Edexcel

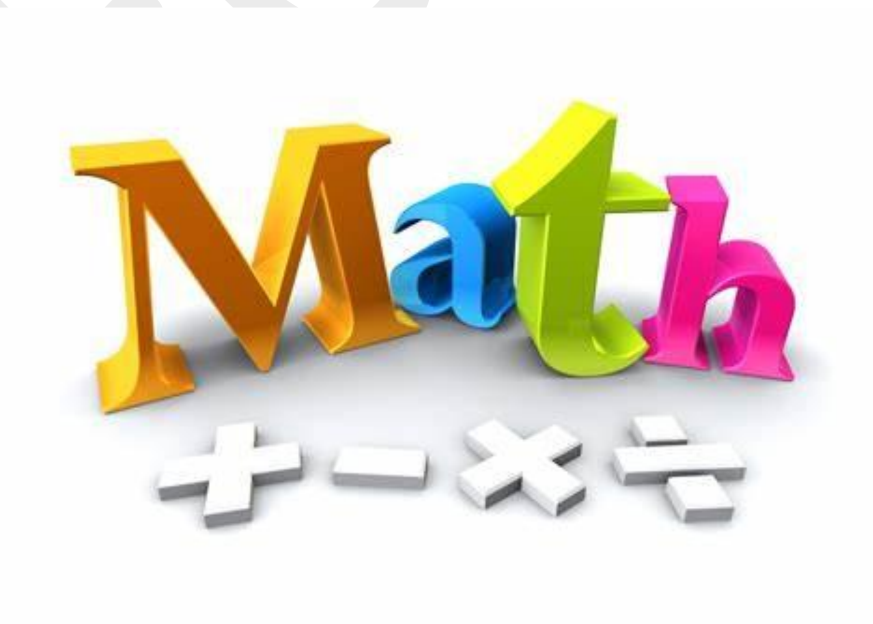
OL IGCSE

Mathematics

CODE: (4CP0)

Unit 05

4MA1



Number 05

BASIC PRINCIPLES

The word **BIDMAS** will help you perform the operations in the right order.

B Brackets **I** Indices **D** Division **M** Multiplication **A** Addition **S** Subtraction

CALCULATORS

√ 2 3 = 4.79583152

The answer can be **rounded** to various degrees of accuracy as shown in the table.

DEGREE OF ACCURACY	SIGNIFICANT FIGURES	DECIMAL PLACES
5	4.7958	4.79583
3	4.80	4.796
1	5	4.8

BIDMAS is helpful with the order of calculations. More complex calculations will require the use of brackets. Be familiar with your calculator and use the instructions that came with it when you bought it.

CALCULATOR FUNCTIONS

- To calculate 6^7, press 6 $x^{\blacksquare}$ 7 = 279936
- To enter 2.5×10^{-3} (written in standard form), press 2 . 5 ×10ˣ (−) 3 =
- To enter $\frac{8}{9}$, press 8 a b/c 9 =
- To enter $3\frac{1}{5}$, press 3 a b/c 1 a b/c 5 =

EXAMPLE 1

SKILLS REASONING

Calculate these **correct to** 3 **significant figures**.

a $\sqrt{12.5^2 - 2.5^2}$ **b** $3\frac{4}{5} - 2.5 \times 10^{-2}$ **c** $\sqrt{\frac{7.75^4}{11.3 + 27.7}}$

a √ (1 2 . 5 x^2 − 2 . 5 x^2) = The answer is 12.2 (to 3 s.f.)

b 3 a b/c 4 a b/c 5 − 2 . 5 ×10ˣ (−) 2 = The answer is 3.78 (to 3 s.f.)

c This is a long calculation, so write it out first: $\sqrt{7.75^4 \div (11.3 + 27.7)} =$

√ (7 . 7 5 $x^{\blacksquare}$ 4 ÷ (1 1 . 3 + 2 7 . 7)) =

The answer is 9.62 (to 3 s.f.)

ESTIMATING

On many occasions it is acceptable and desirable not to calculate an exact answer, but to make a sensible estimate. You have already been approximating numbers to a certain number of decimal places (2.47 = 2.5 to 1 d.p.), significant figures (34.683 = 34.7 to 3 s.f.) and rounding to a fixed suitable value (12752 = 13000 to the nearest 1000). Estimation is usually done without a calculator and is performed in a way that makes the working simple.

EXAMPLE 2

SKILLS

REASONING

Estimate the answers to these calculations.

a 19.7×3.1

b 121.3×98.6

c $252.03 \div 81.3$

d $(11.1 \times (7.8 - 5.1))^2$

a $19.7 \times 3.1 \approx 20 \times 3$
$= 60$ (exact answer is 61.07)

b $121.3 \times 98.6 \approx 120 \times 100$
$= 12\,000$ (exact answer is 11 960.18)

c $252.03 \div 81.3 \approx 240 \div 80$
$= 3$ (exact answer is 3.1)

d $(11.1 \times (7.8 - 5.1))^2 \approx (10 \times (8 - 5))^2$
$= (30)^2$
$= 900$ (exact answer is 898.2009)

ESTIMATING USING STANDARD FORM

It is often useful to use **standard form** to work out an estimate. Make sure you can write a number in standard form.

EXAMPLE 3

SKILLS

REASONING

Use standard form to calculate an estimate for $0.067\,68 \times 38\,750$.

Change to 1 s.f. — Write in standard form — Use rules of indices

$$0.067\,68 \times 38\,750 \approx 0.07 \times 40\,000 = 7 \times 10^{-2} \times 4 \times 10^{4} \approx 30 \times 10^{-2+4} = 30 \times 10^{2} = 3000$$

EXAMPLE 4

SKILLS

REASONING

Use standard form to calculate an estimate for $\sqrt{3.3 \times 10^7}$.

$\sqrt{3.3 \times 10^7} = \sqrt{33 \times 10^6}$ (Change 33 to the nearest square whole number)

$\approx \sqrt{36 \times 10^6}$

$\approx \sqrt{36} \times \sqrt{10^6} = 6 \times 1000 = 6000$

EXAMPLE 5

SKILLS

REASONING

By writing these numbers in standard form correct to 1 significant figure, work out an estimate for $(4.5 \times 10^7) + (4.5 \times 10^6)$.

Write your answer, correct to 1 s.f. in standard form.

$(4.5 \times 10^7) + (4.5 \times 10^6) = (45 \times 10^6) + (4.5 \times 10^6)$ (Make the **index** numbers the same)

$= 49.5 \times 10^6$ (Add **like terms**)

$\approx 5 \times 10^7$

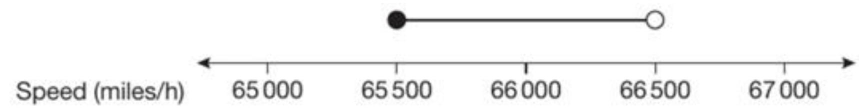
ROUNDING, UPPER AND LOWER BOUNDS

Rounding is often necessary as it can be impossible to give values exactly. Knowing the degree of accuracy of the numbers is useful. Unless requested, it is important not to round too much, otherwise the final answer can be too inaccurate.

We are often given information that is not exact.

For example, a scientific publication may state that 'the Earth is moving around the Sun at 66000 miles/hour'. If this figure has been rounded to the nearest 1000 miles/hour, the exact speed will be between 65500 miles/hour and 66 500 miles/hour.

This range can be shown on a number line.



The range can also be written as 66000 $\pm$ 500 miles/hour.

Numbers that can be expressed as $a \pm b$ have a given tolerance. The tolerance in the example above is 500.

This is a convenient way to express a number that can lie within a set range.

If x is given as 12 $\pm$ 0.5, x can lie between 11.5 and 12.5 (or $11.5 \leq x \leq 12.5$).

KEY POINTS

- Measurements rounded to the nearest unit can be up to half a unit smaller or larger than the rounded value.

  If $x = 7$ (nearest **integer**), the exact value lies in the region $6.5 \leq x < 7.5$.

  If $y = 7.5$ (1 d.p.), the exact value lies in the region $7.45 \leq y < 7.55$.
- The upper bound is half a unit larger than the rounded measurement.
- The lower bound is half a unit smaller than the rounded measurement.

  If $z = 20$ (nearest 10), the exact value lies in the region: (lower bound) $15 \leq z < 25$ (upper bound).

Algebra 05

BASIC PRINCIPLES

- **Simplifying** algebraic expressions such as $x^2 - 4x + x - 4$.
- Multiplying out brackets such as $2(3 + x)$.
- Finding **factors** of numbers.
- Changing word problems into mathematical equations.

MULTIPLYING BRACKETS

TWO LINEAR BRACKETS

ACTIVITY 1

Finding the area of a rectangle involves multiplying two numbers together.

Multiplying $(x + 2)$ by $(x + 4)$ can also be done by finding the area of a rectangle.

This rectangular poster has sides $(x + 2)$ and $(x + 4)$.

Notice that the diagram shows the area of each part.

The total area is

$(x + 2)(x + 4) = x^2 + 4x + 2x + 8 = x^2 + 6x + 8$

	x	4
x	x^2	$4x$
2	$2x$	8

Draw similar diagrams to calculate these:

$(x + 5)(x + 2)$ $\quad$ $(x + 1)(x + 1)$

A very common mistake is to say that $(x + 2)^2 = x^2 + 2^2$.

Show that $(x+2)^2 \neq x^2 + 2^2$ by substituting various numbers for x.

Are there any values of x for which $(x + 2)^2 = x^2 + 2^2$?

What does $(x + 2)^2$ equal?

Remember that $(x + 2)^2 = (x + 2)(x + 2)$.

With imagination, this method can be extended to deal with negative numbers.

$(x + 2)(x - 5) = x^2 - 5x + 2x - 10 = x^2 - 3x - 10$

	x	-5
x	x^2	$-5x$
2	$2x$	-10

Use diagrams to calculate these:

$(x + 4)(x - 3)$ $\quad$ $(x - 3)^2$ $\quad$ $(x + 2)(x - 2)$

FIRST – OUTSIDE – INSIDE – LAST

Brackets can be multiplied without drawing diagrams:

$(x + 2) \times a = xa + 2a$

$\Rightarrow \quad (x + 2) \times (x + 4) = x(x + 4) + 2(x + 4)$

giving $\quad (x + 2)(x + 4) = x^2 + 4x + 2x + 8 = x^2 + 6x + 8$

KEY POINTS

The word (mnemonic) FOIL will remind you of the stages for multiplying out brackets.

FOIL stands for First, Outside, Inside, Last.

From each bracket:

- multiply the First terms
- multiply the Outside terms
- multiply the Inside terms
- multiply the Last terms.

  Then add the four terms.

$$(a + b)(c + d) = ac + ad + bc + bd$$

THREE LINEAR BRACKETS

EXAMPLE 4

Multiply out and simplify $x(x + 3)(x - 4)$.

First do $(x + 3)(x - 4) = x^2 - 4x + 3x - 12 = x^2 - x - 12$.

Then do $x(x^2 - x - 12) = x^3 - x^2 - 12x$.

The order of multiplying does not matter. You get the same answer if you first do $x(x + 3)$ and then multiply the answer by $(x - 4)$.

EXAMPLE 5

Expand and simplify $(x + 1)(x - 3)(x + 2)$.

First do $(x - 3)(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$

Then do $(x + 1)(x^2 - x - 6) = x^3 - x^2 - 6x + x^2 - x - 6 = x^3 - 7x - 6$

The order of multiplying the brackets does not matter.

KEY POINT

- The order of multiplication does not matter.

FACTORISING QUADRATIC EXPRESSIONS

FACTORISING QUADRATIC EXPRESSIONS WITH TWO TERMS

Factorising quadratic expressions such as $x^2 + 2x$ is easy because x is always a **common factor**.

EXAMPLE 6

Factorise $x^2 - 12x$.

x is a common factor, and so

$x^2 - 12x = x(x - 12)$

Factorising $x^2 - 9$ is a little more difficult.

EXAMPLE 7

Expand $(x - 3)(x + 3)$ using FOIL.

$x^2 + 3x - 3x - 9 = x^2 - 9$

So factorising $x^2 - 9$ gives $(x - 3)(x + 3)$.

KEY POINTS

- $x^2 + ax = x(x + a)$ and $x^2 - ax = x(x - a)$.
- $x^2 - a^2 = (x + a)(x - a)$ This is called 'the difference of two squares'.
- $x^2 + a^2$ cannot be factorised.
- Always check your factorisation by multiplying out.

FACTORISING QUADRATIC EXPRESSIONS WITH THREE TERMS

Expanding $(x + 2)(x - 5)$ using FOIL gives $x^2 - 3x - 10$.

Factorising is the reverse process. $x^2 - 3x - 10$ factorises to $(x + 2)(x - 5)$.

EXAMPLE 8

Find a if $x^2 + 5x + 6 = (x + 3)(x + a)$.

Using FOIL, the last terms in each bracket are multiplied to give 6.

So $3 \times a = 6$ and $a = 2$.

Check: $(x + 3)(x + 2) = x^2 + 2x + 3x + 6$
$= x^2 + 5x + 6$

EXAMPLE 9

Find a if $x^2 + x - 12 = (x + 4)(x + a)$.

Using FOIL, the last terms in each bracket are multiplied to give -12.

So $4 \times a = -12$ and $a = -3$.

Check: $(x + 4)(x - 3) = x^2 + 4x - 3x - 12$
$= x^2 + x - 12$

KEY POINT

- If the last **sign** in the expression is +, then the numbers in both brackets will have the same sign as the middle term.

EXAMPLE 11

Factorise $x^2 - 7x + 6$.

The last sign is +, and so both brackets will have the same sign as $-7x$, giving $(x - \_)(x - \_)$.

The missing numbers are both negative, multiply to give +6, and add to -7.

The numbers are -1 and -6.

So $x^2 - 7x + 6 = (x - 1)(x - 6)$.

KEY POINT

- If the last sign in the expression is −, then the numbers in the brackets will have opposite signs.

SOLVING QUADRATIC EQUATIONS BY FACTORISATION

ACTIVITY 5

SKILLS
ANALYSIS

If $a \times b = 12$ what can be said about either a or b?

If $a \times b = 0$ what can be said about either a or b?

A little thought should convince you that either $a = 0$ or $b = 0$ (or both are zero).

EXAMPLE 13

Solve $(x + 2)(x - 3) = 0$.

Either $(x + 2) = 0$ or $(x - 3) = 0$.

If $(x + 2) = 0$, then $x = -2$.

If $(x - 3) = 0$, then $x = 3$.

There are two solutions: $x = -2$ or $x = 3$.

EXAMPLE 14

Solve $(x - 5)^2 = 0$.

$(x - 5)^2 = 0$ is the same as $(x - 5)(x - 5) = 0$.

If the first bracket $(x - 5) = 0$, then $x = 5$.

If the second bracket $(x - 5) = 0$, then $x = 5$.

There is one solution: $x = 5$.

EXAMPLE 15

Solve $x^2 + 5x + 6 = 0$.

$x^2 + 5x + 6 = 0$ factorises to $(x + 3)(x + 2) = 0$. (See Example 10)

$\Rightarrow x = -3$ or $x = -2$.

EXAMPLE 16

Solve $x^2 - 7x + 6 = 0$.

$x^2 - 7x + 6 = 0$ factorises to $(x - 1)(x - 6) = 0$. (See Example 11)

$\Rightarrow x = 1$ or $x = 6$.

KEY POINT

- To solve a **quadratic equation**, rearrange it so that the right-hand side is zero. Then factorise the left-hand side.

EXAMPLE 17

Solve $x^2 - 5x = 6$.

$x^2 - 5x = 6$ must first be rearranged to $x^2 - 5x - 6 = 0$.

Then $x^2 - 5x - 6 = 0$ factorises to $(x + 1)(x - 6) = 0$. (See Example 12)

$\Rightarrow x = -1$ or $x = 6$.

PROBLEMS LEADING TO QUADRATIC EQUATIONS

EXAMPLE 20

The **product** of two **consecutive** even numbers is 120. What are the numbers?

Let the first number be x. The second even number is two more than x and can be written as $x + 2$.

Then $x \times (x + 2) = 120$ (Multiply out the bracket)

$x^2 + 2x = 120$ (Rearrange to equal zero)

$x^2 + 2x - 120 = 0$

$(x - 10)(x + 12) = 0$ (Factorise)

$x = 10$ or -12

So, the numbers are 10 and 12 or −12 and −10.

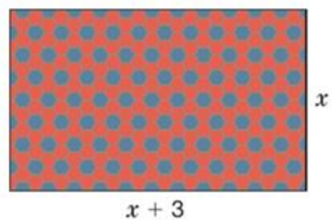
There are two possible answers. Answering the question by 'trial and improvement' would find the positive answer, but probably not the negative answer.

EXAMPLE 21

SKILLS

PROBLEM SOLVING AND REASONING

The length of a rectangular terrace is 3 m more than the width. If the area is $28\,m^2$, find the length and width of the terrace.



Let x be the width in metres. Then the length is $(x + 3)$ metres.

The area is $x \times (x + 3) = 28$ (Multiply out the bracket)

$x^2 + 3x = 28$ (Rearrange to equal zero)

$x^2 + 3x - 28 = 0$ (Factorise)

$(x + 7)(x - 4) = 0$

$x = -7$ or $x = 4$

As the answer cannot be negative, the width is 4 m and the length is 7 m.

KEY POINTS

To form and solve a quadratic equation in a problem-solving context:

- If relevant, draw a diagram and write all the information on it.
- Use x to represent one unknown and define the other **variables** from this.
- Form an equation to represent the situation.
- Solve to find the unknown(s).

If x represents a measurement such as length, it cannot have a negative value.

Graphs 05

BASIC PRINCIPLES

- Solving **inequalities** and showing the result on a number line: e.g. Solve $3x \geq x + 1$.
- Positive and negative **gradients**: e.g. Find the gradient of the line passing through (1, 2) and (3, –4).
- **Sketching** and plotting straight-line graphs: e.g. Sketch $y = 2x + 5$.
- Rearranging equations: e.g. Make x the subject of $y = 3x - 7$.

REPRESENTING INEQUALITIES GRAPHICALLY

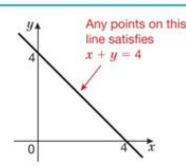
Inequalities in two **variables** can be represented on a graph. This makes them much easier to understand and solve.

EXAMPLE 1

Find the region representing $x + y \leq 4$.

First draw the line $x + y = 4$. This line divides the graph into two regions.

One of these regions **satisfies** $x + y \leq 4$.

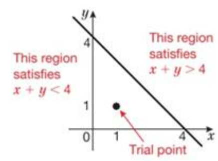


To decide which region satisfies $x + y \leq 4$, take any point in one of the regions, for example (1, 1).

Substitute $x = 1$ and $y = 1$ into $x + y$ to see if the result is less than or equal to 4.

$1 + 1 \leq 4$

So (1, 1) is in the required region, because it satisfies $x + y \leq 4$.

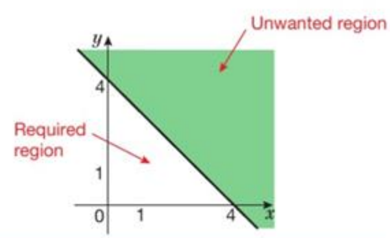


Therefore the required region is below the line $x + y = 4$.

Therefore this is the solution.

The line $x + y = 4$ is drawn as a solid line.

The unwanted region is shown shaded in the diagram.



Shading the unwanted region makes it much easier to indentify regions when more than one inequality is involved. Sometimes you might be asked to shade the wanted region, so read the question carefully. In the work that follows, the unwanted region is always shown shaded.

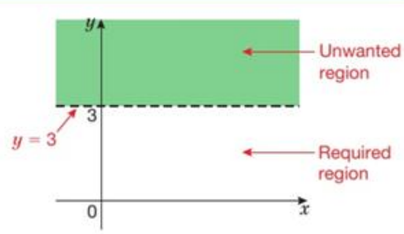
Inequalities in one variable can also be represented on a graph.

EXAMPLE 2

Find the region that represents $y < 3$.

Draw the line $y = 3$ as a dotted line because points on the line do not satisfy $y < 3$.

Points below the line $y = 3$ have y values less than 3, so the required region is below the line, and the unwanted region above the line is shaded.



KEY POINTS

- Find the line representing the equality.
- If points on the line are required, draw a solid line. Otherwise, draw a dotted line.
- Find the required region by using any point that is not on the line.

PERPENDICULAR LINES

There is a simple connection between the gradients of **perpendicular** lines.

ACTIVITY 2

SKILLS
REASONING

On graph paper draw axes with the x-axis labelled from 0 to 10 and the y-axis labelled from 0 to 6. Use the same **scale** for each axis.

Plot the points A (1, 0), B (3, 2) and C (0, 5). Join A to B and B to C.

Measure the angle between the lines AB and BC.

Find m_1, the gradient of AB and m_2, the gradient of CB. Multiply the two gradients together and put your results in a copy of this table.

Point A	Point B	Point C	m_1	m_2	$m_1 \times m_2$
(1, 0)	(3, 2)	(0, 5)			
(5, 0)	(3, 4)	(5, 5)			
(10, 3)	(7, 2)	(6, 5)			

Repeat for the two other sets of points in the table. Comment on your results.

Investigate if your conclusion is true for some other points.

Copy and complete this statement.
Lines which are .... have $m_1 \times m_2 = -1$.

KEY POINTS

- If $m_1 \times m_2 = -1$ then the two lines are perpendicular.
- If one line has a gradient m_1, the gradient of any perpendicular line is $m_2 = -\frac{1}{m_1}$.
  This is found by making m_2 the subject of $m_1 \times m_2 = -1$.
- If $m_1 = m_2$ then the lines are parallel.

MID-POINTS

Example 8 shows how to find the point midway between two points.

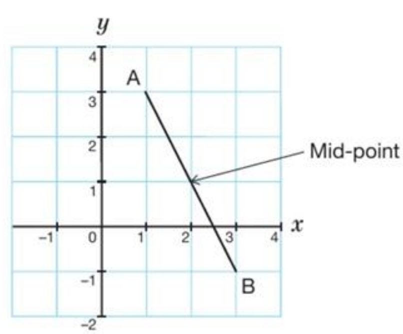
EXAMPLE 8

Find the point midway between A (1, 3) and B (3, –1).

The x-coordinate of the **mid-point** is the **mean** of the x-coordinates of A and B $= \frac{1+3}{2} = 2$

The y-coordinate of the mid-point is the mean of the y-coordinates of A and B $= \frac{3+-1}{2} = 1$

The mid-point is (2, 1).



KEY POINTS

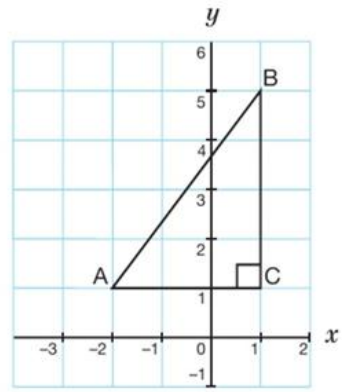
- The mid-point between (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- Check your answer using a **sketch**.

USING PYTHAGORAS' THEOREM

Pythagoras' Theorem can be used to find the distance between two points on a line.

EXAMPLE 10

Find the distance between A (–2, 1) and B (1, 5).



The diagram shows a right-angled triangle with AB as the **hypotenuse**.

AC is found by subtracting the x-coordinates.

AC = 1 – (–2) = 3

BC is found by subtracting the y-coordinates.

BC = 5 – 1 = 4

$AB^2 = 3^2 + 4^2 = 9 + 16 = 25 \Rightarrow AB = \sqrt{25} = 5$

KEY POINTS

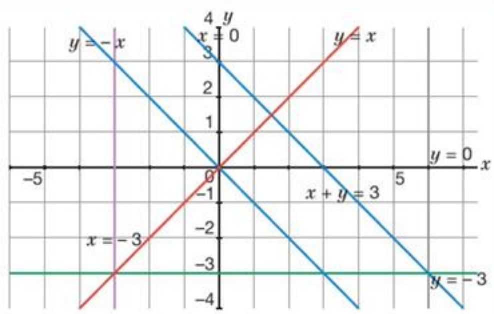
- The mean of the coordinates gives the coordinates of the mid-point.
- Use Pythagoras' Theorem to find the distance between two points.
- Draw a diagram to check if your answer is reasonable.

Shape and space

TRANSFORMATIONS

BASIC PRINCIPLES

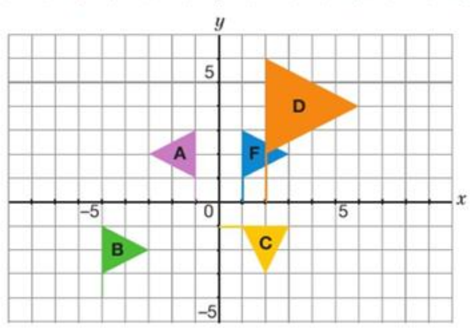
It is important that you are able to recognise certain equations of lines on a Cartesian graph. Many of these are shown here.



There are four basic transformations that you need to understand and be able to describe: translations, reflections, rotations and enlargements.

Flag F (object) goes through the following transformations to give the images A to D.

Object	Image	Transformation	Notes
Flag F	Flag A	Reflection in y-axis	All points are the same distance from the mirror line.
Flag F	Flag B	Translation by **vector** $\begin{pmatrix} -6 \\ -4 \end{pmatrix}$	The orientation of the image is the same after a translation.
Flag F	Flag C	Rotation of 90° clockwise about 0	+ angles are anti-clockwise – angles are clockwise.
Flag F	Flag D	Enlargement of **scale factor** 2 about 0	If the scale factor k is defined ($0 < k < 1$) then the object is decreased in size.

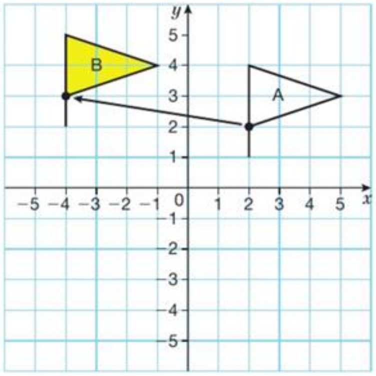


TRANSLATIONS

Transformations move a shape to a different position. Translation is a type of transformation. In a translation, all points of the shape move the same distance in the same direction.

HINT
To describe a translation fully, state the translation vector.

The lengths of the sides of the shape and the angles do not change, so the object and the image are said to be congruent.



Flag A is translated to flag B by a translation by **vector** $\begin{pmatrix}-6\\1\end{pmatrix}$.

Orientation is same, position is changed.

KEY POINT

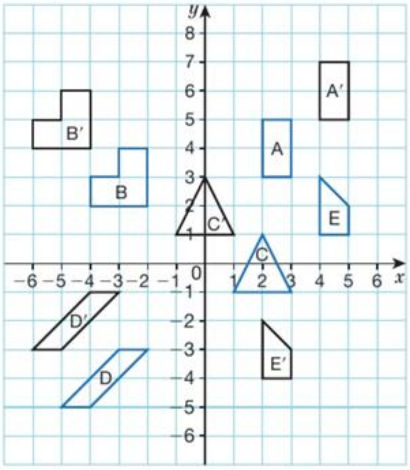
- A column vector $\begin{pmatrix}x\\y\end{pmatrix}$ is used to describe a translation.

The top number gives the movement parallel to the x-axis. The bottom number gives the movement parallel to the y-axis.

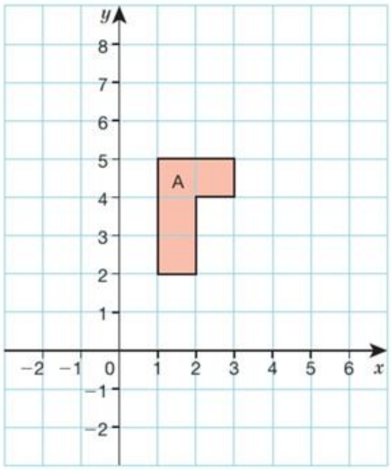
EXERCISE 1

7th

1 ▶ Describe the translation that moves each shape to its image where A is the object, A′ is the image.



2 ▶ Copy this diagram.



Translate shape A by the vector

a $\begin{pmatrix}2\\3\end{pmatrix}$ to B

b $\begin{pmatrix}3\\-4\end{pmatrix}$ to C

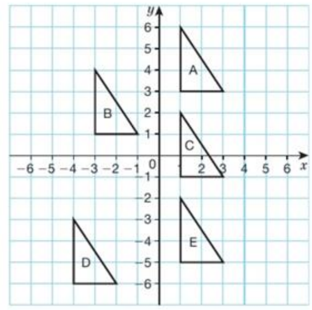
c $\begin{pmatrix}-2\\0\end{pmatrix}$ to D

d $\begin{pmatrix}0\\3\end{pmatrix}$ to E

e $\begin{pmatrix}-1\\-4\end{pmatrix}$ to F.

3 ▶ Describe these translations using column vectors.

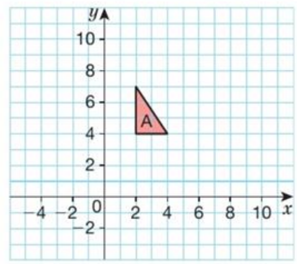
a B to A
b A to C
c B to E
d D to E
e E to D



4 ▶ Copy the diagram.
Translate shape A by the vector

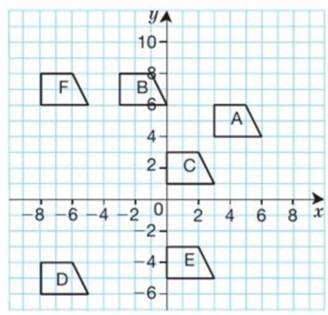
a $\begin{pmatrix}1\\3\end{pmatrix}$ to B **b** $\begin{pmatrix}2\\3\end{pmatrix}$ to C **c** $\begin{pmatrix}0\\-7\end{pmatrix}$ to D

d $\begin{pmatrix}-2\\0\end{pmatrix}$ to E **e** $\begin{pmatrix}-5\\-3\end{pmatrix}$ to F.



5 ▶ Describe these translations using column vectors.

a A to B
b C to E
c D to C
d F to B
e B to F



REFLECTIONS AND ROTATIONS

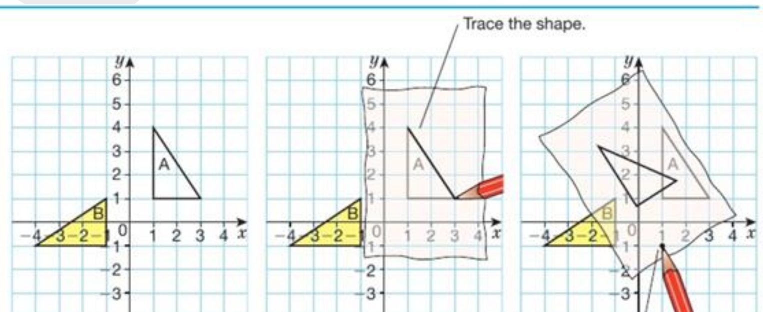
Reflections and rotations are two other types of transformations.
In reflections and rotations, the lengths of the sides of the shape and the angles do not change, so the object and the image are said to be congruent.

EXAMPLE 1
SKILLS
REASONING

Describe the rotation that takes shape A onto shape B.

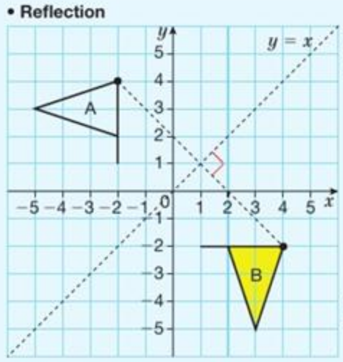
Rotation anti-clockwise 90° about (1, −1)

(Always give the direction, angle and centre of rotation.)



KEY POINTS

- **Reflection**

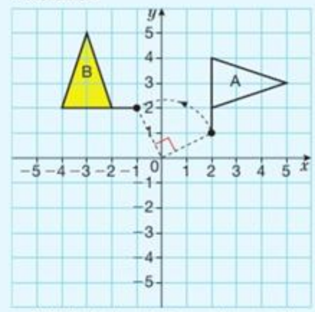


Flag A is reflected to flag B by a reflection in $y = x$.

Orientation and position are changed.

To describe a reflection fully, state the line of reflection.

- **Rotation**



Flag A is rotated to flag B by a rotation of +90° (anti-clockwise) about centre (0, 0).

Orientation and position are changed.

Anti-clockwise rotation angles are '+'
Clockwise rotation angles are '−'

To describe a rotation fully, state the angle of rotation, direction and centre of rotation.

ENLARGEMENTS

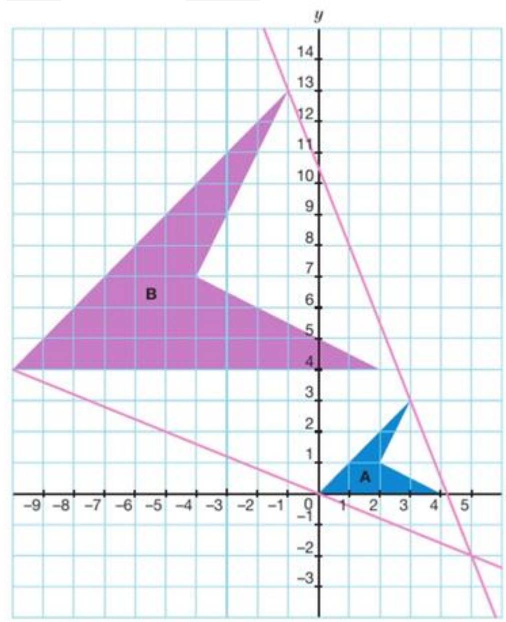
An enlargement changes the size of the object, but not the shape of the object. So the lengths of the sides of the shape change, but the angles of the shape do not change. To enlarge a shape, all the side lengths of the shape are multiplied by the same scale factor.

EXAMPLE 2

SKILLS REASONING

Arrowhead A has been transformed into arrowhead B by an enlargement of scale factor 3 about centre (5, −2).

Note: all the points from the object (arrowhead A) have had their distances from the centre trebled (multiplied by 3) to transform onto their image points on arrowhead B.



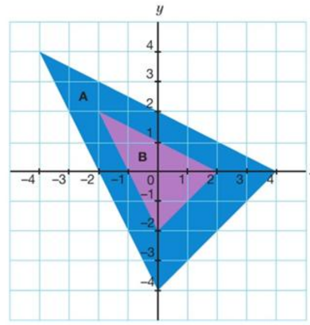
EXAMPLE 3

SKILLS

REASONING

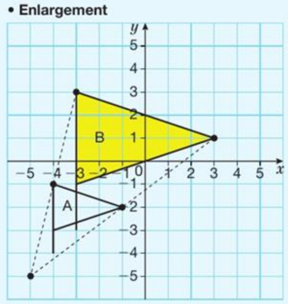
Triangle A has been transformed into triangle B by an enlargement of scale factor $\frac{1}{2}$ about the origin.

Notes:
The distance between each point of the image and the origin is half the original distance.
Even though the image is smaller than the object, the transformation is still called 'an enlargement'.



KEY POINTS

- **Enlargement**



Flag A is enlarged to flag B by a scale factor 2 about centre (−5, −5).

HINT To describe an enlargement fully, state the scale factor and centre of enlargement.

The orientation is the same*, the position has changed.

*If scale factor is positive.

COMBINED TRANSFORMATIONS

Combined transformations are the result of a number of successive transformations one after the other. The final image can be difficult to recognise when compared to the original object.

Note: if an operation is performed in reverse it is called the **inverse**. For example, the inverse of an enlargement of scale factor 3 about O is an enlargement of $\frac{1}{3}$ about O.

EXAMPLE 4

SKILLS

PPROBLEM SOLVING

a Plot points (1, 2), (1, 4) and (2, 4) to form triangle P.

Transformation A is a translation of $\begin{pmatrix}-5\\-4\end{pmatrix}$.

Transformation B is a reflection in $y = x$.

Transformation C is a clockwise rotation of 90° about centre (1, 1).

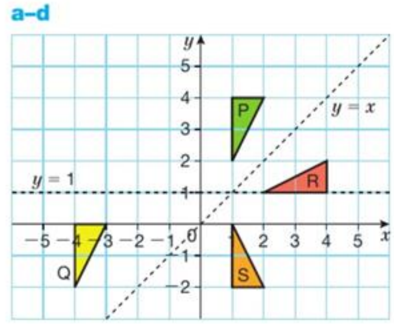
b Draw triangle P after it has been transformed by A and label this image Q.

c Draw triangle P after it has been transformed by B and label this image R.

d Draw triangle R after it has been transformed by C and label this image S.

e Describe fully the single transformation that maps P onto S.

a–d



e S maps onto P by a reflection in $y = 1$.

EXAMPLE 5

SKILLS

PROBLEM SOLVING

Triangle T is shown in the diagram.

a T is reflected in line M to form image A. Draw triangle A.

b A is reflected in the line $y = -1$ to form image B. Draw triangle B.

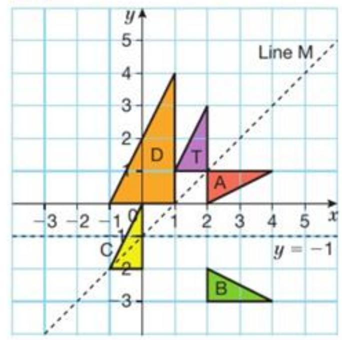
c T goes through a translation with vector $\begin{pmatrix}-2\\-3\end{pmatrix}$ to form image C. Draw triangle C.

d T is enlarged by a scale factor of 2 with centre (3, 2) to form image D. Draw triangle D.

e Describe fully the transformation which maps T onto B.

f Describe fully the transformation which maps C onto D.

a–d



e The single transformation that maps T onto B is a clockwise rotation of 90° about centre (0, −1).

f The single transformation that maps C onto D is an enlargement of scale factor 2 about centre (−1, −4).

Handling data 04

PROBABILITY – SINGLE EVENTS

BASIC PRINCIPLES

Consider these statements. They all involve a degree of uncertainty, which could be estimated through experiment or using previous knowledge.

- I doubt I will ever win the lottery.
- My dog will probably not live more than 5 years.
- It is unlikely to snow in the Sahara Desert.
- Roses will probably never grow at the South Pole.

EXPERIMENTAL PROBABILITY

It is possible to find the experimental probability, p(A), of A occurring through an experiment. The experiment should include a number of **trials** to see how often event A happens.

KEY POINTS

- p(A) means the probability of event A happening.
- p(A') means the probability of event A not happening.
- $\text{p}(A) = \dfrac{\text{number of times } A \text{ occurs}}{\text{total number of trials}}$

EXAMPLE 1

SKILLS
REASONING

Event A is that the same bird lands on Mrs Leung's bird table before 9am each day. It does this on 40 days over a period of 1 year (365 days).

a Estimate the probability that tomorrow event A happens.

b Estimate the probability that tomorrow event A does not happen.

a $p(A) = \frac{40}{365} = \frac{8}{73}$

b $p(A') = \frac{325}{365} = \frac{65}{73}$

Note: $p(A) + p(A') = 1$

RELATIVE FREQUENCY

KEY POINT

- **Relative frequency** or experimental probability $= \frac{\text{number of successes}}{\text{total number of trials}}$

EXAMPLE 2

SKILLS
MODELLING

Bill is interested in analysing the probability that, when a piece of toast with butter falls, it will land with the buttered side facing up. He thinks that this event, A, is unlikely to happen.

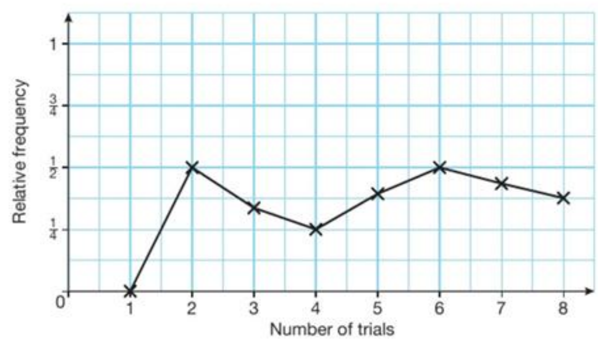
He then carries out eight trials, with the results shown in this table.

Trial number	1	2	3	4	5	6	7	8
Butter lands upwards	✗	✓	✗	✗	✓	✓	✗	✗
Relative frequency	$\frac{0}{1} = 0$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{3}{6}$	$\frac{3}{7}$	$\frac{3}{8}$

He plots the results on a relative frequency diagram.

From these eight trials, Bill estimates that the probability of his toast landing buttered-side up is $\frac{3}{8}$.

How could he improve his estimation of p(A)? Comment on Bill's initial theory.



THEORETICAL PROBABILITY

If all possible outcomes are equally likely, it is possible to find out how many of these outcomes should be event A. This means to calculate the theoretical probability of event A which is written as p(A).

KEY POINT

- $p(A) = \dfrac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$

EXAMPLE 3

SKILLS PROBLEM SOLVING

A fair die is rolled. Calculate the probability of a **prime number** being rolled.

The prime numbers for this experiment are 2, 3 and 5.

Event A is the event of a prime number appearing.

$p(A) = \frac{3}{6}$ (3 is the number of desired outcomes, and 6 is the total number of possible outcomes)

$= \frac{1}{2}$

EXAMPLE 4

SKILLS PROBLEM SOLVING

A coin with heads on both sides is tossed. If event A is defined as the coin landing head side up, calculate $p(A)$ and $p(A')$.

$p(A) = \frac{2}{2} = 1$ (a certainty)

$p(A') = \frac{0}{2} = 0$ (an impossibility)

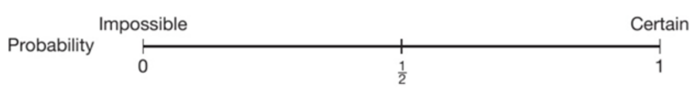
KEY POINT

- If A is an event, $0 \le p(A) \le 1$.

SKILLS ADAPTIVE LEARNING

ACTIVITY 2

Copy this **scale** across your page.



Label the scale, marking approximately where you think the probability of these five events A–E should be placed.

- A hockey captain wins the toss at the start of a match (A).
- A heart is taken out from a pack of playing cards (B).
- A heart is not taken out from a pack of playing cards (C).
- You will be taken away by aliens on your way home from school today (D).
- Your teacher will be wearing shoes for your next geography lesson (E).

If A is an event, it either occurs (A) or it does not occur (A').

It is certain that nothing else can happen.

EXAMPLE 5

SKILLS PROBLEM SOLVING

A card is randomly selected from a pack of 52 playing cards. Calculate the probability that a queen is not chosen.

Event Q is defined as a queen is chosen.

$p(Q') = 1 - p(Q) = 1 - \frac{4}{52} = \frac{48}{52} = \frac{12}{13}$

KEY POINTS

- $p(A) + p(A') = 1$

or, perhaps more usefully,

- $p(A') = 1 - p(A)$

EXPECTED FREQUENCY

If a die is rolled 60 times it would be reasonable to expect ten 6's to turn up.

This figure is obtained by multiplying the number of trials by the probability of the event.

Expected number of 6's = $60 \times \frac{1}{6} = 10$

Ten is not necessarily the exact number of 6's that will occur every time, but it is the most likely number.

KEY POINTS

- Expected frequency = number of trials × probability of the event

SAMPLE SPACE

A sample space (sometimes called a probability space) is a diagram showing all the possible outcomes. This enables probabilities to be easily calculated. This is useful when there are not too many outcomes to consider.

EXAMPLE 6

SKILLS MODELLING

The diagram shows the probability space for a pack of 52 playing cards containing the four suits of hearts, diamonds, spades and clubs.

Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
♥	♥	♥	♥	♥	♥	♥	♥	♥	♥	♥	♥	♥
Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
♦	♦	♦	♦	♦	♦	♦	♦	♦	♦	♦	♦	♦
Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
♠	♠	♠	♠	♠	♠	♠	♠	♠	♠	♠	♠	♠
Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
♣	♣	♣	♣	♣	♣	♣	♣	♣	♣	♣	♣	♣

From the diagram it is a clear that if a single card is randomly taken from this 52-card pack then:

p(red card) = $\frac{26}{52} = \frac{1}{2}$ p(black king) = $\frac{2}{52} = \frac{1}{26}$

p(odd diamond) = $\frac{5}{52}$ p(black prime) = $\frac{8}{52} = \frac{2}{13}$

Revision questions

1.

Train tickets

day return £6.45

monthly saver £98.50

2.

Sue goes to work by train.

Sue worked for 18 days last month.
She bought a day return ticket each day she worked.

A monthly saver ticket is cheaper than 18 day return tickets.
How much cheaper?

The diagram shows a patio in the shape of a rectangle.

3.

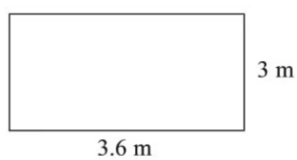


Diagram **NOT** accurately drawn

The patio is 3.6m long and 3 m wide.

Matthew is going to cover the patio with paving slabs.
Each paving slab is a square of side 60 cm.

Matthew buys 32 of the paving slabs.

Does Matthew buy enough paving slabs to cover the patio?
You must show all your working.

4.

Steve wants to put a hedge along one side of his garden.

He needs to buy 27 plants for the hedge.
Each plant costs £5.54.

Steve has £150 to spend on plants for the hedge.

Does Steve have enough money to buy all the plants he needs?

5.

Tom is going to buy 25 plants to make a hedge.

Here is information about the cost of buying the plants.

Kirsty's Plants	**Hedge World**
£2.39 each	Pack of 25
	£52.50 plus VAT at 20%

Tom wants to buy the 25 plants as cheaply as possible.

Should Tom buy the plants from Kirsty's Plants or from Hedge World?
You must show all your working.

6.

The lowest temperature recorded at Scott Base in Antarctica is -57.0 °C.
The highest temperature recorded at Scott Base is 63.8 °C more than this.

What is the highest temperature recorded at Scott Base?

7.

a is a prime number.

b is an even number.

$N = a^2 + ab$

Circle the correct statement about N.

could be even or odd	always even
always prime	always odd

8. Tom is tiling a wall.

He needs to buy at least 100 tiles.
The tiles are sold in large packs and small packs.

Large pack 40 tiles £18
Small pack 28 tiles £14
Special offer
25% reduction when you buy 3 or more **large** packs

Work out the cheapest cost for Tom to buy the packs of tiles he needs.

9.

Tom is also tiling a floor.

The floor is a rectangle with length 600 cm and width 240 cm
Each tile is a square with side 40 cm

Tom uses this method to work out the number of tiles he needs.

Number of tiles that will fit along the length = 600 ÷ 40
= 15
Number of tiles that will fit along the width = 240 ÷ 40
= 6
Total number of tiles needed = 15 + 6
= 21

Give a reason why Tom's method is wrong.

10.

Insert one pair of brackets to make this calculation correct.

$$7 - 5 - 3 + 4 = 9$$