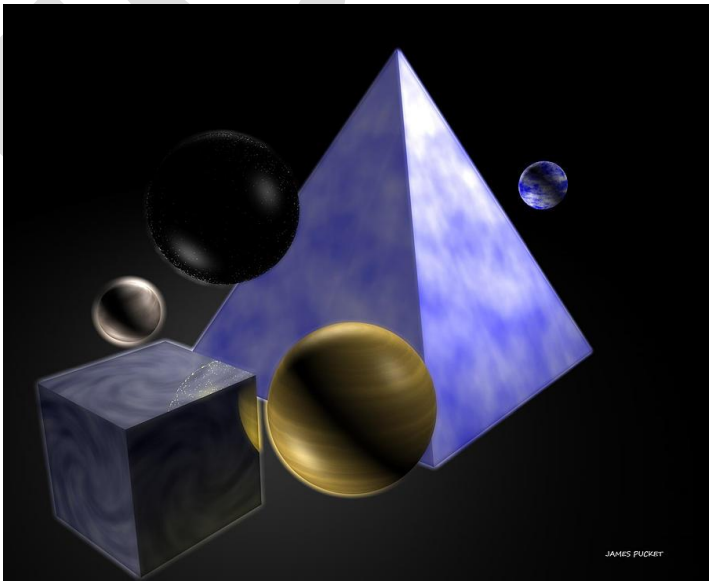
Edexcel OL Mathematics

CODE: (4MA1)

Unit 2

Shape And Space



LEARNING OBJECTIVES

- Find the length of the hypotenuse in a right-angled triangle
- Understand and use facts about the angle in a semicircle
- Find the length of a shorter side in a right-angled triangle
- Solve problems using Pythagoras' Theorem
- Use the properties of angles in a circle
- Use the properties of tangents to a circle
- Understand and use facts about chords being a right angle
- Understand and use facts about angles subtended at the centre and the circumference of circles
- Understand and use facts about cyclic quadrilaterals
- Solve angle problems using circle theorems

BASIC PRINCIPLES

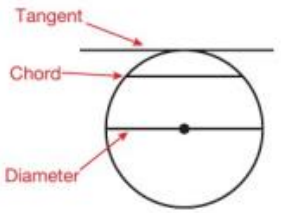
- Solve equations for x such as:

$5^2 = 3^2 + x^2 \Rightarrow x^2 = 5^2 - 3^2 = 16$

$\Rightarrow x = \sqrt{16}$

$\Rightarrow x = 4$

- Straight lines can **intersect** a circle to form a **tangent**, a **diameter** or a **chord**.



- A tangent touches the circle. It is **perpendicular** to the **radius** at the point of contact.

- **Isosceles triangles** frequently occur in circle theorem questions.

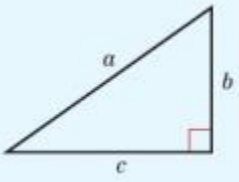
PYTHAGORAS' THEOREM

In a right-angled triangle, the longest side is called the **hypotenuse**. It is the side opposite the right angle. Pythagoras' Theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the **sum** of the squares of the other two sides.

KEY POINT

- $a^2 = b^2 + c^2$

Side a is always the hypotenuse.



EXAMPLE 01

SKILLS: REASONING

Calculate the side a

From Pythagoras' theorem

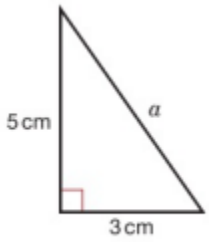
$a^2 = b^2 + c^2$

$a^2 = 3^2 + 5^2$

$= 34$

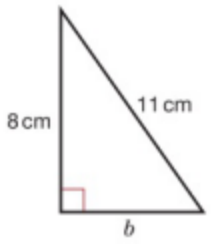
$a = \sqrt{34}$

$a = 5.38\,\text{cm}$ (3 s.f.)



EXAMPLE 02
SKILLS: REASONING
Calculate the side b.
From Pythagoras' theorem

$$a^2 = b^2 + c^2$$
$$11^2 = b^2 + 8^2$$
$$b^2 = 11^2 - 8^2$$
$$= 57$$
$$b = \sqrt{57}$$
$$b = 7.55 \text{ cm (3 s.f.)}$$



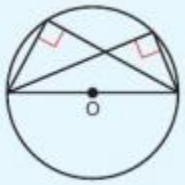
CIRCLE THEOREMS

Circle geometry has existed for a long time. Euclid (350- 300BC) was a Greek mathematician who is often called the **'Father of Geometry'**. His book, called **Elements,** contained many theorems on circles that we study in this section.

ANGLES IN A SEMICIRCLE AND TANGENTS

KEY POINT

- An angle in a semicircle is always a right angle.



KEY POINTS

Writing out reasons for geometrical questions

- Your calculations must be supported by a reason for each step. It is helpful to label points, angles and lengths carefully.
- It is normal practice to write calculations on the left-hand side of the page followed by reasons on the right-hand side as in Activity 2.

Typical reasons:

Angle sum of a straight line = 180°

Angle sum of a triangle = 180°

Isosceles triangles have equal base angles

Angle sum around a point = 360°

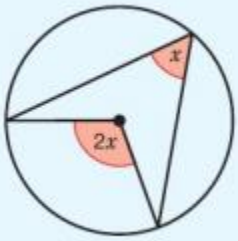
Alternate angles *are equal*

Vertically opposite angles are equal

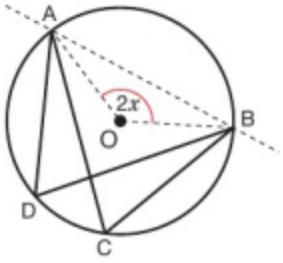
ANGLE AT CENTRE OF CIRCLE IS TWICE THE ANGLE AT CIRCUMFERENCE

KEY POINT

- The angle subtended at the centre of a circle is twice the angle at the circumference.



EXAMPLE 3



SKILL: REASONING

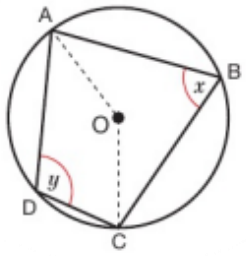
Show that <ADB = <ACB, namely that 'the angles in the same segment are equal'.

Chord AB splits the circle into two **segments**.

Points C and D are in the same **segment.**

Calculations	**Reasons**
<AOB = 2x	General angle chosen
<ADB = x	Angle at centre of circle = 2 x angle at circumference
<ACB = x	Angle at centre of circle = 2 x angle at circumference
<ABD = <ACB	As required

EXAMPLE 4



SKILL: REASONING

Show that <ABC + <ADC= 180°, namely that 'the sum of the opposite angles of a cyclic **quadrilateral** = 180°.

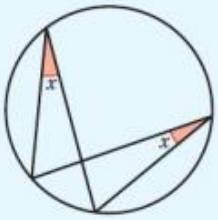
ABCD is a cyclic quadrilateral with OA and OC as radii of the circle.

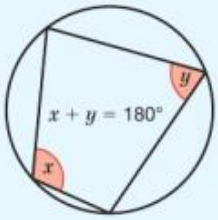
Calculations	Reasons
<ABC = x	General angle chosen
<ADC = y	General angle chosen
<AOC = 2x	Angle at centre of circle = 2 x angle at circumference
<AOC = 2y (reflex angle)	Angle at centre of circle = 2 x angle at circumference
2x + 2y = 360°	Angle sum at a point = 360°
x + y = 180°	As required

Angle sum of a quadrilateral = 360°, so sum of the remaining two angles = 180°.

KEY POINTS

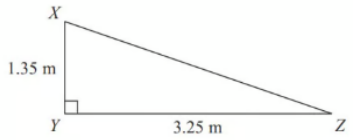
- Angles in the same segment are equal.
- Opposite angles of a cyclic quadrilateral (a quadrilateral with all four vertices on the circumference of a circle) sum to 180°.



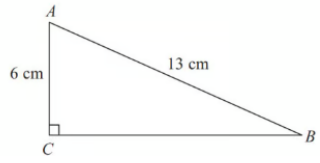


Revision questions

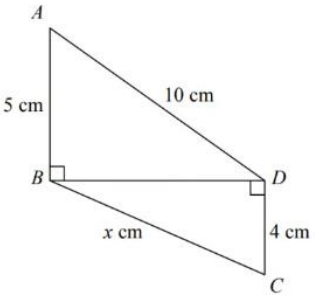
1) XYZ is a right-angle triangle
Calculate the length of XY.
Give your answer correct to 3 significant figures.



2)ABC is a right-angled triangle. AC=6cm
AB = 13 cm
Work out the length of BC.
Give your answer correct to 3 significant figures.

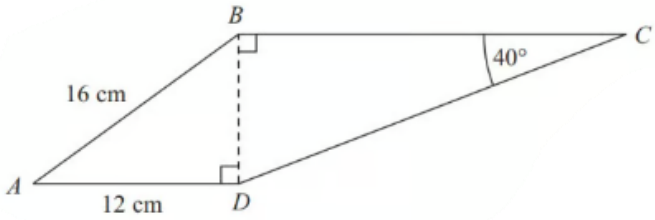


3) triangles ABD and the BCD are right angle triangles
Work out the value of X.
Give your answer 2 correct decimal points.

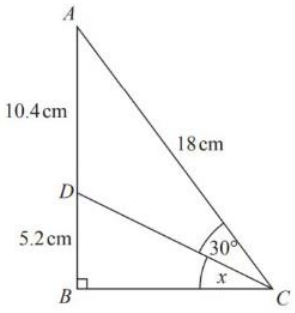


4) The diagram shows the quadrilateral ABCD.

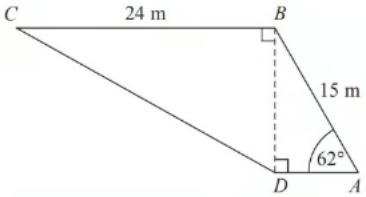
AB = 16 cm. AD = 12 cm.
Angle BCD = 40°.
Angle ADB = angle CBD = 90°.
Calculate the length of CD.
Give your answer correct to 3 significant figures.



5)ABC is a right-angled triangle.
D is a point on AB.
Angle ACD = 30°
AD = 10.4 cm
DB = 5.2 cm
AC = 18 cm
Work out the size of the angle marked x. Give your answer correct to 1 decimal place.

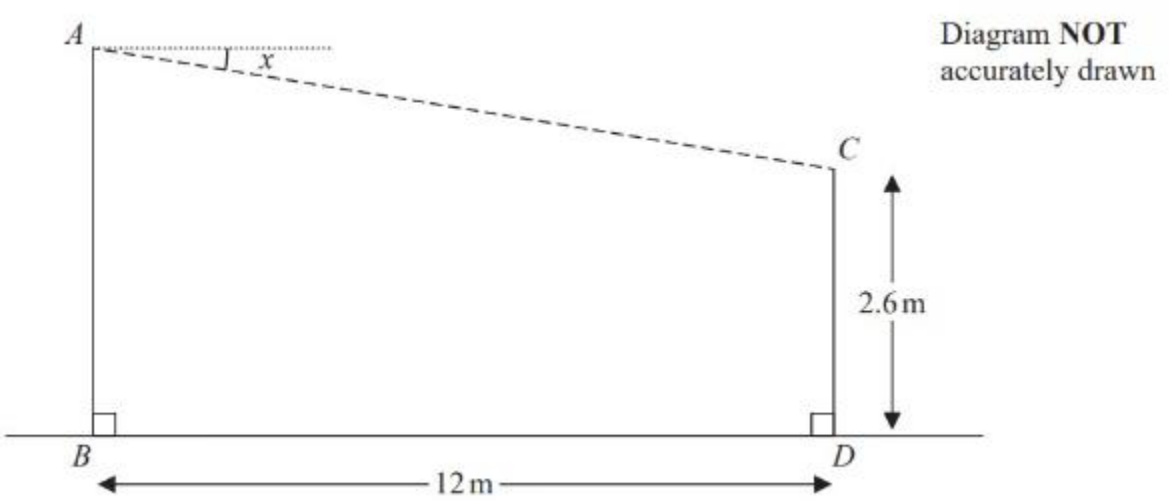


6)AB=15m BC=24m
Angle BAD = 62°
Work out the size of angle BCD.
Give your answer correct to 1 decimal place.

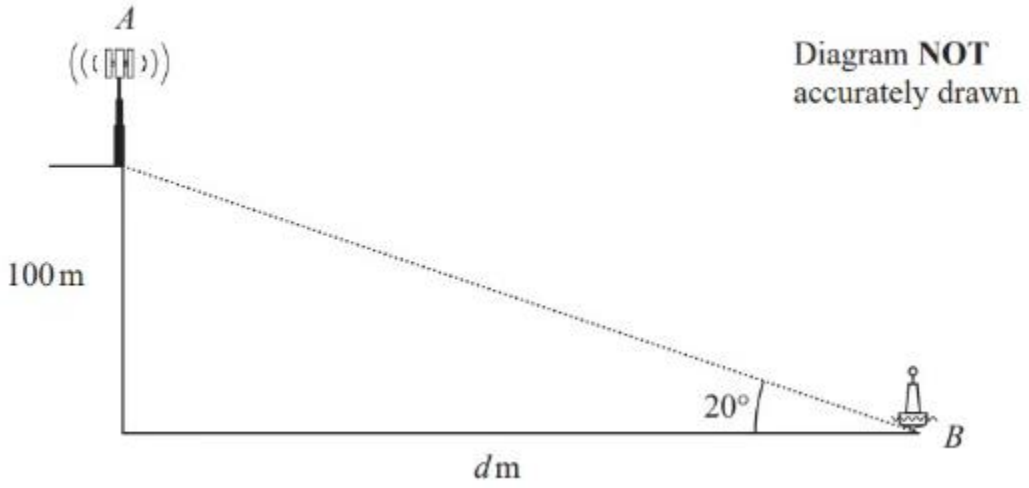


7) A sip wire is shown as the dashed line AC in the diagram

The zip wire is supported by two vertical posts AB and CD standing on horizontal ground.
CD = 2.6 m
BD = 12 m
The zip wire makes an angle x with the horizontal, as shown in the diagram. The design of the zip wire requires the angle x to be at least 5°
Work out the least possible height of the post AB Give your answer correct to 3 significant figures.



8) The diagram shows a vertical cliff with a vertical radio mast on top of the cliff and a buoy in the sea



The height of the cliff is 100 metres.
The buoy is at the point B that is d metres from the base of the cliff.
The angle of elevation from B to the top of the cliff is 20°
Calculate the value of d.
Give your answer correct to 3 significant figures.