

Edexcel OL
Mathematics
CODE: (4MA1)

Unit 2
Number 02



LEARNING OBJECTIVES

- Write a number in standard form
- Calculate with numbers in standard form
- Work out a percentage increase and decrease
- Solve real-life problems involving percentages

BASIC PRINCIPLES

- $10^2 \times 10^3 = 10^5 \Rightarrow 10^m \times 10^n = 10^{m+n}$
- $10^2 \div 10^3 = \frac{1}{10^1} = 10^{-1} \Rightarrow 10^m \div 10^n = 10^{m-n}$
- $(10^2)^3 = 10^6 \Rightarrow (10^m)^n = 10^{mn}$

STANDARD FORM

Standard form is used to express large and small numbers more efficiently.

KEY POINT

- **Standard form** is always written as $a \times 10^b$, where a is between 1 and 10, but is never equal to 10 and b is an **integer** which can be positive or negative.

STANDARD FORM WITH POSITIVE INDICES

EXAMPLE 1

SKILL: REASONING

Write 8250000 in standard form.

$$8250000 = 8.25 \times 1000000 = 8.25 \times 10^6$$

EXAMPLE 2

SKILL: REASONING

Write 3.75×10^5 as an ordinary number. $3.75 \times 10^5 = 3.75 \times 100000 = 375000$

STANDARD FORM WITH NEGATIVE INDICES

EXAMPLE 3

SKILL: REASONING

Write these **powers** of 10 as decimal numbers: **a** 10^{-2} **b** 10^{-6}

$$\text{a } 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$

$$\text{b } 10^{-6} = \frac{1}{10^6} = \frac{1}{1000000} = 0.000001$$

KEY POINT

$$\bullet 10^{-n} = \frac{1}{10^n}$$

EXAMPLE 4

SKILL: REASONING

Write 0.987 in standard form.

Write the number between 1 and 10 first.

$$0.987 = 9.87 \times \frac{1}{10} = 9.87 \times 10^{-1}$$

EXAMPLE 5

SKILL: REASONING

Write 3.75×10 as an ordinary number.

Write the number between 1 and 10 first.

$$3.75 \times 10^{-3} = 3.75 \times \frac{1}{10^3} = 0.00375$$

PERCENTAGES

Percentages are numbers without a dimension that help us make fast judgements. Values are scaled to be out of 100. Percentages appear frequently in everyday life. They can be used to compare quantities and work out a percentage change such as profit or loss.

EXAMPLE 6

SKILL: REASONING

Calculate \$5 as a percentage of \$80.

Express the ratio as a fraction and multiply by 100.

$$\text{\$5 as a percentage of \$80} = \frac{5}{80} \times 100 = 6.25\%$$

KEY POINT

- To calculate x as a percentage of y : $\frac{x}{y} \times 100$

EXAMPLE 7

SKILL: REASONING

Calculate 5% of 80 kg.

$$1\% \text{ of } 80 \text{ kg} = \frac{80}{100} \text{ so } 5\% = 5 \times \frac{80}{100} = 80 \times \frac{5}{100} = 80 \times 0.05 = 4 \text{ kg}$$

KEY POINT

- To calculate x percent of y : $1\% \text{ of } y = \frac{y}{100}$ so $x\% \text{ of } y = x \times \frac{y}{100} = y \times \left(\frac{x}{100}\right)$

The $\left(\frac{x}{100}\right)$ part of the last expression is the **multiplying factor**.

5% of a quantity can be found by using a multiplying factor of 0.05.

95% of a quantity can be found by using a multiplying factor of 0.95 and so on.

PERCENTAGE CHANGE

EXAMPLE 8

SKILL: REASONING

Olive measures Salma's height as 95 cm. Sometime later she measures her height as 1.14m.

Work out the percentage increase in Salma's height.

$$\text{Percentage change} = \frac{\text{value of change}}{\text{original value}} \times 100 = \frac{114 - 95}{95} \times 100 = +20\%$$

Salma's height has changed by +20%.

To compare units, it is necessary to be consistent. In the above example, centimetres were the units used.

EXAMPLE 9

SKILL: REASONING

Kerry improves her 400 m running time from 72s to 63s.

What was Kerry's percentage improvement?

KEY POINT

$$\bullet \text{ Percentage change} = \frac{\text{value of change}}{\text{original value}} \times 100$$

PERCENTAGE INCREASE AND DECREASE

To increase a value by $R\%$ it is necessary to have the original value plus $R\%$.

Therefore, we multiply it by a **factor** of $(1 + \frac{R}{100})$.

EXAMPLE 10

SKILL: REASONING

In 2015, the Kingda Ka Roller Coaster at Six Flags (USA) had the largest vertical drop of 139 m. If the designers want to increase this height by 12%, what will the new height be?

$$\text{New height} = \text{original height} \times (1 + \frac{12}{100}) = 139 \times 1.12 = 155.68 \text{ m}$$

To decrease a value by $R\%$ it is necessary to have the original value minus $R\%$.

Therefore, we multiply it by a factor of $(1 - \frac{R}{100})$.

EXAMPLE 11

SKILL: REASONING

In 2015, the world record for the 100m swimming butterfly in the female Paralympian S12 class was held by Joanna Mendak (Poland) with a time of 65.1 secs.

If this world record is reduced by 5%, what will the new time be?

$$\text{New time} = \text{original time} \times (1 - \frac{5}{100}) = 65.1 \times 0.95 = 61.845 \text{ s} = 61.85 \text{ s (2 d.p.)}$$

Note: this is the same calculation as finding 95% of the original time, so reducing a quantity by 25% is the same as finding 75% of the value and so on.

KEY POINTS

- To increase a quantity by $R\%$, multiply it by $1 + \frac{R}{100}$
- To decrease a quantity by $R\%$, multiply it by $1 - \frac{R}{100}$

PERCENTAGE CHANGE	MULTIPLYING FACTOR
+25%	1.25
+75%	1.75
-25%	0.75
-75%	0.25

PERCENTAGE INCREASE AND DECREASE

If a quantity gains value over time it has **appreciated** or gone through an inflation. It can happen for several reasons, often a **greater demand** or a **smaller supply** can push prices up.

If a quantity loses value over time, it has **depreciated** or gone through a deflation. It can happen for several reasons, often a **smaller demand** or a **greater supply** can push prices down.

Revision questions

1) One sheet of paper is 9×10^{-3} cm thick.

Mark wants to put 500 sheets of paper into the paper tray of his printer. The paper tray is 4 cm deep.

Is the paper tray deep enough for 500 sheets of paper?

You must explain your answer.

2) Bill's weight decreases from 64.8 kg to 59.3 kg.

Calculate the percentage decrease in Bill's weight. Give your answer correct to 3 significant figures.

b) Ria is going to buy a caravan.

The total cost of the caravan is £7000 plus VAT at 20%.

Ria pays a deposit of £3000.

She pays the rest of the total cost in 6 equal monthly payments.

Work out the amount of each monthly payment.

3) a) The table shows some information about eight planets.

Find the difference between the mass of Venus and the mass of Mercury.

b) Nishat says that Neptune is over a hundred times further away from Earth than Venus is.

Is Nishat, right?

You must show how you get your answer.

Planet	Distance from Earth (km)	Mass (kg)
Earth	0	5.97×10^{24}
Jupiter	6.29×10^8	1.898×10^{27}
Mars	7.83×10^7	6.42×10^{23}
Mercury	9.17×10^7	3.302×10^{23}
Neptune	4.35×10^9	1.024×10^{26}
Saturn	1.28×10^9	5.68×10^{26}
Uranus	2.72×10^9	8.683×10^{25}
Venus	4.14×10^7	4.869×10^{24}

4)a) Calculate $9 \times 10^4 \times 3 \times 10^3$.

Give your answer in standard form.

b) Work out the value of $(9 \times 10^{-4}) \times (3 \times 10^7)$. Give your answer in standard form.

c) Work out the value of 25^{-3} .

d) Work out the value of 3503. Give your answer in standard form.

5) Tom is a ski jumper.

The maximum length of skis he can use is 145% of his height. Tom's height is 1.80 m.

Work out the maximum length of skis Tom can use.

6) George wants to watch all 23 games that a football team will play at home next season.

He can buy.

a season ticket costing £425.

or 23 separate tickets costing £24 each ticket.

What percentage of the total cost of 23 separate tickets does George save by buying a season ticket?

7) Emily buys a pack of 12 bottles of water. The pack costs £5.64

Emily sells all 12 bottles for 50p each.

Work out Emily's percentage profit.

Give your answer correct to 1 decimal place.

8) Gladys buys a table for \$465 to sell in her shop.

She sells the table for \$520.

Work out the percentage profit that Gladys makes from the sale of the table.

Give your answer correct to 3 significant figures.

b) In 2018, the population of Sydney was 5.48 million. This was 22% of the total population of Australia. Work out the total population of Australia in 2018 Give your answer correct to 3 significant figures.

9)Railtickets and Cheaptrains are two websites selling train tickets.
Each of the websites adds a credit card charge and a booking fee to the ticket price.

Railtickets	Cheaptrains
Credit card charge: 2.25% of ticket price	Credit card charge: 1.5% of ticket price
Booking fee: 80 pence	Booking fee: £1.90

Nadia wants to buy a train ticket.
The ticket price is £60 on each website.
Nadia will pay by credit card.
Will it be cheaper for Nadia to buy the train ticket from Railtickets or from Cheaptrains?

10)A company sells boxes to factories. Fred buys boxes.
The boxes are sold in packs of 1000. Each pack costs £193.86
Fred orders 3 packs of boxes.
He gets a discount on his total order.
The table shows the discount he will get.

Total Order	Discount
£100 - £300	5%
£301 - £400	10%
£401 and above	15%

Work out the total cost of the order after the discount. You must show your working.