms to be eliminated are the same and of different sign.
- Eliminate by adding the resulting equations. Solve the resulting equation.
- Substitute your answer into one of the original equations to find the other answer.
- Check by substituting both answers into the other original equation.

SOLVING PROBLEMS USING SIMULTANEOUS EQUATIONS

Tickets at a concert cost either £10 or £15. The total takings from sales of tickets were £8750. Sales of £10 tickets were two times the sales of £15 tickets. How many tickets were sold?

SKILLS: MODELLING

Let x be the number of £10 tickets sold, and y the number of £15 tickets sold.

The total takings were £8750, and so

Let x be the number of £10 tickets sold, and y the number of £15 tickets sold.

The total takings was £8750, and so

$$10x + 15y = 8750$$

Divide by 5 to simplify.

$$2x + 3y = 1750 \qquad (1)$$

Sales of £10 tickets were two times the sales of £15 tickets, and so

$$x = 2y \qquad (2)$$

To check that equation (2) is correct, substitute simple numbers that obviously work, such as $x = 10$, $y = 5$.

Substituting (2) into (1) gives

$$4y + 3y = 1750$$
$$7y = 1750$$
$$y = 250$$

and so $x = 500$, from (2).

750 tickets were sold in total.

Check: In (1), $1000 + 750 = 1750$.

KEY POINTS

- Define your variables.
- Write equations to represent each sentence from the question.
- Solve the equations by using the substitution method or the elimination method. Choose the method which seems the most suitable.

EXAMPLE 16

Ahmed makes a camel journey of 20 km. The camel travels at 12 km/h for the first part of the journey, but then conditions become worse, and the camel can only travel at 4 km/h for the second part of the journey. The journey takes 3 hours. Find the distance of each part of the journey.

Let x be the distance in km of the first part of the journey, and y be the distance in km of the second part.

Let x be the distance in km of the first part of the journey, and y be the distance in km of the second part.

$$x + y = 20 \qquad (1) \qquad \text{(Total distance is 20 km)}$$

Use the formula $\text{time} = \dfrac{\text{distance}}{\text{speed}}$

$$\frac{x}{12} + \frac{y}{4} = 3 \qquad (2) \qquad \text{(Total time taken is 3 hours)}$$

Multiply equation (2) by the lowest common denominator = 12 to clear the fractions.

$$x + 3y = 36 \qquad (3)$$
$$x + y = 20 \qquad (1)$$

Subtract equation (1) from equation (3) to eliminate the terms in x.

$$2y = 16$$
$$y = 8$$

From equation (1), if $y = 8$ then $x = 12$, so the first part is 12 km and the second part is 8 km.

Check: These values work in equations (1) and (2).

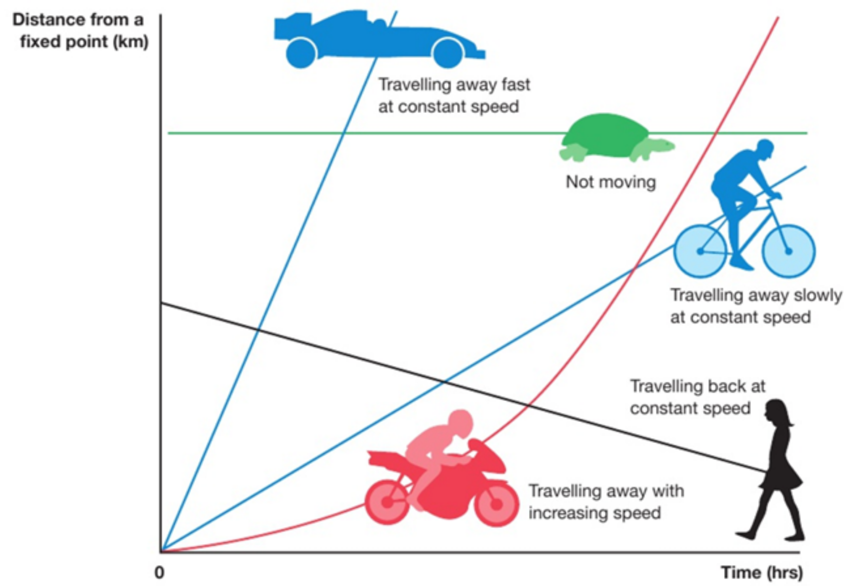
Graphs 03

LEARNING OBJECTIVES

■Draw and interpret distance-time graphs Draw and interpret speed-time graphs

BASIC PRINCIPLES

The motion of an object can be simply described by a graph of distance travelled against the time taken. The examples shown illustrate how the shape of the graph can describe the speed of the object.



DISTANCE-TIME GRAPHS

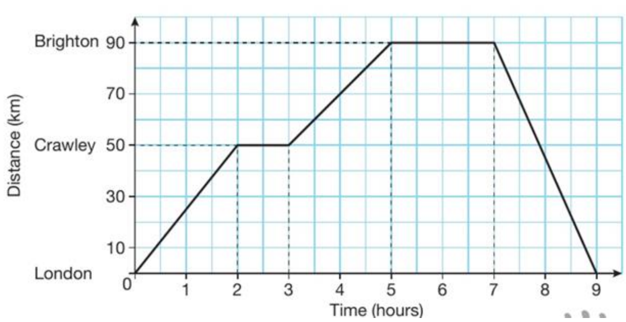
Travel graphs show motion. They make understanding how things move when compared to time much clearer by using diagrams.

EXAMPLE 01

A vintage car goes from London to Brighton for a car show, and then returns to London.

Here is a graph representing the distance in relation to the time of the journey.

SKILLS
INTERPRETATION



a. What is the speed of the car from London to Crawley?

b. The car breaks down at Crawley. For how long does the car break down?

c. What is the speed of the car from Crawley to Brighton?

d. The car is transported by a recovery vehicle back to London from Brighton. At what speed is the car transported?

a The speed from London to Crawley is $\frac{50 \text{ km}}{2 \text{ h}} = 25 \text{ km/h}$.

b The car is at Crawley for 1 hour.

c The speed from Crawley to Brighton is $\frac{40 \text{ km}}{2 \text{ h}} = 20 \text{ km/h}$.

d The speed from Brighton to London is $\frac{90 \text{ km}}{2 \text{ h}} = 45 \text{ km/h}$.

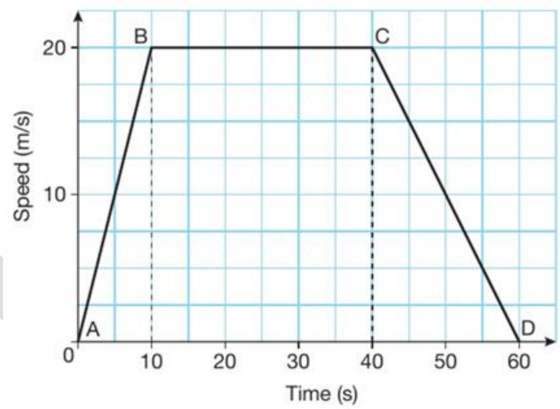
KEY POINTS

On a distance–time graph:

- The vertical axis represents the distance from the starting point.
- The horizontal axis represents the time taken.
- A horizontal line represents no movement.
- The **gradient** of the slope gives the speed (a straight line implies a constant speed).
- A positive gradient represents the outbound journey.
- A negative gradient represents the return journey.

SPEED-TIME GRAPHS

Travel graphs of speed against time can be used to find out more about speed changes and distances travelled.
A train changes speed as shown in the speed-time graph.
The train's speed is increasing between A and B, so it is accelerating.
The train's speed is decreasing between C and D, so it is decelerating (retarding).
The train's speed is constant at 20 m/s (and therefore the acceleration is zero) between B and C for 30s. It has travelled 600m (20 x 30m).
This is the area under the graph between B and C.



a Find the total distance travelled by the train, and therefore find the average speed for the whole journey.

b Find the train's acceleration between A and B, B and C, and C and D.

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a Total distance travelled = area under graph

$$= \left(\tfrac{1}{2} \times 10 \times 20\right) + (30 \times 20) + \left(\tfrac{1}{2} \times 20 \times 20\right) = 900$$

Therefore, average speed $= \frac{900 \text{ m}}{60 \text{ s}} = 15 \text{ m/s}$

b Acceleration between A and B = gradient of line AB

$$= \frac{20 \text{ m/s}}{10 \text{ s}} = 2 \text{ m/s}^2$$

Between B and C the speed is constant, so the acceleration is zero.

Acceleration between C and D = gradient of line CD

$$= \frac{-20 \text{ m/s}}{20 \text{ s}} = -1 \text{ m/s}^2$$

(The – sign indicates retardation or deceleration.)

KEY POINTS

On a speed–time graph:

- The vertical axis represents speed.
- The horizontal axis represents time.
- The gradient of the slope gives the acceleration.
- Acceleration = $\frac{\text{change in speed}}{\text{time}}$
- A positive gradient represents acceleration.
- A negative gradient represents deceleration or **retardation**.
- The area under the graph gives the distance travelled.

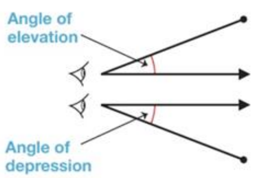
Shape and space 03

LEARNING OBJECTIVES

Use the tangent ratio to find a length and an angle in a right-angled triangle

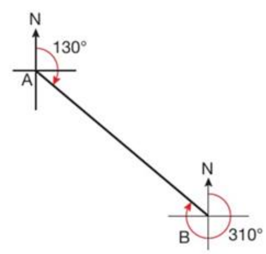
- ■ Use angles of elevation and depression
- ■ Use the tangent ratio to solve problems

BASIC PRINCIPLES



Bearings are measured

- clockwise
- from north.



A is 310° from B. B is 130° from A.

TANGENT RATIO

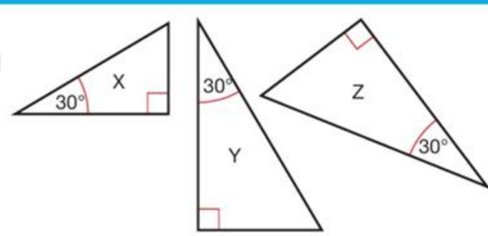
Skills : reasoning

ACTIVITY 1

Triangles X, Y and Z are **similar**. For each triangle, measure the sides opposite (o) and adjacent (a) to the 30° angle in millimetres.

Calculate the **ratio** of $\frac{o}{a}$ to 2 **decimal places** for X, Y and Z.

What do you notice?



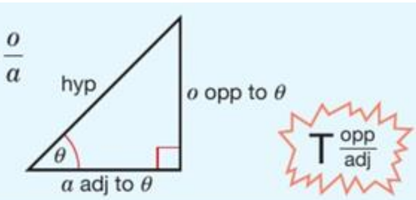
In Activity 1, you should have found that the ratio o: for the 30° angle is the same for all three triangles. This is the case for any similar right-angled triangle with a 30° angle; this should not surprise you because you were calculating the **gradient** of the same slope each time.

The actual value of $\frac{\text{opposite}}{\text{adjacent}}$ for 30° is 0.577 350 (to 6 d.p.).

The ratio $\frac{\text{opposite}}{\text{adjacent}}$ for a given angle θ is a fixed number. It is called the **tangent** of θ, or **tan θ**.

KEY POINT

- $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{o}{a}$



SKILLS; ANYLISIS

ACTIVITY 2

You can find the tangent ratio on your calculator.

Make sure your calculator is in **degree mode**.

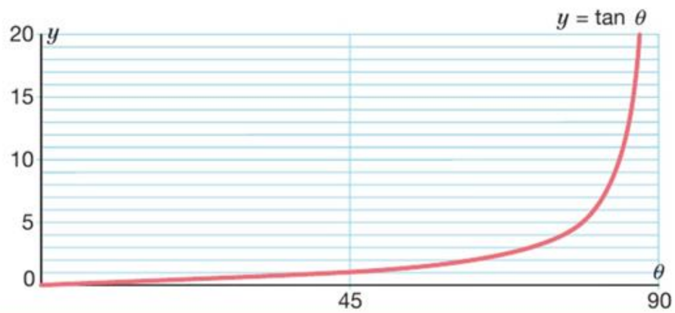
Press the tan button followed by the value of the angle. Then press =.

Copy and complete the table, **correct to 3 significant figures**.

θ (°)	0°	15°	30°	45°	60°	75°	89°	90°
tan θ			0.577			3.73		

Why is tan 89° so large?

Why can tan 90° not be found?



CALCULATING SIDES

EXAMPLE 01

SKILLS: PROBLEM SOLVING

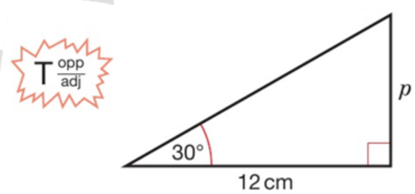
Find the length of the side p correct to 3 significant figures.

$$\tan 30° = \frac{p}{12}$$

$$12 \times \tan 30° = p$$

$$p = 6.93\text{ cm (to 3 s.f.)}$$

1 2 × tan 3 0 = 6.92820 (to 6 s.f.)



EXAMPLE 02

SKILLS: PROBLEM SOLVING

PQ represents a 25m tower, and R is an engineer's mark p m away from Q. The angle of elevation of the top of the tower from the engineer's mark R on level ground is 60°.

Find the distance RQ correct to 3 significant figures.

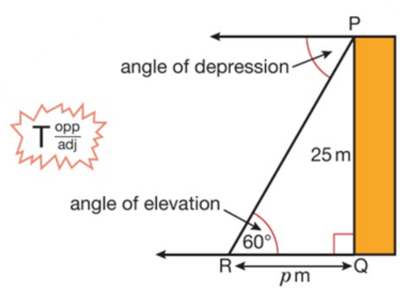
$$\tan 60° = \frac{25}{p}$$

$$p \times \tan 60° = 25$$

$$p = \frac{25}{\tan 60°}$$

$$p = 14.4\text{ m (to 3 s.f.)}$$

2 5 ÷ tan 6 0 = 14.4338 (to 6 s.f.)



CALCULATING ANGLES

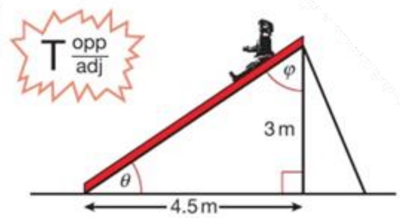
If you know the adjacent and opposite sides of a right-angled triangle, you can find the angles in the triangle. For this 'inverse' operation, you need to use the **INV tan** buttons on your calculator.

EXAMPLE 03

SKILLS; PROBLEM SOLVING

The diagram shows a child on a slide in a playground.

Find angles and to the nearest degree.



$\tan \theta = \frac{3}{4.5}$ and $\tan \phi = \frac{4.5}{3}$

INV tan (3 ÷ 4 . 5) = 33.6901 (to 6 s.f.)

INV tan (4 . 5 ÷ 3) = 56.3099 (to 6 s.f.)

So $\theta = 34°$ and $\phi = 56°$ (to the nearest degree).

Finding one angle in a right-angled triangle allows the third angle to be found as the sum of the angles in a triangle is 180°.

KEY POINT

- To calculate an angle from a tangent ratio, use the INV tan or SHIFT tan buttons.

Handling data 02

LEARNING OBJECTIVES

■ Estimate the mean and range from a grouped frequency table

■ Find the modal class and the group containing the median

BASIC PRINCIPLES

- To collect and find patterns in large amounts of data, it is necessary to group the information together and use frequency tables.

A quick way to do this is by tally tables that allow fast calculation of frequency.

Tally marks are arranged into groups of five to make counting faster, allowing frequencies to be displayed.

Note: 𝍸 represents 5, |||| represents 4, ||| represents 3 etc.

TYPE OF PET	TALLY	FREQUENCY
Dog	𝍸 𝍸 |	11
Cat	𝍸 ||	7
Goldfish	𝍸 |	6
Guinea pig	|||	3
Hamster	||	2
Lizard	|	1
Tortoise	|	1
Rabbit	|||	3

- **Mean** = $\frac{\text{total of all values}}{\text{total number of values}}$
- **Median** = value of the middle number when data is ordered in ascending or descending order
- **Mode** = number that occurs most frequently
- Discrete data can only be **integer** values (number of people, goals, boats…).
- Continuous data can have any value in a particular **range** (time, speed, weight…).
- The symbol sigma $\sum$ is used many times in statistics as a quick way to write 'adding up' of a particular quantity.

FREQUENCY TABLES

Data can be summarised efficiently in a frequency table.

DISCRETE DATA

EXAMPLE 1
SKILLS
ANALYSIS

A teacher records how many times 20 pupils are late to class in a school year.

1	5	3	4	2	0	0	4	5	5
3	4	4	5	1	3	3	4	5	5

a Work out the mean number of late arrivals.
b Work out the median number of late arrivals.
c Write down the mode of the data.
d Draw a **bar chart** for the data.

x represents the recorded number of late arrivals.

x	TALLY	FREQUENCY f	$f \times x$
0	||	2	$2 \times 0 = 0$
1	||	2	$2 \times 1 = 2$
2	|	1	$1 \times 2 = 2$
3	||||	4	$4 \times 3 = 12$
4	~~||||~~	5	$5 \times 4 = 20$
5	~~||||~~ |	6	$6 \times 5 = 30$
		$\sum f = 20$	$\sum fx = 66$

The fourth column ($f \times x$) produces a value for the 0's and the 1's and the 2's etc.

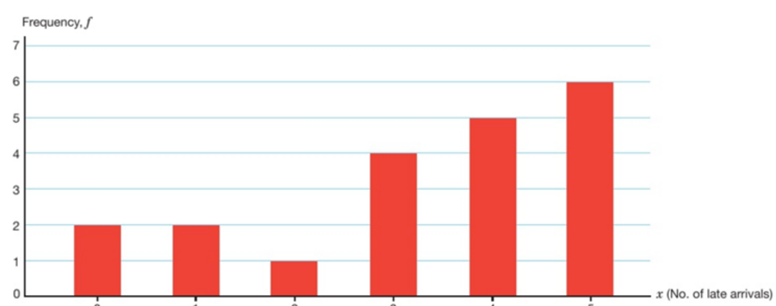
Therefore the **sum** of the fx column, $\sum fx$ = the total of all the values.

a Mean = $\frac{\text{sum of all values}}{\text{number of values}} = \frac{\sum fx}{\sum f} = \frac{66}{20}$ = 3.3 days per pupil.

b Median = 4 days per pupil (10th value = 4 and 11th value = 4).

c Mode = 5 days per pupil (5 is the value with the highest frequency of 6).

d The bar chart for this data is shown.



CONTINUOUS DATA

EXAMPLE O2
SKILLS: PROBLEM SOLVING
The national flower of Malaysia is the Chinese Hibiscus. A botanist takes a sample of 50 of these plants and produces a frequency table of their heights, h m. The exact heights are not recorded, but the values are grouped in **classes** with exact boundaries.
a. Work out an estimate of the mean height.
b. Work out which class contains the median height.
c. Write down which class contains the mode of the data.
d. Draw a **frequency polygon** for the data.

To calculate useful values (mean, median and mode) it is necessary to add to the two columns of height and frequencies as shown below.

Let the value of the Chinese Hibiscus height be *h* m.

h	FREQUENCY f	MID-POINT x	$f \times x$
$0 \le h < 0.5$	3	0.25	$3 \times 0.25 = 0.75$
$0.5 \le h < 1$	5	0.75	$5 \times 0.75 = 3.75$
$1 \le h < 1.5$	7	1.25	$7 \times 1.25 = 8.75$
$1.5 \le h < 2$	15	1.75	15 1.75 = 26.25
$2 \le h < 2.5$	11	2.25	$11 \times 2.25 = 24.75$
$2.5 \le h \le 3$	9	2.75	$9 \times 2.75 = 24.75$
	$\sum f = 50$		$\sum fx = 89$

The **mid-point**, x, is used for each group since it is the best estimate for the mean of all the heights in each group.

The value of $f \times x$ is the best estimate for all the heights in each class.

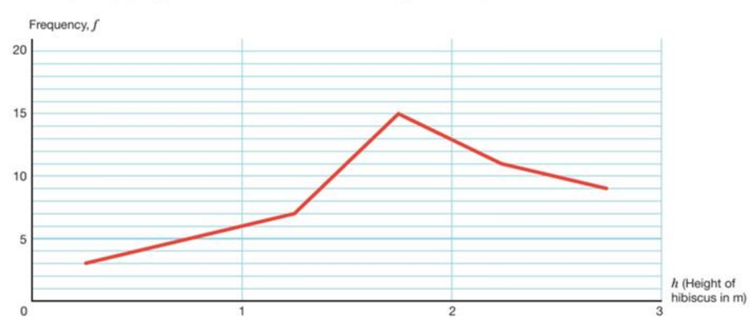
So, the sum of the fx column , $\sum fx$ = the best estimate total of all the heights.

a Estimate of mean $= \frac{\text{sum of all values}}{\text{number of values}} = \frac{\sum fx}{\sum f} = \frac{89}{50} = 1.78\text{m}$.

b Median is in the $1.5 \le h < 2$ class (as this contains the 25th value).

c **Modal class** is $1.5 \le h < 2$ as these values have the highest frequency of 15.

d The frequency polygon for this data is shown using the mid-points of each class.



KEY POINT

- If data is distributed with a frequency distribution table, the mean is given by

  Mean $= \frac{\sum fx}{\sum f}$

  Discrete data: x values are the exact scores.

  Continuous data: x values are the mid-points of each class.

  $\sum$ is a Greek letter 'sigma' which means add up all the values.

Revision questions

(1) Buses to Acton leave a bus station every 24 minutes.
Buses to Barton leave the same bus station every 20 minutes.
A bus to Acton and a bus to Barton both leave the bus station at 9.00 am.
When will a bus to Acton and a bus to Barton next leave the bus station at the same time?

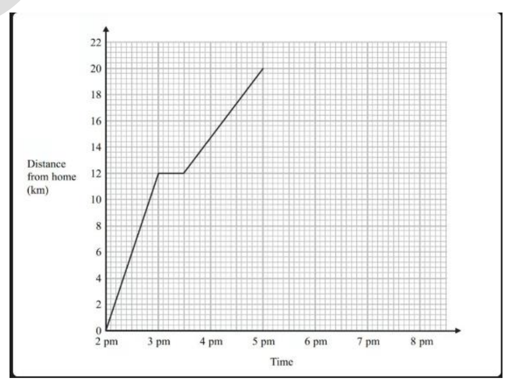
(2) Rita is going to make some cheeseburgers for a party.
She buys some packets of cheese slices and some boxes of burgers.
There are 20 cheese slices in each packet.
There are 12 burgers in each box.
Rita buys the same number of cheese slices and burgers.
i)How many packets of cheese slices and how many boxes of burgers does she buy?
Rita wants to put one cheese slice and one burger into each bread roll.
She wants to use all the cheese slices and all the burgers.
ii) How many bread rolls does Rita need?

(3) Matt and Dan cycle around a cycle track.
Each lap Matt cycles takes him 50 seconds.
Each lap Dan cycles takes him 80 seconds.
Dan and Matt start cycling at the same time at the start line.
Work out how many laps they will each have cycled when they are next at the start line together.

(4) In a box of pens, there are
three times as many red pens as green pens
and two times as many green pens as blue pens.
For the pens in the box, write down
the ratio of the number of red pens to the number of green pens to the number of blue pens.

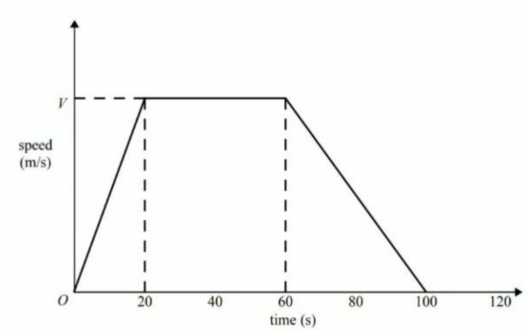
(5) i)factorise $x^2 + 3x - 10$
ii) $y^2 - 10y - 16$
iii) $x^2 - 12x + 27$

(6) Simon went for a cycle ride.
He left home at 2 pm.
The travel graph represents part of Simon's cycle ride.



At 3 pm Simon stopped for a rest. How many minutes did he rest?

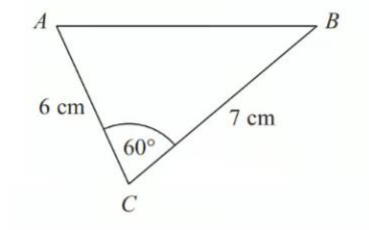
(7) Here is a speed-time graph for a car journey. The journey took 100 seconds.



The car travelled 1.75km in the 100 seconds.
Work out the value of V.

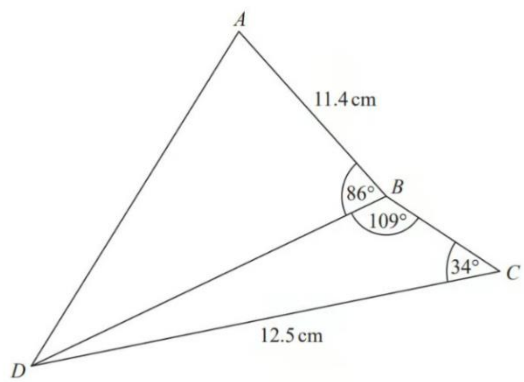
Describe the acceleration of the car for each part of this journey.

(8) ABC is a triangle.

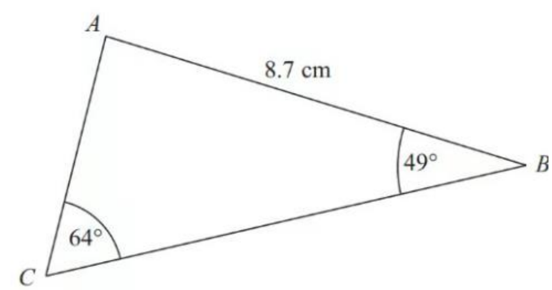


Work out the area of triangle ABC.
Give your answer correct to 3 significant figures.

(9)
Work out the length of AD.
Give your answer correct to 3 significant figures.



(10)
ABC is a triangle.
AB = 8.7 cm. Angle ABC = 49°.
Angle ACB = 64°.
Calculate the area of triangle ABC.
Give your answer correct to 3 significant figures.



(11)
Calculate the length of LN.
Give your answer correct to 3 significant figures.