Edexcel

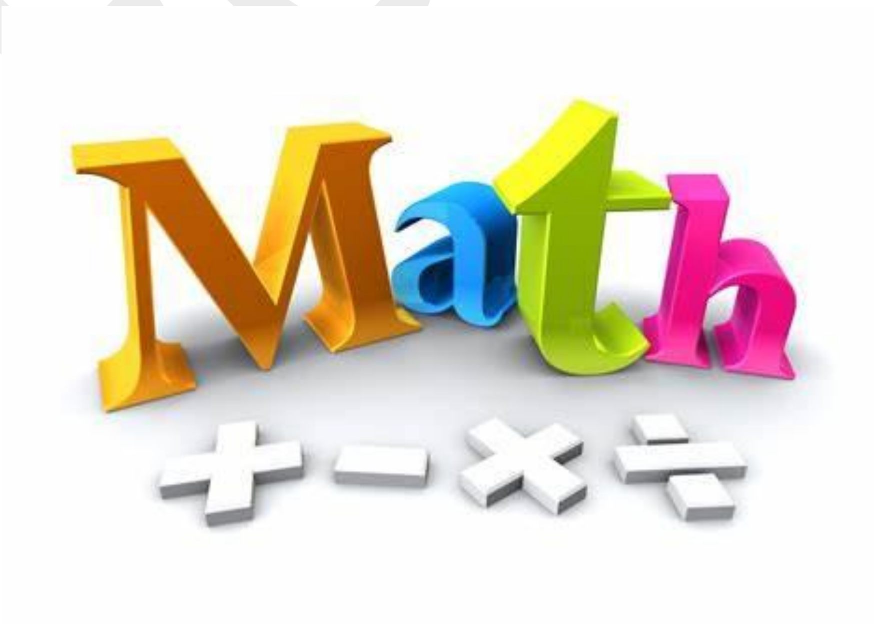
OL IGCSE

Mathematics

CODE: (4CP0)

Unit 08

4MB1



NUMBER 8

BASIC PRINCIPLES

- Rearrange formulae
- You will be expected to remember these conversions:
  - 1000 metres (m) = 1 kilometre (km)
  - 1000 millimetres (mm) = 1 metre (m)
  - 100 centimetres (cm) = 1 metre (m)
  - 10 millimetres (mm) = 1 centimetre (cm)
  - 1 millilitre (ml) = 1 cm^3
  - 1 litre = 1000 ml = 1000 cm^3
  - 1 kilogram (kg) = 1000 grams (g)
  - 1 tonne (t) = 1000 kilograms (kg)

CONVERTING BETWEEN UNITS OF LENGTH

EXAMPLE 1

SKILLS
PROBLEM SOLVING

The **circumference** of the Earth is approximately 40 075 km.
Change 40 075 km to mm.

40 075 km $= 40\,075 \times 1000$ m (as 1 km = 1000 m)
$= 40\,075 \times 1000 \times 1000$ mm (as 1 m = 1000 mm)
$= 4.0075 \times 10^{10}$ mm

EXAMPLE 2

SKILLS
PROBLEM SOLVING

The thickness of paper is around 0.05 mm. Change 0.05 mm to km.

0.05 mm $= \frac{0.05}{1000}$ m (as 1000 mm = 1 m)
$= \frac{0.05}{1000 \times 1000}$ km (as 1000 m = 1 km)
$= 5 \times 10^{-8}$ km

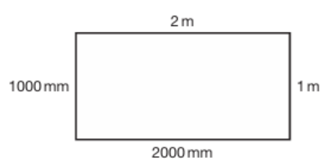
CONVERTING BETWEEN UNITS OF AREA

A diagram is useful, as is shown in the following examples.

EXAMPLE 3

A rectangle measures 1 m by 2 m. Find the area in mm^2.

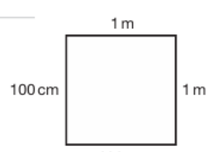
1 m is 1000 mm
2 m is 2000 mm
So the diagram is as shown.
So the area is 1000×2000 mm^2
$= 2\,000\,000$ mm^2
$= 2 \times 10^6$ mm^2



EXAMPLE 4

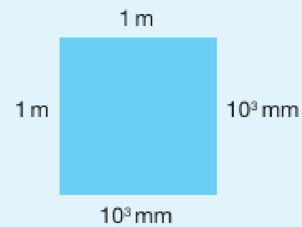
Change 30 000 cm^2 to m^2.

$1\ m^2 = 1\ m \times 1\ m$
$= 100\ cm \times 100\ cm$
$= 10\,000\ cm^2$
So $30\,000\ cm^2 = \frac{30\,000}{10\,000}\ m^2$
$= 3\ m^2$



KEY POINTS

- A diagram can help convert areas.
- $1\text{ m}^2 = 10^6\text{ mm}^2$



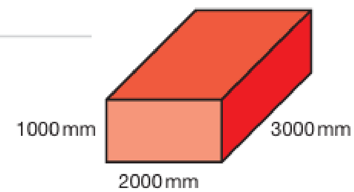
CONVERTING BETWEEN UNITS OF VOLUME

Diagrams are very useful.

EXAMPLE 5

A **cuboid** measures 1 m by 2 m by 3 m.

Find the volume in mm^3.



1 m is 1000 mm

2 m is 2000 mm

3 m is 3000 mm

So the volume is $1000 \times 2000 \times 3000 = 6 \times 10^9\text{ mm}^3$

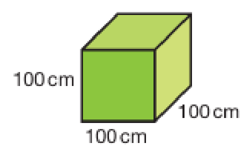
EXAMPLE 6

Change 10^7 cm^3 to m^3.

$$\begin{aligned} 1\text{ m}^3 &= 1\text{ m} \times 1\text{ m} \times 1\text{ m} \\ &= 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm} \\ &= 10^6\text{ cm}^3 \end{aligned}$$

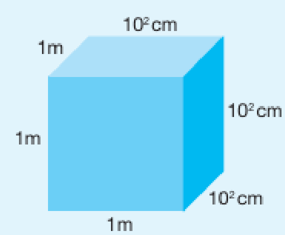
So $10^7\text{ cm}^3 = \dfrac{10^7}{10^6}\text{ m}^3$

$= 10\text{ m}^3$



KEY POINTS

- A diagram can help convert volumes.
- $1\text{ m}^3 = 10^6\text{ cm}^3$



COMPOUND MEASURES

A compound measure is made up of two or more different measurements and is often a measure of a **rate** of change.

Examples of a compound measure are speed, density and pressure.

Speed is a compound measure because speed $= \dfrac{\text{distance}}{\text{time}}$

Density is a compound measure because density $= \dfrac{\text{mass}}{\text{volume}}$

Pressure is a compound measure because pressure $= \dfrac{\text{force}}{\text{area}}$

EXAMPLE 7

A car is traveling at 36 km/hr.
What is its speed in m/s?

36 km/hr = 36 × 1000 m/hr = 36 000 m/hr
(to convert km/hr to m/hr × 1000)

36 000 m/hr = 36 000 ÷ 60 m/min = 600 m/min
(to convert m/hr to m/min ÷ 60)

600 m/min = 600 ÷ 60 m/s = 10 m/s
(to convert m/min to m/s ÷ 60)

Alternatively

distance = 36 km = 36 × 1000 m

time = 1 hour = 60 × 60 seconds

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{36 \times 1000}{60 \times 60} = 10\,\text{m/s}$$

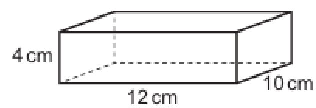
Density is the **mass** of a substance contained in a certain volume. It is usually measured in grams per cubic centimetre (g/cm^3). If a substance has a density of 4 g/cm^3 then 1 cm^3 has a mass of 4 g, 2 cm^3 has a mass of 8 g and so on.

EXAMPLE 8

The diagram shows a block of wood in the shape of a cuboid.

The density of wood is 0.6 g/cm^3.

Work out the mass of the block of wood.



The formula to calculate density is

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

The density is given and the mass needs to be found.

So work out the volume in cm^3:

Volume of block = $l \times w \times h$

= 12 × 10 × 4 = 480 cm^3

Substitute into the formula:

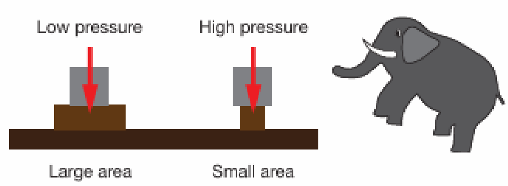
$$0.6 = \frac{\text{mass}}{480}$$

Multiply both sides by 480:

$$0.6 \times 480 = \frac{\text{mass}}{480} \times 480$$

Mass = 288 g

Pressure is the force applied over a certain area. It is usually measured in newtons per cm^2 (N/cm^2) or newtons per m^2 (N/m^2). If a force of 12 N is applied to an area of 4 cm^2 then the pressure is 12 ÷ 4 = 3 N/cm^2.



EXAMPLE 9

The piston of a bicycle pump has an area of $7\,cm^2$.
A force of 189 N is applied to the piston. What is the pressure generated?

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

$$\text{pressure} = 189 \div 7 = 27\,N/cm^2$$

KEY POINTS

- $\text{speed} = \frac{\text{distance}}{\text{time}}$ usually measured in m/s or km/hr
- $\text{density} = \frac{\text{mass}}{\text{volume}}$ usually measured in g/cm^3 or kg/m^3
- $\text{pressure} = \frac{\text{force}}{\text{area}}$ usually measured in N/cm^2 or N/m^2

ALGEBRA 8

BASIC PRINCIPLES

- Use function machines to find the output for different inputs (number functions, and those containing a **variable** such as x).
- Use function machines to find inputs for different outputs.
- Write an expression to describe a **function** (given as a function machine).
- Find the function of a function machine given the outputs for a set of inputs.
- Draw **mapping diagrams** for simple functions (to show, e.g., the inputs/outputs for a function machine).
- Substitute positive and negative **integers** and fractions into expressions, including those involving small **powers**.
- Solve simple linear equations.
- Solve **quadratic equations** by **factorising**.
- Rearrange simple formulae.
- Recognise that $\sqrt{x}$ means the positive square root of x.
- Identify **inverse** operations.
- Draw linear and **quadratic graphs**.

FUNCTIONS

IDENTIFYING FUNCTIONS

The keys on your calculator are divided into two main classes, operation keys and function keys. The operation keys, such as [+] and [×], operate on two numbers to give a single result.

The function keys, such as [x^2] and [$x!$], operate on one number to give a single result.

With a function key it is possible for two different inputs to give the same output, for example $2^2 = 4$ and $(-2)^2 = 4$

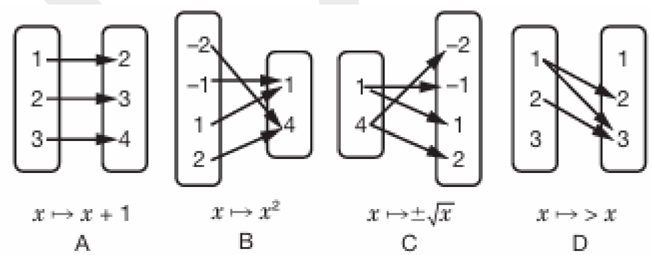
Relationships that are 'one to one' and 'many to one' are functions. A mapping diagram makes it easy to decide if it is a function or not. If only one arrow leaves each member of the **set** then the relationship is a function.

Example 01

State, giving a reason, whether each mapping shows a function or not.

SKILLS

INTERPRETATION



A is a function as each element of the first set maps to exactly one element of the second set. It is called a 'one to one' function.

B is a function as each element of the first set maps to exactly one element of the second set. Since the elements in the second set have more than one element from the first set mapped to them, it is called a 'many to one' function.

C is not a function as each element of the first set maps to more than one element of the second set.

D is not a function as at least one element of the first set maps to more than one element of the second set

A mapping diagram can be used if the size of the sets in the diagram is small. If the sets in the diagram are infinite (for example the set of all real numbers) then the vertical line test on a graph is used.

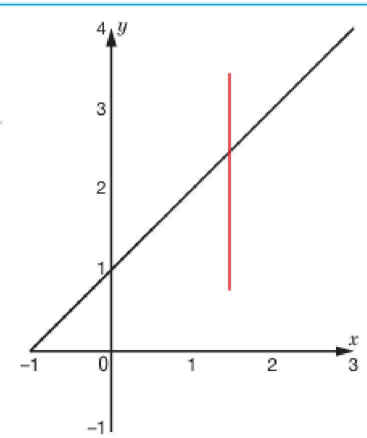
EXAMPLE 2

SKILLS

INTERPRETATION

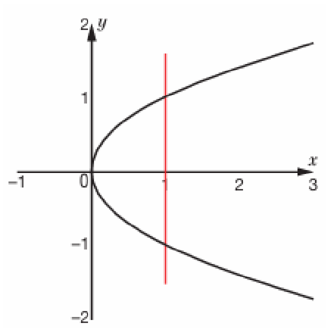
Use the vertical line test to decide if the graph shows a function or not, giving a reason. If it is a function, state if it is 'one to one' or 'many to one'.

Wherever the red vertical line is placed on the graph, it will only **intersect** at one point, showing, for example, that $1.5 \mapsto 2.5$, so this is a function. It is an example of a 'one to one' function.

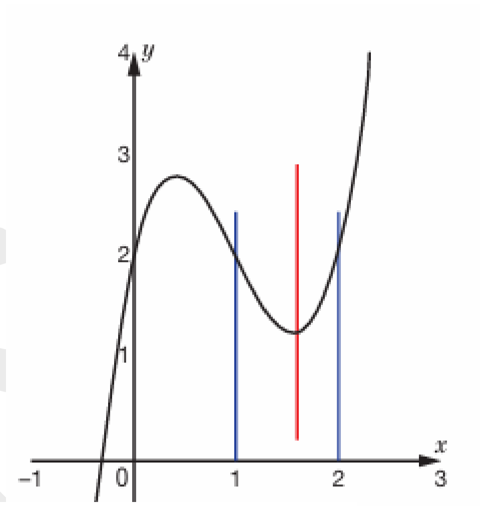


The vertical lines only intersect the graph on the right at one point wherever they are placed, so this is also a function.

The two blue lines intersect the graph at the same y value, showing that 1 ↦ 2 and 2 ↦ 2 so this is an example of a 'many to one' function.



The red vertical line intersects the graph on the left at two points, showing, for example, that 1 ↦ 1 and 1 ↦ −1, so this is not a function.



KEY POINTS

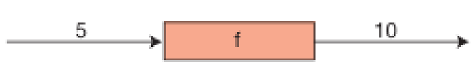
- Relationships that are 'one to one' and 'many to one' are functions.
- If only one arrow leaves each member of the set then the relationship is a function.
- If a vertical line placed anywhere on a graph intersects the graph at only one point then the graph shows a function.

FUNCTION NOTATION

A function is a set of rules for turning one number into another. Functions are very useful, for example, they are often used in computer spreadsheets. A function is a mathematical computer, an imaginary box that turns an input number into an output number.

If the function doubles every input number then

A letter can be used to represent the rule. If we call the doubling function f then



f has operated on 5 to give 10, so we write f(5) = 10

If x is input then $2x$ is output, so we write $f(x) = 2x$ or $f: x \mapsto 2x$

EXAMPLE 3

SKILLS
INTERPRETATION

a If $h : x \mapsto 3x - 2$ find

i h(2)
ii h(y)

b If $g(x) = \sqrt{5 - x}$, find g(–4)

a **i** $h(2) = 3 \times 2 - 2 = 4$
ii $h(y) = 3y - 2$

b $g(-4) = \sqrt{5 - (-4)} = \sqrt{9} = 3$

EXAMPLE 4

SKILLS
INTERPRETATION

a If $f(x) = 2 + 3x$ and $f(x) = 8$, find x

b If $g(x) = \frac{1}{x-3}$ and $g(x) = \frac{1}{2}$, find x

a $2 + 3x = 8 \quad \Rightarrow \quad 3x = 6 \quad \Rightarrow \quad x = 2$

b $\frac{1}{x-3} = \frac{1}{2} \quad \Rightarrow \quad x - 3 = 2 \quad \Rightarrow \quad x = 5$

KEY POINT

- A function is a rule for turning one number into another.

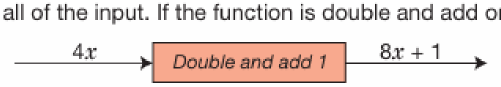
EXAMPLE 5

SKILLS
INTERPRETATION

Given $f(x) = 7x + 5$ and $h(x) = 6 + 2x$,
find the value of x such that $f(x) = h(x)$.

$7x + 5 = 6 + 2x \quad \Rightarrow \quad 5x = 1 \quad \Rightarrow \quad x = \frac{1}{5}$

A function operates on all of the input. If the function is double and add one then if $4x$ is input, $8x + 1$ is output.



EXAMPLE 6

SKILLS
INTERPRETATION

If $f(x) = 3x + 1$, find

a $f(2x)$ **b** $2f(x)$ **c** $f(x - 1)$ **d** $f(-x)$

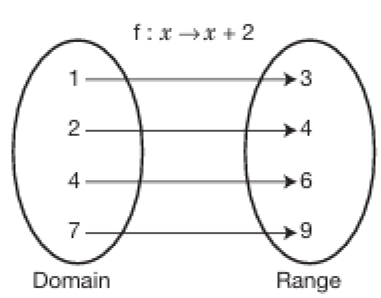
a $f(2x) = 3(2x) + 1 = 6x + 1$
b $2f(x) = 2(3x + 1) = 6x + 2$
c $f(x - 1) = 3(x - 1) + 1 = 3x - 2$
d $f(-x) = 3(-x) + 1 = 1 - 3x$

KEY POINT

- A function operates on all of its input.

DOMAIN AND RANGE

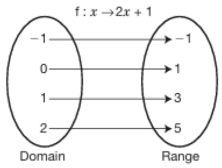
Here the only numbers the function can use are {1, 2, 4, 7}. This set is called the domain of the function. The set {3, 4, 6, 9} produced by the function is called the range of the function



EXAMPLE 7

Find the range of the function $f: x \mapsto 2x + 1$ if the domain is $\{-1, 0, 1, 2\}$.

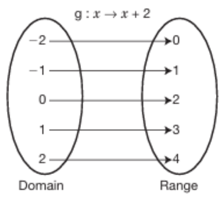
The diagram shows that the range is $\{-1, 1, 3, 5\}$.



EXAMPLE 8

Find the range of the function $g(x) = x + 2$ if the domain is $\{x: x \geq -2, x \text{ is an integer}\}$.

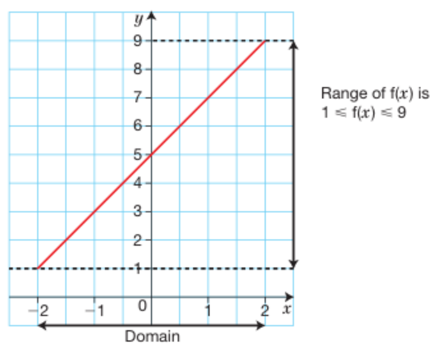
The diagram shows some of the domain. As the function changes an integer into another integer, the range is $\{y: y \geq 0, y \text{ is an integer}\}$.



EXAMPLE 9

If $f(x) = 2x + 5$ has a domain of $-2 \leq x \leq 2$, find the range of $f(x)$.

Range of $f(x)$ is $1 \leq f(x) \leq 9$.



EXAMPLE 10

Find the range of

a $h: x \mapsto x^2$ **b** $f: x \mapsto x^2 + 2$

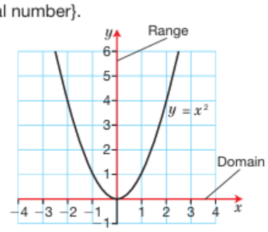
if the domain of both functions is all real numbers.

a Since $x^2 \geq 0 \Rightarrow$ the range is $\{y: y \geq 0, y \text{ a real number}\}$

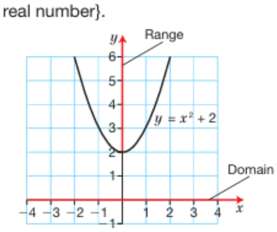
b Since $x^2 \geq 0$, then $x^2 + 2 \geq 2 \Rightarrow$ the range is $\{y: y \geq 2, y \text{ a real number}\}$

The graph of the function gives a useful picture of the domain and range. The domain corresponds to the x-axis, and the range to the y-axis. The graphs of the functions used in Example 10 are shown below.

The first graph ($y = x^2$) shows that all the y values are greater than or equal to zero, meaning that the range is $\{y: y \geq 0, y \text{ a real number}\}$.



The second graph ($y = x^2 + 2$) shows that all the y values are greater than or equal to 2, meaning that the range is $\{y: y \geq 2, y \text{ a real number}\}$.



KEY POINTS

- The graph of the function gives a useful picture of the domain and range.
- The domain corresponds to the x-axis, and the range to the y-axis.

VALUES EXCLUDED FROM THE DOMAIN

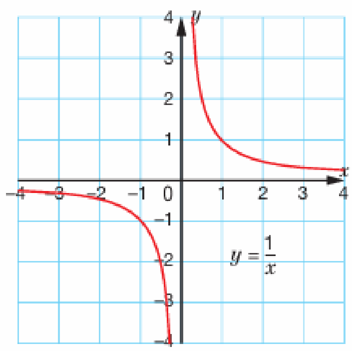
Sometimes there are values which cannot be used for the domain as they lead to impossible operations, usually division by zero or the square root of a negative number.

EXAMPLE 11

SKILLS ANALYSIS

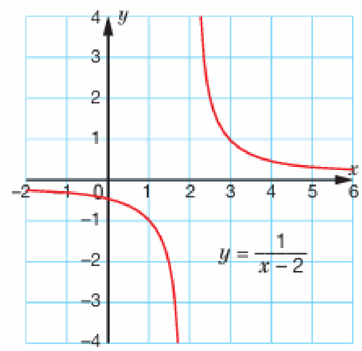
State which values (if any) must be excluded from the domain of these functions.

a $f(x) = \frac{1}{x}$ **b** $g(x) = \frac{1}{x-2}$



a Division by zero is not allowed, so $x = 0$ must be excluded from the domain of f (see graph).

b Division by zero is not allowed, so $x - 2 \neq 0$ which means $x = 2$ must be excluded from the domain of g (see graph).

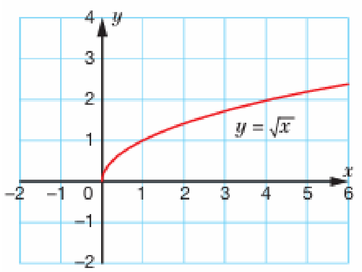


EXAMPLE 12

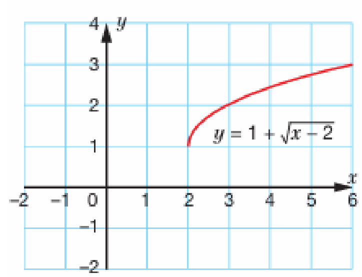
State which values (if any) must be excluded from the domain of these functions.

a $f(x) = \sqrt{x}$ **b** $g(x) = 1 + \sqrt{x-2}$

a The square root of a negative number is not allowed, though it is possible to square root zero, so $x < 0$ must be excluded from the domain of f (see graph).



b If $x - 2 < 0$ then x must be excluded from the domain. $x - 2 < 0 \Rightarrow x < 2$, so $x < 2$ must be excluded from the domain of g (see graph).



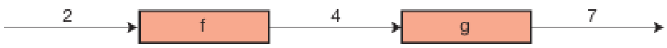
KEY POINTS

- Some numbers cannot be used for the domain as they lead to impossible operations.
- These operations are usually division by zero or the square root of a negative number.

COMPOSITE FUNCTIONS

When one function is followed by another, the result is a composite function.

If $f : x \mapsto 2x$ and $g : x \mapsto x + 3$, then



If the order of these functions is changed, then the output is different:

If x is input then

$g[f(x)]$ is usually written without the square brackets as $gf(x)$.

$gf(x)$ means do f first, followed by g.

Note that the domain of g is the range of f.

In the same way, $fg(x)$ means do g first, followed by f.

EXAMPLE 13

SKILLS

INTERPRETATION

If $f(x) = x^2$ and $g(x) = x + 2$, find

a $fg(3)$ **b** $gf(3)$ **c** $fg(x)$ **d** $gf(x)$ **e** $gg(x)$

a $g(3) = 5$, so $fg(3) = f(5) = 25$

b $f(3) = 9$, so $gf(3) = g(9) = 11$

c $g(x) = x + 2$, so $fg(x) = f(x + 2) = (x + 2)^2$

d $f(x) = x^2$, so $gf(x) = g(x^2) = x^2 + 2$

e $gg(x) = g(x + 2) = x + 4$

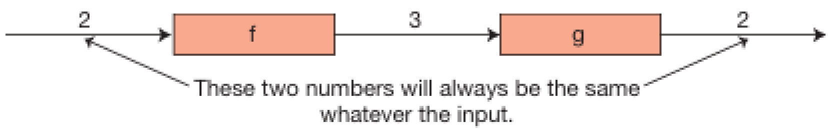
KEY POINTS

- $fg(x)$ and $gf(x)$ are composite functions.
- To work out $fg(x)$, first work out $g(x)$ and then substitute the answer into $f(x)$.
- To work out $gf(x)$, first work out $f(x)$ and then substitute the answer into $g(x)$.

INVERSE FUNCTION

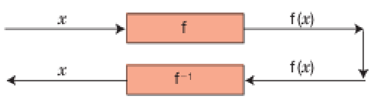
The inverse function undoes whatever has been done by the function. If the function is travel 1 km North, then the inverse function is travel 1 km South.

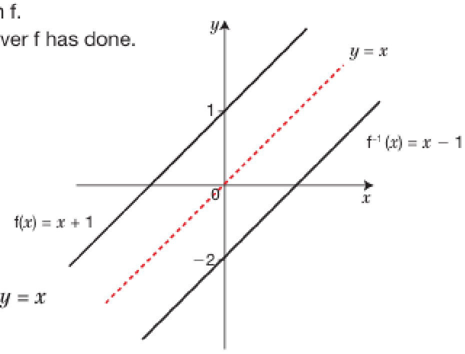
If the function is 'add one' then the inverse function is 'subtract one'. These functions are $f : x \mapsto x + 1$ ('add one') and $g : x - 1$ ('subtract one'). If f is followed by g then whatever number is input is also the output.



If x is the input then x is also the output.

The function g is called the inverse of the function f.
The inverse of f is the function that undoes whatever f has done.
The notation f^{-1} is used for the inverse of f.





Note that graphically $f^{-1}(x)$ is a reflection of $f(x)$ in $y = x$

FINDING THE INVERSE FUNCTION

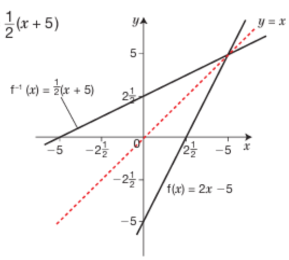
If the inverse function is not obvious, the following steps will find the inverse function. Step 1 Write the function as y = … Step 2 Change any x to y and any y to x. Step 3 Make y the subject giving the inverse function and use the correct f−1(x) notation

EXAMPLE 14 Find the inverse of $f(x) = 2x - 5$

Step 1 $y = 2x - 5$

Step 2 $x = 2y - 5$

Step 3 $x = 2y - 5 \Rightarrow 2y = x + 5 \Rightarrow y = \frac{1}{2}(x + 5) \Rightarrow f^{-1}(x) = \frac{1}{2}(x + 5)$



The graph shows that $f^{-1}(x) = \frac{1}{2}(x + 5)$ is a reflection of $f(x) = 2x - 5$ in the line $y = x$

EXAMPLE 15 Find the inverse of $g(x) = 2 + \frac{3}{x}$

Step 1 $y = 2 + \frac{3}{x}$

Step 2 $x = 2 + \frac{3}{y}$

Step 3 $x - 2 = \frac{3}{y} \Rightarrow y = \frac{3}{x - 2} \Rightarrow g^{-1}(x) = \frac{3}{x - 2}$

KEY POINT

- To find the inverse function:
  - Step 1 Write the function as $y = \dots$
  - Step 2 Change any x to y and any y to x.
  - Step 3 Make y the subject giving the inverse function and use the correct $f^{-1}(x)$ notation.
- The graph of the inverse function is the reflection of the function in the line $y = x$.

ACTIVITY 1

SKILLS
INTERPRETATION

If f : $x \mapsto x + 1$ and g : $x - 1$, show that f is the inverse of g.

If f(x) = $2x$, show that g(x) = $\frac{x}{2}$ is the inverse of f.

Is f also the inverse of g?

If f(x) = $4 - x$, show that f is the inverse of f.

(Functions like this are called 'self inverse'.)

ACTIVITY 2

SKILLS
PROBLEM SOLVING

ANCIENT CODING

One of the first people that we know of who used codes was Julius Caesar, who invented his own. He would take a message and move each letter three places down the alphabet, so that a $\mapsto$ d, b $\mapsto$ e, c $\mapsto$ f and so on. At the end of the alphabet he imagined the alphabet starting again so w $\mapsto$ z, x $\mapsto$ a, y $\mapsto$ b and z $\mapsto$ c. Spaces were ignored.

a Copy and complete this table where the first row is the alphabet and the second row is the coded letter.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
D	E	F																				Z	A	B	C

b Decode 'lkdwhpdwkv'.

c Code your own message and ask someone in your class to decode it.

Julius Caesar was using a function to code his message and the inverse function to decode it. The system is simple and easy to break, even if each letter is moved more than three places down the alphabet.

d Code another message, moving the letters by more than three places.

e Ask someone in your class to decode your message. Do not tell them how many places you moved the letters.

Modern computers make it easy to deal with codes like this. If you can program a spreadsheet, try using one to code and decode messages, moving the letters more than three places.

(The spreadsheet function =CODE(letter) returns a number for the letter while the function =CHAR(number) is the inverse function giving the letter corresponding to the number.)

The ideal function to use is a 'trapdoor function' which is one that is simple to use but with an inverse that is very difficult to find unless you know the key. Modern functions use the problem of factorising the **product** of two huge **prime** numbers, often over forty digits long. Multiplying the primes is straightforward, but even with the fastest modern computers the factorising can take hundreds of years!

GRAPHS 7

BASIC PRINCIPLES

- Plot graphs of linear, **quadratic**, **cubic** and **reciprocal** functions using a table of values.
- Use graphs to solve **quadratic equations** of the form $ax^2 + bx + c = 0$
- Solve a pair of **linear simultaneous equations** graphically (recognising that the solution is the point of intersection).

USING GRAPHS TO SOLVE QUADRATIC EQUATIONS

An accurately drawn graph can be used to solve equations that may be difficult to solve by other methods. The graph of y = x^2 is easy to draw and can be used to solve many quadratic equations

EXAMPLE 1

SKILLS PROBLEM SOLVING

Here is the graph of $y = x^2$. By drawing a suitable straight line on the graph, solve the equation $x^2 - x - 3 = 0$, giving answers **correct to** 1 d.p.

Rearrange the equation so that one side is x^2.

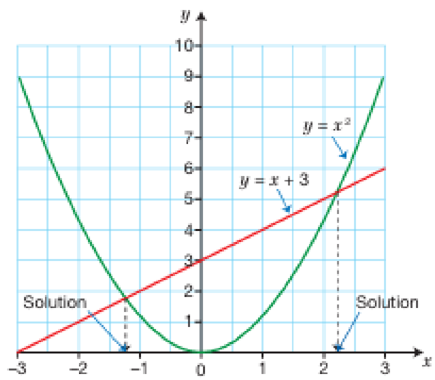
$x^2 - x - 3 = 0$

$x^2 = x + 3$

Draw the line $y = x + 3$

Find where $y = x^2$ **intersects** $y = x + 3$

The graph shows the solutions are $x = -1.3$ or $x = 2.3$



EXAMPLE 2

SKILLS PROBLEM SOLVING

Here is the graph of $y = x^2$. By drawing a suitable straight line on the graph, solve the equation $2x^2 + x - 8 = 0$, giving answers correct to 1 d.p.

Rearrange the equation so that one side is x^2.

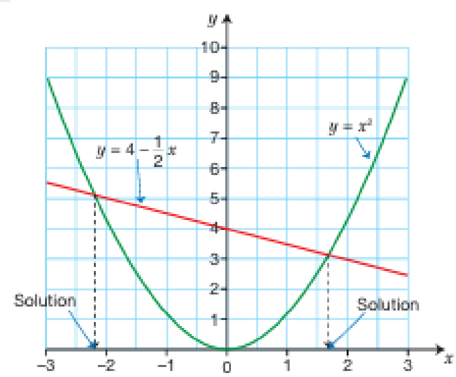
$2x^2 + x - 8 = 0$

$x^2 = 4 - \frac{1}{2}x$

Draw the line $y = 4 - \frac{1}{2}x$

Find where $y = x^2$ intersects $y = 4 - \frac{1}{2}x$

The graph shows the solutions are $x = -2.3$ or $x = 1.8$



KEY POINTS

- The graph of $y = x^2$ can be used to solve quadratic equations of the form $ax^2 + bx = c = 0$
- Rearrange the equation so that $x^2 = f(x)$, where $f(x)$ is a linear **function**.
- Draw $y = f(x)$ and find the x co-ordinates of the intersection points of the curve $y = x^2$ and the line $y = f(x)$

EXAMPLE 3

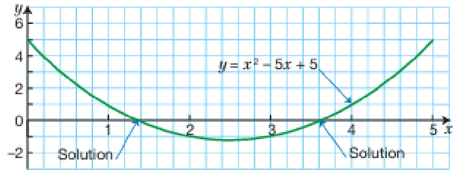
SKILLS

PROBLEM SOLVING

Here is the graph of $y = x^2 - 5x + 5$ for $0 \le x \le 5$

By drawing suitable straight lines on the graph, solve these equations, giving answers to 1 d.p.

a $0 = x^2 - 5x + 5$
b $0 = x^2 - 5x + 3$
c $0 = x^2 - 4x + 4$



a Find where $y = x^2 - 5x + 5$ intersects $y = 0$ (the x-axis).

The graph shows the solutions are $x = 1.4$ and $x = 3.6$ to 1 d.p.

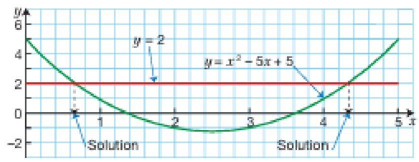
b Rearrange the equation so that one side is $x^2 - 5x + 5$

$0 = x^2 - 5x + 3$ (Add 2 to both sides)

$2 = x^2 - 5x + 5$

Find where $y = x^2 - 5x + 5$ intersects $y = 2$

The graph shows the solutions are $x = 0.7$ and $x = 4.3$ to 1 d.p.



c Rearrange the equation so that one side is $x^2 - 5x + 5$

$0 = x^2 - 4x + 4$ (Add 1 to both sides)

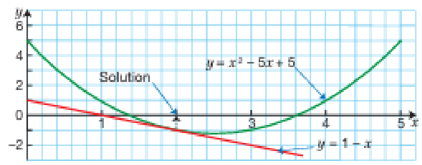
$1 = x^2 - 4x + 5$ (Subtract x from both sides)

$1 - x = x^2 - 5x + 5$

Find where $y = x^2 - 5x + 5$ intersects $y = 1 - x$

The graph shows the solution is $x = 2$ to 1.d.p.

Note: If the line does not cut the graph, there will be no real solutions.



KEY POINT

- The graph of one quadratic equation can be used to solve other quadratic equations with suitable rearrangement.

USING GRAPHS TO SOLVE OTHER EQUATIONS

EXAMPLE 4

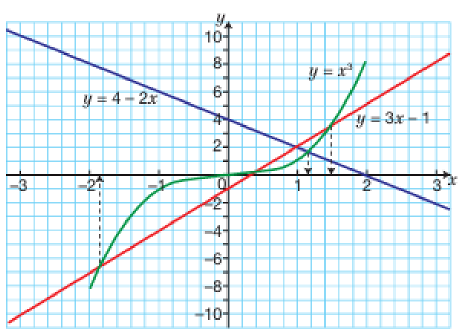
SKILLS

PROBLEM SOLVING

Here is the graph of $y = x^3$

By drawing suitable straight lines on the graph, solve these equations, giving the answers to 1 d.p.

a $x^3 + 2x - 4 = 0$
b $x^3 - 3x + 1 = 0$



a Rearrange the equation so that one side is x^3

$x^3 + 2x - 4 = 0$ (Add 4 to both sides)

$x^3 + 2x = 4$ (Subtract $2x$ from both sides)

$x^3 = 4 - 2x$

Find where $y = x^3$ and $y = 4 - 2x$ intersect.

The graph shows that there is only one solution.

The graph shows the solution is $x = 1.2$ to 1 d.p.

b Rearrange the equation so that one side is x^3

$x^3 - 3x + 1 = 0$ (Subtract 1 from both sides)

$x^3 - 3x = -1$ (Add $3x$ to both sides)

$x^3 = 3x - 1$

Find where $y = x^3$ and $y = 3x - 1$ intersect.

The graph shows that there are three solutions.

The graph shows the solutions are $x = -1.9$, $x = 0.4$ or $x = 1.5$ to 1 d.p.

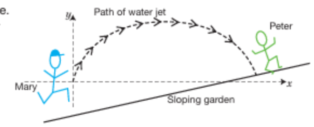
USING GRAPHS TO SOLVE NON-LINEAR SIMULTANEOUS EQUATIONS

You can use a graphical method to solve a pair of simultaneous equations where one equation is linear and the other is non-linear.

ACTIVITY 1

SKILLS
MODELLING

Mary is watering her garden with a hose. Her little brother, Peter, is annoying her so she tries to spray him with water.



The path of the water jet is given by
$y = 2x - \frac{1}{4}x^2$
The slope of the garden is given by
$y = \frac{1}{4}x - 1$
Peter is standing at (8, 1)
The origin is the point where the water leaves the hose, and units are in metres.
Copy and complete these tables.

x	0	2	4	6	8	10
$2x$			8			
$-\frac{1}{4}x^2$				−9		
$y = 2x - \frac{1}{4}x^2$		3				

x	0	4	8
$\frac{1}{4}x$			2
$y = \frac{1}{4}x - 1$	−1		

On one set of axes, draw the two graphs representing the path of the water and the slope of the garden.
Does the water hit Peter? Give a reason for your answer.
Mary changes the angle of the hose so that the path of the water is given by $y = x - 0.1x^2$
Draw in the new path. Does the water hit Peter this time?

In Activity 1, the simultaneous equations $y = 2x - \frac{1}{4}x^2$ and $y = \frac{1}{4}x - 1$ were solved graphically by drawing both graphs on the same axes and finding the x co-ordinates of the points of intersection.
Some non-linear simultaneous equations can be solved algebraically and this is the preferred method as it gives accurate solutions. When this is impossible then graphical methods must be used.

EXAMPLE 5

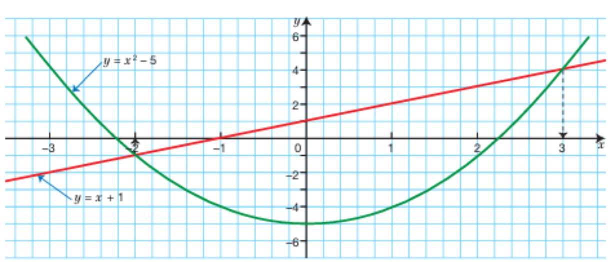
SKILLS
PROBLEM SOLVING

Solve the simultaneous equations $y = x^2 - 5$ and $y = x + 1$ graphically.

Construct a tables of values and draw both graphs on one set of axes.

x	−3	−2	−1	0	1	2	3
$x^2 - 5$	4	−1	−4	−5	−4	−1	4

x	−3	0	3
$x + 1$	−2	1	4



The co-ordinates of the intersection points are (−2, −1) and (3, 4) so the solutions are
$x = -2$, $y = -1$ or $x = 3$, $y = 4$

KEY POINT

- To solve simultaneous equations graphically, draw both graphs on one set of axes. The co-ordinates of the intersection points are the solutions of the simultaneous equations.

SHAPE AND SPACE 8

BASIC PRINCIPLES

- Understand and use of **column vectors**.
- Know what a **resultant vector** is.
- A knowledge of **bearings**.
- Understand and use of Pythagoras' theorem.
- **Sketch** 2D shapes.
- Knowledge of the properties of **quadrilaterals** and **polygons**.

VECTORS AND VECTOR NOTATION

A vector has both magnitude (size) and direction and can be represented by an arrow. A scalar has only size.

Vectors: Force, **velocity**, 10 km on a 060° bearing, **acceleration**…

Scalars: Temperature, time, area, length…

In this book, vectors are written as bold letters (such as **a**, **p** and **x**) or capitals covered by an arrow (such as $\overrightarrow{AB}$, $\overrightarrow{PQ}$ and $\overrightarrow{XY}$). In other books, you might find vectors written as bold italic letters (***a***, ***p***, ***x***). When hand-writing vectors, they are written with a wavy or straight underline ($\underset{\sim}{a}$, $\underset{\sim}{p}$, $\underset{\sim}{x}$ or $\underline{a}$, $\underline{p}$, $\underline{x}$).

On co-ordinate axes, a vector can be described by a column vector, which can be used to find the magnitude and angle of the vector.

The magnitude of a vector is its length. So, the magnitude of the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is $\sqrt{x^2+y^2}$

The magnitude of the vector **a** is often written as |**a**|.

EXAMPLE 1

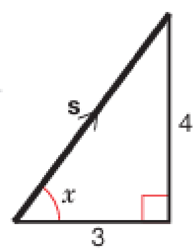
SKILLS

PROBLEM SOLVING

a Express vector **s** as a column vector.

b Find the magnitude of vector **s**.

c Calculate the size of angle x.



a $\mathbf{s} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

b $|\mathbf{s}| = \sqrt{3^2+4^2} = \sqrt{25} = 5$

c $\tan x = \frac{4}{3} \Rightarrow x = 53.1°$ (to 3 s.f.)

EXAMPLE 2

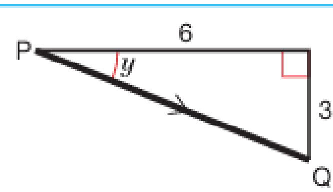
SKILLS

PROBLEM SOLVING

a Express vector $\overrightarrow{PQ}$ as a column vector.

b Find the magnitude of vector $\overrightarrow{PQ}$.

c Calculate the size of angle y.



a $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

b Length of $\overrightarrow{PQ} = \sqrt{6^2+(-3)^2} = \sqrt{45} = 6.71$ (to 3 s.f.)

c $\tan y = \frac{3}{6} \Rightarrow y = 26.6°$ (to 3 s.f.)

KEY POINT

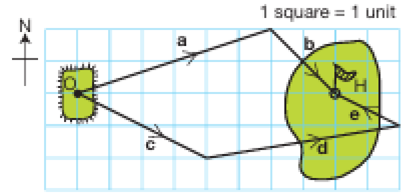
- The magnitude of the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is its length: $\sqrt{x^2+y^2}$

ACTIVITY 1

SKILLS
PROBLEM SOLVING

Franz and Nina are playing golf. Their shots to the hole (H) are shown as vectors.

Copy and complete this table by using the grid on the right.



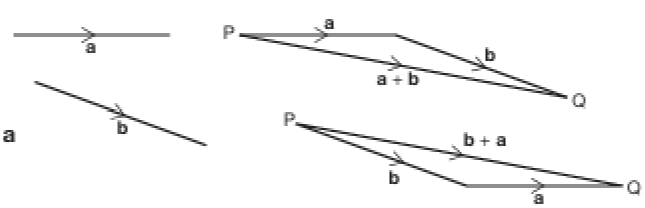
VECTOR	COLUMN VECTOR	MAGNITUDE (TO 3 s.f.)	BEARING
a	$\begin{pmatrix} 6 \\ 2 \end{pmatrix}$	6.32	072°
b			
c			
d			
e			

Write down the vector $\overrightarrow{OH}$ and state if there is a connection between $\overrightarrow{OH}$ and the vectors **a**, **b** and vectors **c**, **d**, **e**.

ADDITION OF VECTORS

The result of adding a set of vectors is the vector representing their total effect. This is the resultant of the vectors. To add a number of vectors, they are placed end to end so that the next vector starts where the last one finished. The resultant vector joins the start of the first vector to the end of the last one.

Vector $\overrightarrow{PQ}$ = **a** + **b** = **b** + **a**

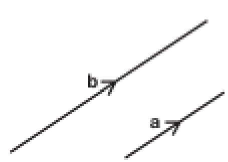


PARALLEL AND EQUIVALENT VECTORS

wo vectors are parallel if they have the same direction but not necessarily equal length. For example, these vectors a and b are parallel.

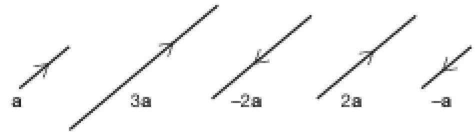
$\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

Two vectors are equivalent if they have the same direction and length.



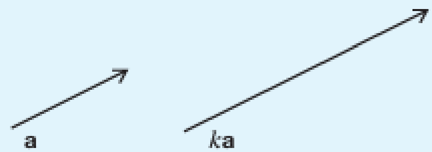
MULTIPLICATION OF A VECTOR BY A SCALAR

When a vector is multiplied by a scalar, its length is multiplied by this number; but its direction is unchanged, unless the scalar is negative, in which case the direction is reversed



KEY POINTS

- **a** = 2**b** ⇒ **a** is parallel to **b** and **a** is twice as long as **b**.
- **a** = k**b** ⇒ **a** is parallel to **b** and **a** is k times as long as **b**.



ADDITION, SUBTRACTION AND MULTIPLICATION OF VECTORS

Vectors can be added, subtracted and multiplied using their components.

EXAMPLE 3

SKILLS ANALYSIS

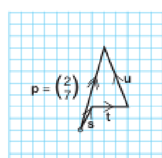
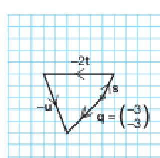
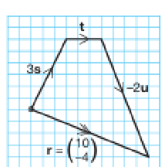
s = $\begin{pmatrix}1\\2\end{pmatrix}$, **t** = $\begin{pmatrix}3\\0\end{pmatrix}$ and **u** = $\begin{pmatrix}-2\\5\end{pmatrix}$

a Express in column vectors

p = **s** + **t** + **u**, **q** = **s** − 2**t** − **u** and **r** = 3**s** + **t** − 2**u**

b Sketch the resultants **p**, **q** and **r** accurately.

c Find their magnitudes.

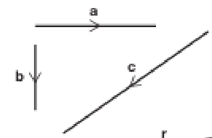
a CALCULATION	b SKETCH	c MAGNITUDE
p = **s** + **t** + **u** $= \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\5\end{pmatrix}$ $= \begin{pmatrix}2\\7\end{pmatrix}$		Length of **p** $= \sqrt{2^2 + 7^2}$ $= \sqrt{53}$ = 7.3 to 1 d.p.
q = **s** − 2**t** − **u** $= \begin{pmatrix}1\\2\end{pmatrix} - 2\begin{pmatrix}3\\0\end{pmatrix} - \begin{pmatrix}-2\\5\end{pmatrix}$ $= \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}-6\\0\end{pmatrix} + \begin{pmatrix}2\\-5\end{pmatrix}$ $= \begin{pmatrix}-3\\-3\end{pmatrix}$		Length of **q** $= \sqrt{(-3)^2 + (-3)^2}$ $= \sqrt{18}$ = 4.2 to 1 d.p.
r = 3**s** + **t** − 2**u** $= 3\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\0\end{pmatrix} - 2\begin{pmatrix}-2\\5\end{pmatrix}$ $= \begin{pmatrix}3\\6\end{pmatrix} + \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}4\\-10\end{pmatrix}$ $= \begin{pmatrix}10\\-4\end{pmatrix}$		Length of **r** $= \sqrt{10^2 + (-4)^2}$ $= \sqrt{116}$ = 10.8 to 1 d.p.

VECTOR GEOMETRY

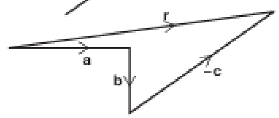
Vector methods can be used to solve geometric problems in two dimensions and produce simple geometric proofs.

EXAMPLE 4
SKILLS
ANALYSIS

Given vectors **a**, **b** and **c** as shown, draw the vector **r** where **r** = **a** + **b** − **c**

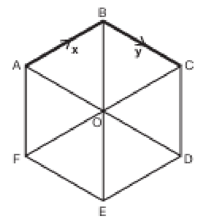


Here is the resultant of **a** + **b** − **c** = **r**



EXAMPLE 5
SKILLS
ANALYSIS

ABCDEF is a regular hexagon with centre O.
$\overrightarrow{AB} = \mathbf{x}$ and $\overrightarrow{BC} = \mathbf{y}$. Express the vectors $\overrightarrow{ED}$, $\overrightarrow{DE}$, $\overrightarrow{FE}$, $\overrightarrow{AC}$, $\overrightarrow{FA}$ and $\overrightarrow{AE}$ in terms of **x** and **y**.



$\overrightarrow{ED} = \mathbf{x}$ $\quad \overrightarrow{DE} = -\mathbf{x}$ $\quad \overrightarrow{FE} = \mathbf{y}$

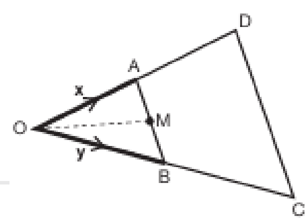
$\overrightarrow{AC} = \mathbf{x} + \mathbf{y}$ $\quad \overrightarrow{FA} = \mathbf{x} - \mathbf{y}$ $\quad \overrightarrow{AE} = 2\mathbf{y} - \mathbf{x}$

PARALLEL VECTORS AND COLLINEAR POINTS

Simple vector geometry can be used to prove that lines are parallel and points lie on the same straight line (collinear).

EXAMPLE 6
SKILLS
ANALYSIS

In ΔOAB, the mid-point of AB is M.
$\overrightarrow{OA} = \mathbf{x}, \overrightarrow{OB} = \mathbf{y}, \overrightarrow{OD} = 2\mathbf{x}$ and $\overrightarrow{OC} = 2\mathbf{y}$



a Express $\overrightarrow{AB}$, $\overrightarrow{OM}$ and $\overrightarrow{DC}$ in terms of **x** and **y**.

b What do these results show about AB and DC?

a
$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB}\\ &= -\mathbf{x} + \mathbf{y}\\ &= \mathbf{y} - \mathbf{x}\end{aligned}$$

$$\begin{aligned}\overrightarrow{OM} &= \overrightarrow{OA} + \overrightarrow{AM}\\ &= \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AB}\\ &= \mathbf{x} + \tfrac{1}{2}(\mathbf{y} - \mathbf{x})\\ &= \mathbf{x} + \tfrac{1}{2}\mathbf{y} - \tfrac{1}{2}\mathbf{x}\\ &= \tfrac{1}{2}\mathbf{x} + \tfrac{1}{2}\mathbf{y}\\ &= \tfrac{1}{2}(\mathbf{x} + \mathbf{y})\end{aligned}$$

$$\begin{aligned}\overrightarrow{DC} &= \overrightarrow{DO} + \overrightarrow{OC}\\ &= -2\mathbf{x} + 2\mathbf{y}\\ &= 2\mathbf{y} - 2\mathbf{x}\\ &= 2(\mathbf{y} - \mathbf{x})\end{aligned}$$

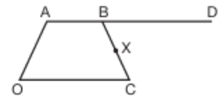
b $\overrightarrow{DC} = 2\overrightarrow{AB}$, so AB is parallel to DC and the length of DC is twice the length of AB.

EXAMPLE 7

SKILLS
ANALYSIS

OABC is a quadrilateral in which $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{a} + 2\mathbf{b}$ and $\overrightarrow{OC} = 4\mathbf{b}$

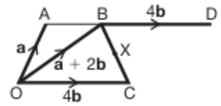
D is the point so that $\overrightarrow{BD} = \overrightarrow{OC}$ and X is the mid-point of BC.



a Find in terms of **a** and **b**

i $\overrightarrow{OD}$ **ii** $\overrightarrow{OX}$

b Explain what your answers to parts **a i** and **a ii** mean.



a i $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$= \mathbf{a} + 2\mathbf{b} + 4\mathbf{b}$

$= \mathbf{a} + 6\mathbf{b}$

ii $\overrightarrow{OX} = \overrightarrow{OC} + \overrightarrow{CX}$

$\overrightarrow{CX} = \frac{1}{2}\overrightarrow{CB}$

$\overrightarrow{CB} = \overrightarrow{CO} + \overrightarrow{OB}$

$= \overrightarrow{OB} - \overrightarrow{OC}$

$= \mathbf{a} + 2\mathbf{b} - 4\mathbf{b}$

$= \mathbf{a} - 2\mathbf{b}$

$\overrightarrow{CX} = \frac{1}{2}(\mathbf{a} - 2\mathbf{b}) = \frac{1}{2}\mathbf{a} - \mathbf{b}$

$\overrightarrow{OX} = 4\mathbf{b} + \frac{1}{2}\mathbf{a} - \mathbf{b} = \frac{1}{2}\mathbf{a} + 3\mathbf{b}$

b $\overrightarrow{OD} = \mathbf{a} + 6\mathbf{b} = 2(\frac{1}{2}\mathbf{a} + 3\mathbf{b})$

$\overrightarrow{OD} = 2\overrightarrow{OX}$

So OD and OX are parallel with a common point. This means that O, X and D lie on the same straight line. The length of OD is 2 times the length of OX. So X is the mid-point of OD.

KEY POINTS

- $\overrightarrow{PQ} = k\overrightarrow{QR}$ shows that the lines PQ and QR are parallel. Also they both pass through point Q so PQ and QR are part of the same straight line.

- P, Q and R are said to be collinear (they all lie on the same straight line).

ACTIVITY 2

SKILLS
INTERPRETATION

A radar-tracking station O is positioned at the origin of x and y axes, where the x-axis points due East and the y-axis points due North.

All the units are in km.

A helicopter is noticed t minutes after midday with position vector **r**, where

$\mathbf{r} = \begin{pmatrix} 12 \\ 5 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

Copy and complete this table and use it to plot the journey of the helicopter.

TIME	12:00 $t = 0$	12:01 $t = 1$	12:02 $t = 2$	12:03 $t = 3$	12:04 $t = 4$
r	$\begin{pmatrix} 12 \\ 5 \end{pmatrix}$				

Calculate the speed of the helicopter in km/hour and its bearing **correct to** 1 **decimal place**.

An international border is described by the line $y = 5x$.

Draw the border on your graph.

Estimate the time the helicopter crosses the border by looking carefully at your graph.

Considering the helicopter's position vector $\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}$, express x and y in terms of t and use these equations with $y = 5x$ to find the time when the border is crossed, correct to the nearest second.

HANDLING DATA 5

BASIC PRINCIPLES

- For equally likely outcomes, probability = $\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$
- P(A) means the probability of event A occurring.
- P(A′) means the probability of event A not occurring.
- $0 \leq P(A) \leq 1$
- $P(A) + P(A') = 1$, so $P(A') = 1 - P(A)$
- P(A|B) means the probability of A occurring given that B has already happened.

LAWS OF PROBABILITY

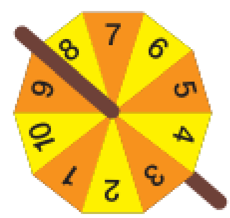
EXAMPLE 1

SKILLS

REASONING

Calculate the probability that a **prime** number is not obtained when a fair 10-sided spinner that is numbered from 1 to 10 is spun.

Let A be the event that a prime number is obtained.



$P(A') = 1 - P(A)$

$= 1 - \frac{4}{10}$ (There are 4 prime numbers: 2, 3, 5, 7)

$= \frac{6}{10} = \frac{3}{5}$

INDEPENDENT EVENTS

If two events have no influence on each other, they are independent events. If it snows in Moscow, it would be unlikely that this event would have any influence on your teacher winning the lottery on the same day. These events are said to be independent

MUTUALLY EXCLUSIVE EVENTS

Two events are mutually exclusive when they cannot happen at the same time. For example, a number rolled on a die cannot be both odd and even.

COMBINED EVENTS

MULTIPLICATION ('AND') RULE

EXAMPLE 2

Two dice are rolled together. One is a fair die numbered 1 to 6. On the other, each face is of a different colour: red, yellow, blue, green, white and purple

What is the probability that the dice will show an odd number and a purple face?

All possible outcomes are shown in this sample space diagram.

	R	Y	B	G	W	P
1	●	●	●	●	●	⊙
2	●	●	●	●	●	●
3	●	●	●	●	●	⊙
4	●	●	●	●	●	●
5	●	●	●	●	●	⊙
6	●	●	●	●	●	●

Let event A be that the dice will show an odd number and a purple face.

By inspection of the sample space diagram, $P(A) = \frac{3}{36} = \frac{1}{12}$

Alternatively:

If event O is that an odd number is thrown: $P(O) = \frac{1}{2}$

If event P is that a purple colour is thrown: $P(P) = \frac{1}{6}$

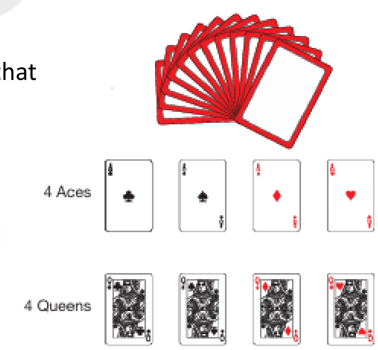
So, $P(A) = P(O \text{ and } P) = P(O) \times P(P) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

KEY POINT

- For two independent events A and B, P(A and B) = P(A) × P(B)

ADDITION ('OR') RULE

A card is randomly selected from a pack of 52 playing cards. Find the probability that either an Ace or a Queen is selected



Event A is an Ace is picked out: $P(A) = \frac{4}{52}$

Event Q is a Queen is picked out: $P(Q) = \frac{4}{52}$

Probability of A or Q: $P(A \text{ or } Q) = P(A) + P(Q)$

$$= \frac{4}{52} + \frac{4}{52}$$
$$= \frac{8}{52}$$
$$= \frac{2}{13}$$

KEY POINT

- For mutually exclusive events A and B, P(A or B) = P(A) + P(B)

The above rule makes sense, as adding fractions produces a larger fraction and the condition of one or other event happening suggests a greater chance

INDEPENDENT EVENTS AND TREE DIAGRAMS

A tree diagram shows two or more events and their probabilities. The probabilities are written on each branch of the diagram.

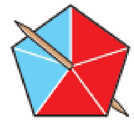
EXAMPLE 4
SKILLS
PROBLEM SOLVING

This fair five-sided spinner is spun twice.

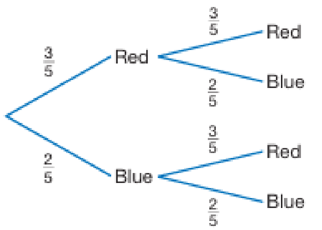
a Draw a tree diagram to show the probabilities.

b What is the probability of both spins landing on red?

c What is the probability of landing on one red and one blue?



a



b Go along the branches for Red, Red. The 1st and 2nd spins are independent so:

$P(R, R) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

c Go along the branches for Red, Blue and Blue, Red:

$P(R, B) = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

$P(B, R) = \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

The outcomes Red, Blue and Blue, Red are mutually exclusive so:

$P(R, B \text{ or } B, R) = \frac{6}{25} + \frac{6}{25} = \frac{12}{25}$

THE 'AT LEAST' SITUATION

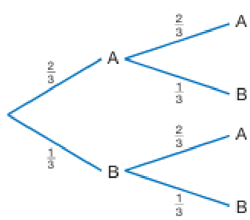
EXAMPLE 5
SKILLS
REASONING

A box contains three letter tiles:
A tile is chosen at random from the box and then replaced.
A second tile is now chosen at random from the box and then returned.

a Draw a tree diagram to show the probabilities.

b Work out the probability that at least one A is chosen.

a First selection Second selection



b Let the number of letter A's taken out be X.
At least one A means that $X \geq 1$

Method 1

List all the possibilities.

$P(X \geq 1) = P(X = 1) + P(X = 2) = P(AB) + P(BA) + P(AA)$

$= \frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{3}$

$= \frac{2}{9} + \frac{2}{9} + \frac{4}{9} = \frac{8}{9}$

Method 2

Use the rule $P(E) + P(E') = 1$

$P(X \geq 1) + P(X = 0) = 1$

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - P(BB)$

$= 1 - \frac{1}{3} \times \frac{1}{3}$

$= 1 - \frac{1}{9} = \frac{8}{9}$

Note: Both methods need to be understood and applied.

CONDITIONAL PROBABILITY

The probability of an event based on the occurrence of a previous event is called a conditional probability. Two events are dependent if one event depends upon the outcome of another event.

For example, removing a King from a pack of playing cards reduces the chance of choosing another King. A conditional probability is the probability of a dependent event. Tree diagrams can be used to solve problems involving dependent events

EXAMPLE 6

SKILLS
INTERPRETATION

Two cards are randomly selected one after the other from a pack of 52 playing cards and **not** replaced.

Calculate the probability that the second card is a King given that the first card is

a a King
b not a King.

a P(second is a King given the first card is a King) = $\frac{3}{51}$
(3 kings left in a pack of 51)

b P(second is a King given the first card is not a King) = $\frac{4}{51}$
(4 kings left in a pack of 51)

EXAMPLE 7

SKILLS
REASONING

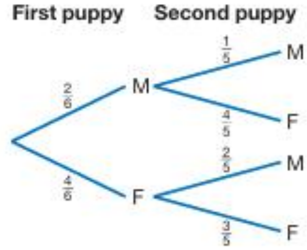
A group of Border Collie puppies contains four females (F) and two males (M). A vet randomly removes one from their basket and it is **not** replaced before another one is chosen.

a Draw a tree diagram to show all the possible outcomes.

b What is the probability that the vet removes

i two males **ii** one male and one female?

a **First puppy** **Second puppy**



b **i** When the second puppy is taken, there are only five left in the basket.

Let event A be that two males are chosen.

P(A) = P(M_1 and M_2) (M_1 means the first puppy is a male, M_2 means the second puppy is a male)

= P(M_1) × P(M_2) (Given by tree diagram route MM)

$= \frac{2}{6} \times \frac{1}{5} = \frac{2}{30} = \frac{1}{15}$

ii Let event B be that a male and a female are chosen.

P(B) = P(M_1 and F_2) or P(F_1 and M_2)

(Given by tree diagram routes MF and FM)

$= \frac{2}{6} \times \frac{4}{5} + \frac{4}{6} \times \frac{2}{5}$

$= \frac{8}{30} + \frac{8}{30} = \frac{16}{30} = \frac{8}{15}$

Revision questions

1.

$$f(x) = 3x^2 - 2x - 8$$

Express $f(x+2)$ in the form $ax^2 + bx$

2.

The function f is such that $f(x) = x^2 - 8x + 5$ where $x \leqslant 4$

Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

3.

Two functions, f and g are defined as

$$f : x \mapsto 1 + \frac{1}{x} \text{ for } x > 0$$

$$g : x \mapsto \frac{x+1}{2} \text{ for } x > 0$$

Given that $h = fg$

express the inverse function h^{-1} in the form $h^{-1} : x \mapsto \ldots$

$h^{-1} : x \mapsto$ ....

4.

$f(x) = 2x - 3$ and $g(x) = x^2$

Show that $f^{-1}(55) = fg(4)$

5.

$f(x) = \frac{x}{3} + 4$ for all values of x.

$g(x) = 6x^2 + 3$ for all values of x.

Work out $fg(x)$.

Give your answer in the form $ax^2 + b$ where a and b are integers.

6.

$f(x) = 3x^2 - 4x + 8$ for all values of x

Jenny says,

"f(10) must equal 2 × f(5), because 10 is 2 × 5"

Is Jenny correct?
Show working to support your answer.

7.

$g(x) = 16 - x \quad h(x) = x^3$

Solve $gh(x) = 24$

8.

$f(x) = \dfrac{x}{x+2} \qquad g(x) = x^2 - 2$

Work out $fg(x)$

Give your answer in the form $a + bx^n$ where a, b and n are integers.

9.

$f(x) = \dfrac{2x + 3}{x - 4}$

Work out $f^{-1}(x)$

10.

$g(x) = 3x + 7$

Solve $g^{-1}(x) = 2x$