Edexcel OL Mathematics

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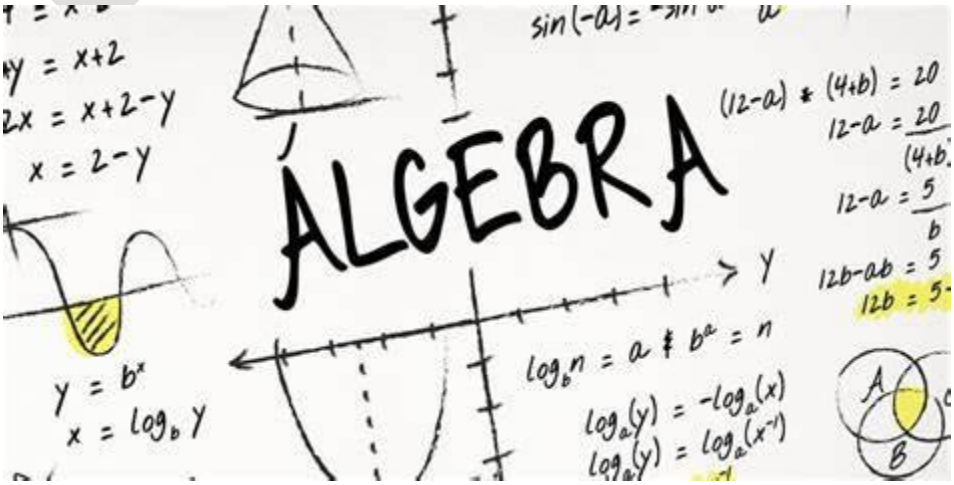
Unit 2

Algebra 2



LEARNING OBJECTIVES

- Multiply and divide algebraic fractions Add and subtract algebraic fractions
- Solve equations with roots and powers.
- Use the rules of indices (to simplify algebraic expressions)
- Solve inequalities and show the solution on a number line

BASIC PRINCIPLES

- Simplifying number fractions: $\frac{9}{12}=\frac{3}{4}$, $\frac{2}{3}\div\frac{1}{3}=\frac{2}{3}\times\frac{3}{1}=2$, $\frac{2}{3}+\frac{1}{4}=\frac{8+3}{12}=\frac{11}{12}$
- Solving equations means doing the same to both sides to get the unknown on one side by itself.
- $10^4=10\times10\times10\times10$
- $x<y$ means 'x is less than y' or 'y is greater than x'.
- $x\geq y$ means 'x is greater than or equal to y' or 'y is less than or equal to x'.

SIMPLIFYING ALGEBRAIC FRACTIONS

Algebraic fractions are simplified in the same way as number fractions.

MULTIPLICATION AND DIVISION

EXAMPLE 1 Simplify $\frac{4x}{6x}$

$$\frac{{}^{2}\cancel{4}\cancel{x}}{{}_{3}\cancel{6}\cancel{x}}=\frac{2\cancel{x}^{1}}{3\cancel{x}_{1}}=\frac{2}{3}$$

EXAMPLE 2 Simplify $\frac{3x^2}{6x}$

$$\frac{3x^2}{6x}=\frac{{}^{1}\cancel{3}\times x\times{}^{1}\cancel{x}}{{}_{2}\cancel{6}\times{}_{1}\cancel{x}}=\frac{x}{2}$$

EXAMPLE 3 Simplify $\left(27xy^2\right)\div(60x)$

$$\left(27xy^2\right)\div(60x)=\frac{27xy^2}{60x}=\frac{{}^{9}\cancel{27}\times{}^{1}\cancel{x}\times y\times y}{{}_{20}\cancel{60}\times{}_{1}\cancel{x}}=\frac{9y^2}{20}$$

EXAMPLE 4 Simplify $\frac{3x^2}{y}\times\frac{y^3}{x}$

$$\frac{3x^2}{y}\times\frac{y^3}{x}=\frac{3\times x\times{}^{1}\cancel{x}}{{}_{1}\cancel{y}}\times\frac{{}^{1}\cancel{y}\times y\times y}{{}_{1}\cancel{x}}=3xy^2$$

EXAMPLE 5 Simplify $\frac{2x^2}{y}\div\frac{2x}{5y^3}$

$$\frac{2x^2}{y}\div\frac{2x}{5y^3}=\frac{{}^{1}\cancel{2}\times x\times{}^{1}\cancel{x}}{{}_{1}\cancel{y}}\times\frac{5\times{}^{1}\cancel{y}\times y\times y}{{}_{1}\cancel{2}\times{}_{1}\cancel{x}}=5xy^2$$

KEY POINT

- To divide by a fraction, turn the fraction upside down and multiply.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

ADDITION AND SUBTRACTION

EXAMPLE 6 Simplify $\frac{a}{4} + \frac{b}{5}$

$$\frac{a}{4} + \frac{b}{5} = \frac{5a + 4b}{20}$$

EXAMPLE 7 Simplify $\frac{3x}{5} - \frac{x}{3}$

$$\frac{3x}{5} - \frac{x}{3} = \frac{9x - 5x}{15} = \frac{4x}{15}$$

EXAMPLE 8 Simplify $\frac{2}{3b} + \frac{1}{2b}$

$$\frac{2}{3b} + \frac{1}{2b} = \frac{4 + 3}{6b} = \frac{7}{6b}$$

EXAMPLE 9 Simplify $\frac{3 + x}{7} - \frac{x - 2}{3}$

$$\frac{3 + x}{7} - \frac{x - 2}{3} = \frac{3(3 + x) - 7(x - 2)}{21} = \frac{9 + 3x - 7x + 14}{21} = \frac{23 - 4x}{21}$$

Remember to use brackets here. Note sign change.

SOLVING EQUATIONS WITH ROOTS AND POWER

EXAMPLE 10
Solve $3x^2+452$.
$3x^2 + 4 = 52$
(Subtract 4 from both sides)
$3x^2 = 48$
(Divide both sides by 3)
$x^2 = 16$
(Square root both sides)
$x = +4$
Check: $3 \times 16 + 4 = 52$
Note: -4 is also an answer because $(-4) \times (-4) = 16$.

EXAMPLE 11
Solve $5\sqrt{x} = 50$.
$5\sqrt{x}=50$
(Divide both sides by 5)
$\sqrt{x}=10$
(Square both sides)
$x = 100$
Check: $5\times \sqrt{100} = 50$

KEY POINT

- To solve equations, do the same operations to both sides.

POSITIVE INTEGER INDICES

10 x 10 x 10 x 10 is written in a shorter form as 104. In the same way, a xaxaxa is written as a^4. To help you to understand how the rules of indices work, look carefully at these examples.

KEY POINTS

OPERATION	EXAMPLE	RULES
Multiplying	$a^4 \times a^2 = (a \times a \times a \times a) \times (a \times a) = a^6 = a^{4+2}$	Add the indices $(a^m \times a^n = a^{m+n})$
Dividing	$a^4 \div a^2 = \frac{a \times a \times a \times a}{a \times a} = a^2 = a^{4-2}$	Subtract the indices $(a^m \div a^n = a^{m-n})$
Raising to a power	$(a^4)^2 = (a \times a \times a \times a) \times (a \times a \times a \times a) = a^8 = a^{4\times2}$	Multiply the indices $(a^m)^n = a^{mn}$

EXAMPLE 13

Use the rules of indices to simplify 63 x 64. Then use your calculator to check the answer.

$6^3 \times 6^4 = 6^7 = 279936$ (Add the indices)

EXAMPLE 14

Simplify $9^5 / 9^2$.

$9^5 / 9^2 = 9^3 = 729$ (subtract the indices)

EXAMPLE 15

Simplify $(4^2)^5 = 4^{10}$

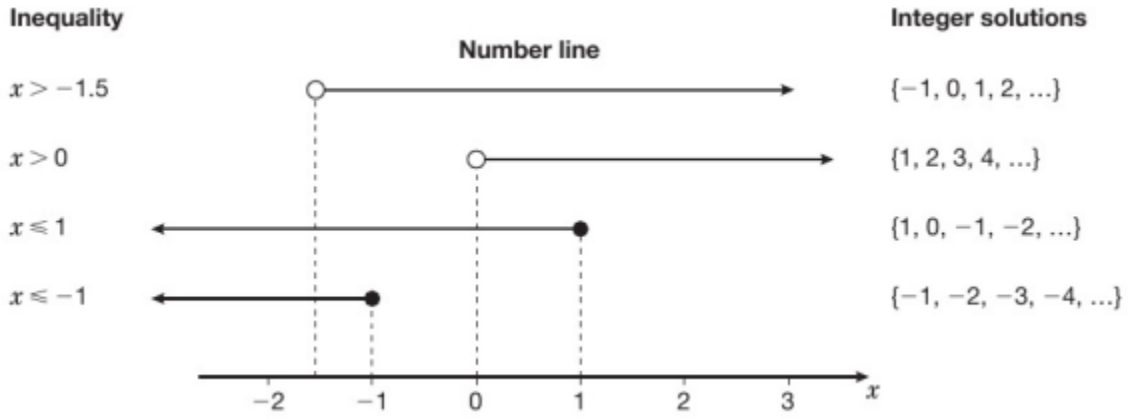
$(4^2)^5 = 4^{10} = 1048576$ (multiply the indices)

INEQUALITIES

NUMBER LINES

These are examples of how to show **inequalities** on a number line.

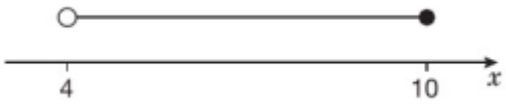
EXAMPLE 16



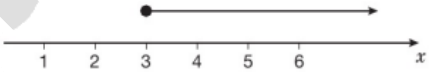
SOLVING LINEAR INEQUALITIES

Inequalities are solved in the same way as algebraic equations, EXCEPT that when multiplying or dividing by a negative number, the inequality sign is reversed.

EXAMPLE 17
Solve the inequality $4 < x < 10$. Show the result on a number line.
$4< x \leq 10$ (Split the inequality into two parts)
$4< x$ and $x \leq 10$
$x > 4$ and $x \leq 10$
Note: x cannot be equal to 4.



EXAMPLE 18
Solve the inequality $4\geq 13-3x$. Show the result on a number line.
$4\geq 13-3x$ (Add 3x to both sides)
$3x + 4 \geq 13$ (Subtract 4 from both sides)
$3x \geq 9$ (Divide both sides by 3)
$x\geq 3$



EXAMPLE 19
Solve the inequality $5-3x < 1$. List the four smallest **integers** in the solution set.
$5-3x <1$ (Subtract 5 from both sides)
$-3x<-4$(Divide both sides by -3, so reverse the inequality sign)

$$x > \frac{-4}{-3}$$

$$x > 1\frac{1}{3}$$

So, the four smallest integers are 2, 3, 4 and 5.

EXAMPLE 20
Solve the inequality $x \leq 5x+1 < 4x+5$. Show the inequality on a number line.
$x\leq 5x+1<4x+5$ (Split the inequality into two parts)
a) $x \leq 5x+1$ (Subtract 5x from both sides)
$-4x \leq 1$ (Divide both sides by -4, so reverse the inequality sign)

$$x \geq -\frac{1}{4}$$

b) $5x+1<4x+5$ (Subtract 4x from both sides)
$x+1<5$ (Subtract 1 from both sides)
$x < 4$

KEY POINTS

- $x > 4$ means that x cannot be equal to 4.
- $x \geq 4$ means that x can be equal to 4 or greater than 4.
- When finding the solution set of an inequality:
  Collect up the algebraic term on one side.
  When multiplying or dividing both sides by a negative number, reverse the inequality sign.

Revision questions

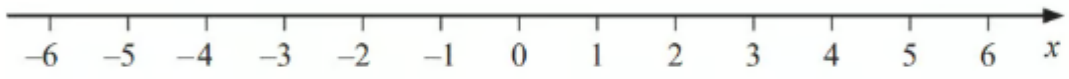
1)a) simplify $5u^2w^4$ x $7uw^3$

b) simplify following,

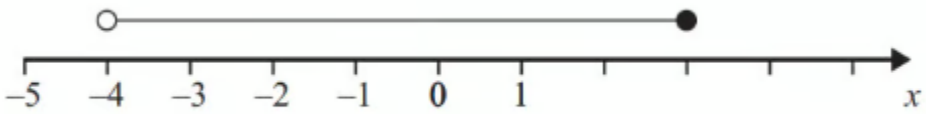
$(9x^8y^3)^{\frac{1}{2}}$ $\quad \frac{45e^6f^8}{5ef^2}$ $\quad \frac{36af^8}{12a^5f^2}$ $\quad (3x^2y^4)^3$

2)a) Solve 14n 11n + 6>

b) On the number line below, show the set of values of x for which -2 < x + 3 < 4



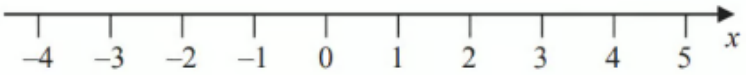
3) a) write down the inequality shown in the diagram



b) n is an integer
write down the possible value of n

$-3 < n \leqslant 1$

4) a) solve the inequality
3p- 7 > 11
b) solve 3x – 5 < 16
c) show the inequality x < 3 on the number line below



5) a) solve the inequality

$x^2 > 3(x + 6)$

b) find the interval for which

$x^2 - 7x + 10 \leqslant 0.$

c) solve the inequality

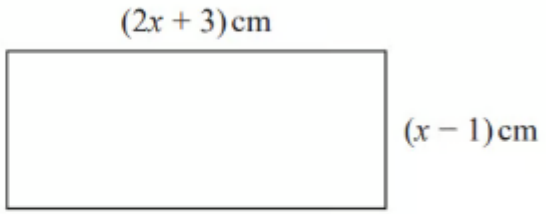
$x^2 - 5x - 6 \leqslant 0$

6) a) solve the inequality

$x^2 > 3(x + 6)$

b)

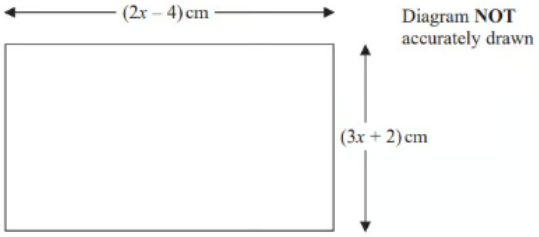
Here is a rectangle.



Given that the area of the rectangle is less than $75\ \text{cm}^2$

find the range of possible values of x

7) The diagram shows a rectangle.



The area of the rectangle is $A\ \text{cm}^2$

Given that $A < 3x + 27$

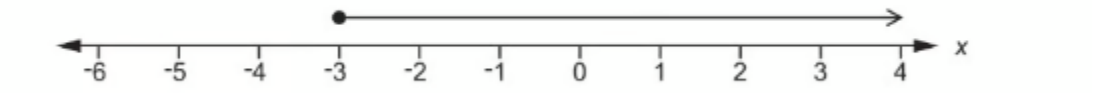
find the range of possible values for x.

8)

n is an integer such that $3n + 2 \leqslant 14$ and $\frac{6n}{n^2 + 5} > 1$

Find all the possible values of n.

9)Martha's solution to the inequality 8x+53x<10 is shown on the number line.



Is her solution correct?
Explain your reasoning.

10) simplify

$$\frac{3}{x} + \frac{4}{x}$$

b) simplify

$$\frac{10x^2 + 23x + 12}{4x^2 - 9}$$