Cambridge

IGCSE

Physics

CODE: (9702)

Chapter 10 and Chapter 11

Kirchhoff 's laws and Resistance and resistivity

Kirchhoff's first law

You should be familiar with the idea that current may divide up where a circuit splits into two separate branches. For example, a current of 5.0 A may split at a junction or a point in a circuit into two separate currents of 2.0 A and 3.0 A.

We would not expect some of the current to disappear, or extra current to appear from nowhere. This is the basis of **Kirchhoff's first law**, which states that:

> The sum of the currents entering any point in a circuit is equal to the sum of the currents leaving that same point.

This is illustrated in Figure 10.3. In the first part, the current into point P must equal the current out, so:

$$I_1 = I_2$$

In the second part of the figure, we have one current coming into point Q, and two currents leaving. The current divides at Q. Kirchhoff's first law gives:

$$I_1 = I_2 + I_3$$

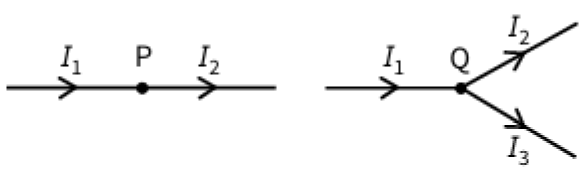

Figure 10.3 Kirchhoff's first law: current is conserved because charge is conserved.

Kirchhoff's first law is an expression of the conservation of **charge.**

The law can be tested by connecting ammeters at different points in a circuit where the current divides. You should recall that an ammeter must be connected in series so the current to be measured passes **through** it.

Formal statement of Kirchhoff's first law

We can write Kirchhoff's first law as an equation:

$$\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$$

Here, the symbol Σ (Greek letter sigma) means 'the sum of all', so ΣI_{in} means 'the sum of all currents entering into a point' and ΣI_{out} means 'the sum of all currents leaving that point'. This is the sort of equation which a computer program can use to predict the behaviour of a complex circuit.

Kirchhoff's second law

This law deals with e.m.f.s and voltages in a circuit. We will start by considering a simple circuit which contains a cell and two resistors of resistances R1 and R2 (Figure 10.8).

we can write the following equation;

$$E = IR_1 + IR_2$$

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e.m.f. of battery = sum of p.d.s across the resistors

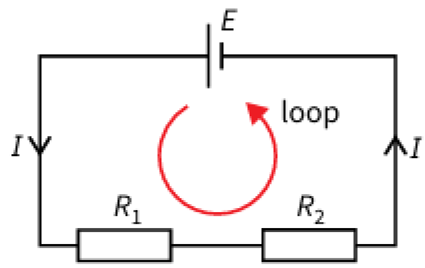


Figure 10.8 A simple series circuit.

You should not find these equations surprising. However, you may not realise that they are a consequence of applying Kirchhoff's second law to the circuit. This law states that:

> The sum of the e.m.f.s around any loop in a circuit is equal to the sum of the p.d.s around the loop.

An equation for Kirchhoff's second law

In a similar manner to the formal statement of the first law, the second law can be written as an equation:

$$\Sigma E = \Sigma V$$

where ΣE is the sum of the e.m.f.s and ΣV is the sum of the potential differences.

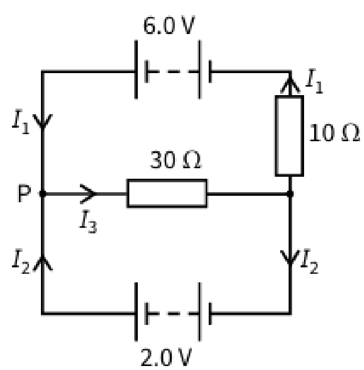


Figure 10.11 Kirchhoff's laws are needed to determine the currents in this circuit.

Signs and directions

Caution is necessary when applying Kirchhoff's second law. You need to take account of the ways in which the sources of e.m.f. are connected and the directions of the currents. Figure 10.12 shows one loop from a larger complicated circuit to illustrate this point.

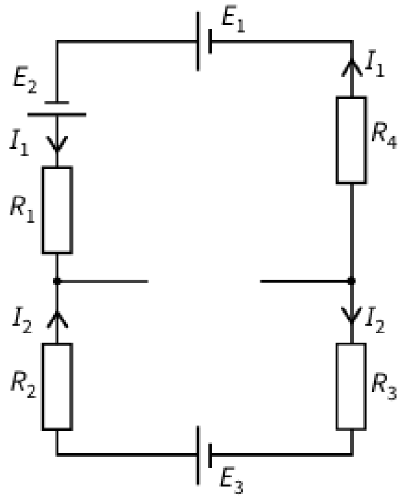


Figure 10.12 A loop extracted from a complicated circuit.

e.m.f.s

Starting with the cell of e.m.f. E_1 and working **anticlockwise** around the loop (because E_1 is 'pushing current' anticlockwise):

$$\text{sum of e.m.f.s} = E_1 + E_2 - E_3$$

Note that E_3 is opposing the other two e.m.f.s.

p.d.s

Starting from the same point, and working **anticlockwise** again:

$$\text{sum of p.d.s} = I_1R_1 - I_2R_2 - I_2R_3 + I_1R_4$$

Note that the direction of current I_2 is clockwise, so the p.d.s that involve I_2 are negative.

Conservation of energy

Kirchhoff's second law is a consequence of the principle of conservation of energy. If a charge, say 1 C, moves around the circuit, it **gains** energy as it moves through each source of e.m.f. and loses energy as it passes through each p.d.

If the charge moves all the way round the circuit, so that it ends up where it started, it must have the same energy at the end as at the beginning.

energy gained passing through sources of e.m.f.
= energy lost passing through components with p.d.s

You should recall that an e.m.f. in volts is simply the energy gained per 1 C of charge as it passes through a source. Similarly, a p.d. is the energy lost per 1 C as it passes through a component.

1 volt = 1 joule per coulomb

Hence we can think of Kirchhoff's second law as:

energy gained per coulomb around loop
= energy lost per coulomb around loop

Resistor combinations

You are already familiar with the formulae used to calculate the combined resistance R of two or more resistors connected in series or in parallel. To derive these formulae we have to make use of Kirchhoff's laws.

Resistors in series

Take two resistors of resistances R1 and R2 connected in series (Figure 10.16). According to Kirchhoff's first law, the current in each resistor is the same. The p.d. V across the combination is equal to the sum of the p.d.s across the two resistors:

$$V = V_1 + V_2$$

Since $V = IR$, $V_1 = IR_1$ and $V_2 = IR_2$, we can write:

$$IR = IR_1 + IR_2$$

Cancelling the common factor of current I gives:

$$R = R_1 + R_2$$

For three or more resistors, the equation for total resistance R becomes:

$$R = R_1 + R_2 + R_3 + \cdots$$

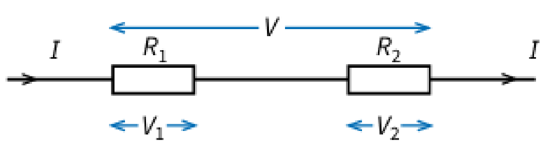


Figure 10.16 Resistors in series.

Resistors in parallel

For two resistors of resistances R1 and R2 connected in parallel (Figure 10.18), we have a situation where the current divides between them. Hence, using Kirchhoff's f irst law, we can write:

$$I = I_1 + I_2$$

If we apply Kirchhoff's second law to the loop that contains the two resistors, we have:

$$I_1R_1 - I_2R_2 = 0\,\text{V}$$

(because there is no source of e.m.f. in the loop).

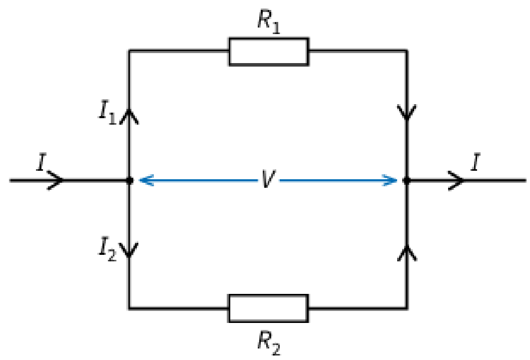


Figure 10.18 Resistors connected in parallel.

This equation states that the two resistors have the same p.d. V across them. Hence we can write

$$I = \frac{V}{R}$$

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

Substituting in $I = I_1 + I_2$ and cancelling the common factor V gives:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

For three or more resistors, the equation for total resistance R becomes:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

To summarise, when components are connected in parallel:

■ ■ all have the same p.d. across their ends

■ ■ the current is shared between them

■ ■ we use the reciprocal formula to calculate their combined resistance.

Solving problems with parallel circuits

Here are some useful ideas which may prove helpful when you are solving problems with parallel circuits (or checking your answers to see whether they seem reasonable).

■ Connecting resistors in parallel results in a lower combined resistance than individual resistances, increasing current drawn from a supply. This is due to the extra pathways provided by the parallel connection, a potential hazard illustrated in Figure 10.19.

■ When components are connected in parallel, they all have the same p.d. across them. This means that you can often ignore parts of the circuit which are not relevant to your calculation.

■ Similarly, for resistors in parallel, you may be able to calculate the current in each one individually, then add them up to find the total current. This may be easier than working out their combined resistance using the reciprocal formula.

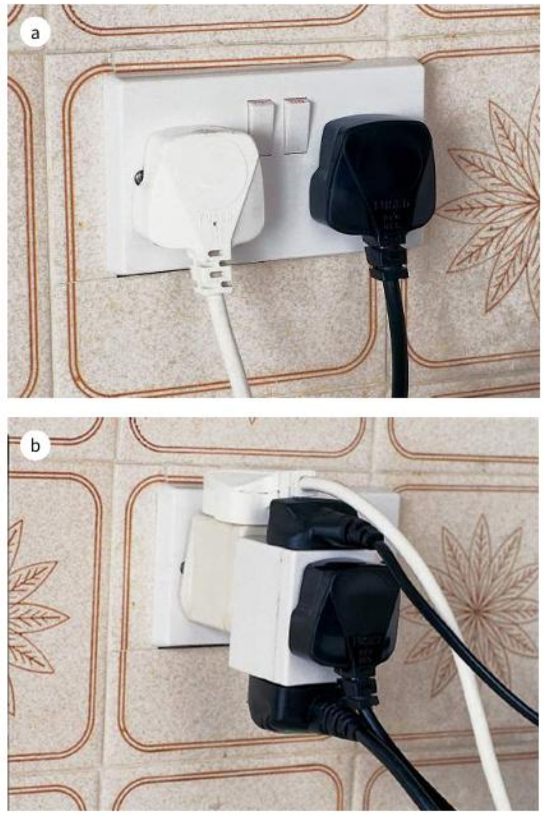


Figure 10.19 **a** Correct use of an electrical socket. **b** Here, too many appliances (resistances) are connected in parallel. This reduces the total resistance and increases the current drawn, to the point where it becomes dangerous.

Chapter 11: Resistance and resistivity

The I–V characteristic for a metallic conductor

Chapter 9 explains how to measure resistance using a voltmeter and ammeter. This section investigates the variation of current and resistance as potential difference across a conductor changes. Variable power supplies or resistors can alter the potential difference, allowing measurements at different potential differences. Figure 11.2 shows an I-V characteristic graph, where current I is directly proportional to voltage V.

The straight-line graph passing through the origin shows that the resistance of the conductor remains constant. If you double the current, the voltage will also double. However, its resistance, which is the ratio of the voltage to the current, remains the same. Instead of using:

$$R = \frac{V}{I}$$

to determine the resistance, for a graph of I against V which is a straight line passing through the origin you can also use:

$$\text{resistance} = \frac{1}{\text{gradient of graph}}$$

You get results similar to those shown in Figure 11.2 for a commercial resistor. Resistors have different resistances, hence the gradient of the I–V graph will be different for different resistors.

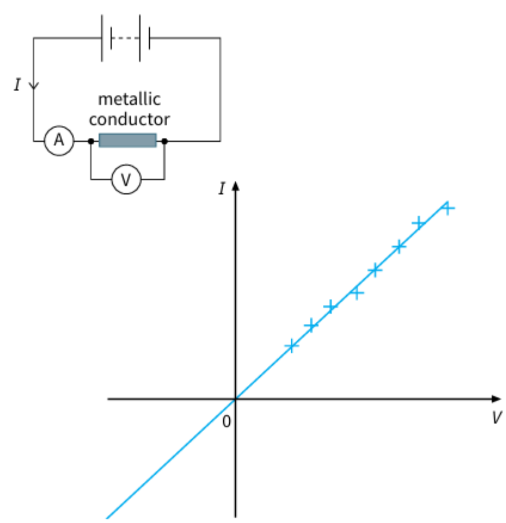


Figure 11.2 To determine the resistance of a component, you need to measure both current and potential difference.

Ohm's law

For the metallic conductor whose I–V characteristic is shown in Figure 11.2, the current in it is directly proportional to the p.d. across it. This means that its resistance is independent of both the current and the p.d.

This is because the ratio V / I is a constant. Any component which behaves like this is described as an ohmic component, and we say that it obeys Ohm's law. The statement of Ohm's law is very precise and you must not confuse this with the equation ' V/ I = R'

Ohm's law

A conductor obeys Ohm's law if the current in it is directly proportional to the potential difference across its ends.

Resistance and temperature

A conductor that does not obey Ohm's law is described as non-ohmic.

Figure 11.3 The metal filament in a lamp glows as the current passes through it. It also feels warm. This shows that the lamp produces both heat and light.

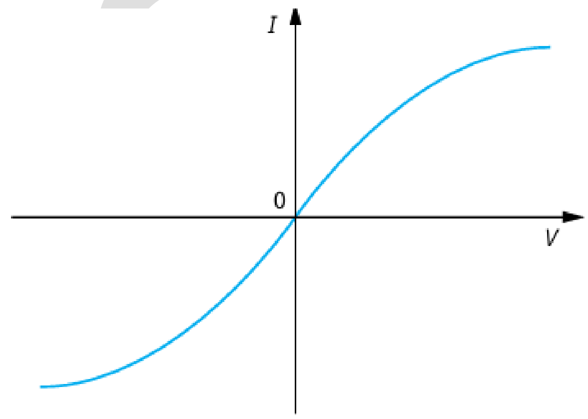


Figure 11.4 The *I–V* characteristic for a filament lamp.

There are some points you should notice about the graph in Figure 11.4:

■ ■ The line passes through the origin (as for an ohmic component).

■ ■ For very small currents and voltages, the graph is roughly a straight line.

■ ■ At higher voltages, the line starts to curve. The current is a bit less than we would have expected from a straight line. This suggests that the lamp's resistance has increased. You can also tell that the resistance has increased because the ratio V/ I is larger for higher voltages than for low voltages.

Thermistors

Thermistors are components that are designed to have a resistance which changes rapidly with temperature. Thermistors ('thermal resistors') are made from metal oxides such as those of manganese and nickel. There are two distinct types of thermistors:

■ ■ Negative temperature coefficient (NTC) thermistors – the resistance of this type of thermistor decreases with increasing temperature. Those commonly used for physics teaching may have a resistance of many thousands of ohms at room temperature, falling to a few tens of ohms at 100 °C. You should become familiar with the properties of NTC thermistors.

■ ■ Positive temperature coefficient (PTC) thermistors – the resistance of this type of thermistor rises abruptly at a definite temperature, usually around 100–150 °C.

The change in their resistance with temperature gives thermistors many uses:

■ ■ Water temperature sensors in cars and ice sensors on aircraft wings – if ice builds up on the wings, the thermistor 'senses' this temperature drop and a small heater is activated to melt the ice.

■ ■ Baby alarms – the baby rests on an air-filled pad, and as he or she breathes, air from the pad passes over a thermistor, keeping it cool; if the baby stops breathing, the air movement stops, the thermistor warms up and an alarm sound.

■ ■ Fire sensors – a rise in temperature activates an alarm.

■ ■ Overload protection in electric razor sockets – if the razor overheats, the thermistor's resistance rises rapidly and cuts off the circuit

Diodes

Figure 11.7 shows the I–V characteristic for a diode. There are some points you should notice about this graph.

■ We have included positive and negative values of current and voltage. This is because, when connected one way round (positively biased), the diode conducts and has a fairly low resistance. Connected the other way round (negatively biased), it allows only a tiny current and has almost infinite resistance.

■ For positive voltages less than about 0.6 V, the current is almost zero and hence the diode has almost infinite resistance. It starts to conduct suddenly at its **threshold voltage**. The resistance of the diode decreases dramatically for voltages greater than 0.6 V.

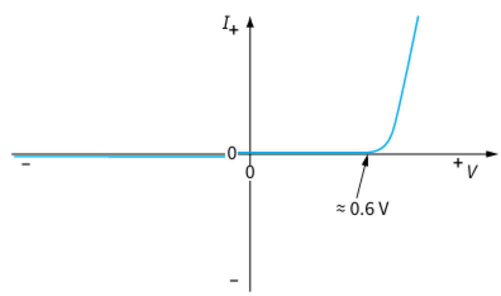


Figure 11.7 The current against potential difference (I–V) characteristic for a diode. The graph is not a straight line. A diode does not obey Ohm's law.

The threshold voltage at which an LED starts to conduct and emit light is higher than 0.6 V and depends on the colour of light it emits, but may be taken to be about 2 V.

Understanding the origin of resistance

The resistance of a pure metal wire increases linearly with temperature from 0°C to 100°C, while an NTC thermistor's resistance decreases dramatically over a narrow temperature range, highlighting the origins of resistance.

Figure 11.8 This torch has seven white LEDs, giving a brighter, whiter light than a traditional filament lamp.

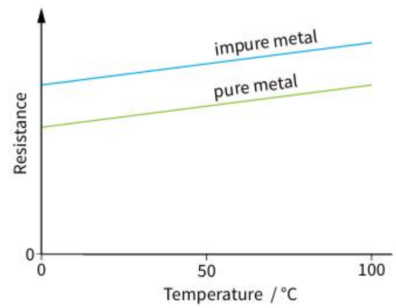


Figure 11.9 The resistance of a metal increases gradually as its temperature is increased. The resistance of an impure metal wire is greater than that of a pure metal wire of the same dimensions.

This suggests that there are two factors which affect the resistance of a metal:

■ ■ the temperature

■ ■ the presence of impurities.

Figure 11.11 shows a simple model which explains what happens in a metal when electrons flow through it.

Electrons lose energy when they collide with vibrating ions or impurity atoms, causing metal to heat up and increase resistance. Conduction in semiconductors occurs when electrons gain enough energy to break free from their atoms, increasing the number of conduction electrons. This makes the material a better conductor, despite more electron-ion collisions.

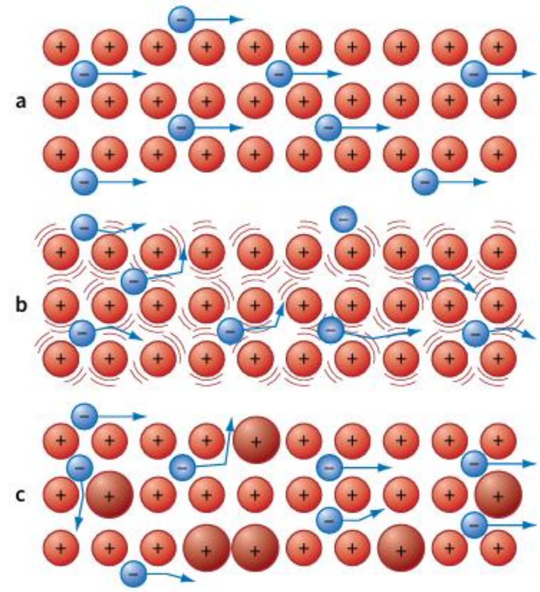


Figure 11.11 A model of the origins of resistance in a metal. **a** At low temperatures, electrons flow relatively freely. **b** At higher temperatures, the electrons are obstructed by the vibrating ions and they make very frequent collisions with the ions. **c** Impurity atoms can also obstruct the free flow of electrons.

Resistivity

For a metal in the shape of a wire, R depends on the following factors:

■ ■ length L

■ ■ cross-sectional area A

■ ■ the material the wire is made from

■ ■ the temperature of the wire.

At a constant temperature, the resistance is directly proportional to the length of the wire and inversely proportional to its cross-sectional area. That is:

$$\text{resistance} \propto \text{length}$$

and

$$\text{resistance} \propto \frac{1}{\text{cross-sectional area}}$$

We can see how these relate to the formulae for adding resistors in series and in parallel:

■ ■ If we double the length of a wire, it is like connecting two identical resistors in series; their resistances add to give double the resistance. The resistance is proportional to the length.

- Doubling the cross-sectional area of a wire is like connecting two identical resistors in parallel; their combined resistance is halved (since $\frac{1}{R_{\text{total}}} = \frac{1}{R} + \frac{1}{R}$). Hence the resistance is inversely proportional to the cross-sectional area.

Combining the two proportionalities for length and cross-sectional area, we get:

$$\text{resistance} \propto \frac{\text{length}}{\text{cross-sectional area}}$$

or

$$R \propto \frac{L}{A}$$

he relevant property of the material is its resistivity, for which the symbol is ρ (Greek letter rho). The word equation for resistance is:

$$\text{resistance} = \frac{\text{resistivity} \times \text{length}}{\text{cross-sectional area}}$$

$$R = \frac{\rho L}{A}$$

$$\text{resistance} = \frac{\text{resistivity} \times \text{length}}{\text{cross-sectional area}}$$

$$R = \frac{\rho L}{A}$$

We can rearrange this equation to give an equation for resistivity. The resistivity of a material is defined by the following word equation:

$$\text{resistivity} = \frac{\text{resistance} \times \text{cross-sectional area}}{\text{length}}$$

$$\rho = \frac{RA}{L}$$

Values of the resistivities of some typical materials are shown in Table 11.2.

Material	Resistivity / Ω m	Material	Resistivity / Ω m
silver	1.60×10^{-8}	mercury	69.0×10^{-8}
copper	1.69×10^{-8}	graphite	800×10^{-8}
nichrome[(a)]	1.30×10^{-8}	germanium	0.65
aluminium	3.21×10^{-8}	silicon	2.3×10^{3}
lead	20.8×10^{-8}	Pyrex glass	10^{12}
manganin[(b)]	44.0×10^{-8}	PTFE[(d)]	10^{13}–10^{16}
eureka[(c)]	49.0×10^{-8}	quartz	5×10^{16}

(a) Nichrome – an alloy of nickel, copper and aluminium used in electric heaters because it does not oxidise at 1000 °C.
(b) Manganin – an alloy of 84% copper, 12% manganese and 4% nickel.
(c) Eureka (constantan) – an alloy of 60% copper and 40% nickel.
(d) Poly(tetrafluoroethene) or Teflon.

Table 11.2 Resistivities of various materials at 20 °C.

Resistivity and temperature

Resistivity, like resistance, depends on temperature. For a metal, resistivity increases with temperature. As we saw above, this is because there are more frequent collisions between the conduction electrons and the vibrating ions of the metal.

Revision questions

1.

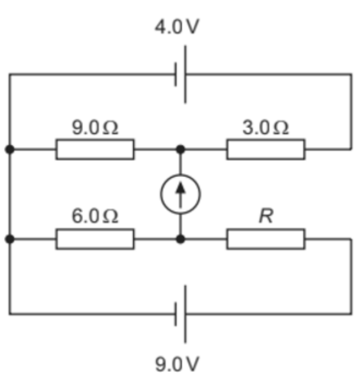
Each of Kirchhoff's two laws presumes that some quantity is conserved.

Which row states Kirchhoff's **first** law and names the quantity that is conserved?

	statement	quantity
A	the algebraic sum of currents into a junction is zero	charge
B	the algebraic sum of currents into a junction is zero	energy
C	the e.m.f. in a loop is equal to the algebraic sum of the product of current and resistance round the loop	charge
D	the e.m.f. in a loop is equal to the algebraic sum of the product of current and resistance round the loop	energy

2.

In the circuit shown, the cells have negligible internal resistance and the reading on the galvanometer is zero.

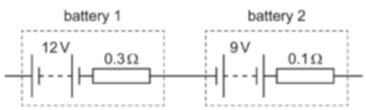


What is the value of resistor *R*?

A 2.0 Ω **B** 6.0 Ω **C** 12 Ω **D** 18 Ω

3.

Two batteries are connected together, as shown.

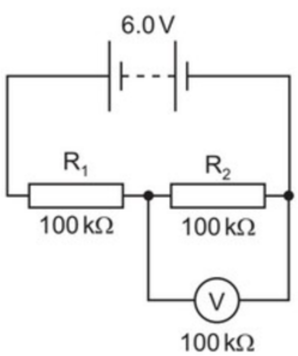


Battery 1 has electromotive force (e.m.f.) 12V and internal resistance 0.3Ω.
Battery 2 has e.m.f. 9V and internal resistance 0.1Ω.
What are the e.m.f. and the internal resistance of a single battery that has the same effect as the combination?

	e.m.f. /V	internal resistance/ Ω
A	3	0.2
B	3	0.4
C	21	0.2
D	21	0.4

4.

In the circuit shown, the 6.0V battery has negligible internal resistance. Resistors R1 and R2 and the voltmeter each have a resistance of 100 $k\omega$.

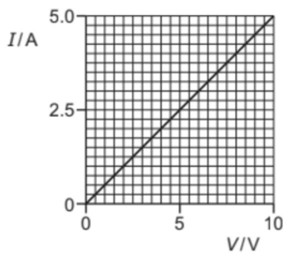


What is the current in the resistor R_2?

A 20 μA B 30 ωA C 40 ωA D 60 ωA

5.

A piece of wire has a length of 0.80 m and a diameter of $5.0 \times 10^{-4} m$. The I–V characteristic of the wire is shown.



What is the resistivity of the metal from which the wire is made?

A $1.2 \times 10^{-7} \Omega m$

B $1.6 \times 10^{-7} \Omega m$

C $4.9 \times 10^{-7} \Omega m$

D $2.0 \times 10^{-6} \Omega m$

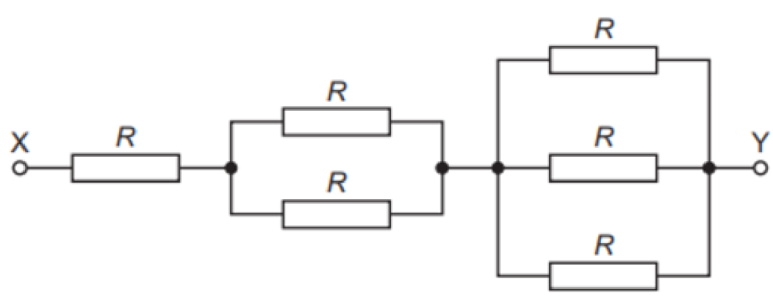
6.

Two copper wires S and T, of equal length, are connected in parallel. Wire S has a diameter of 3.0 mm. Wire T has a diameter of 1.5 mm. A potential difference is applied across the ends of this parallel arrangement. What is the value of the ratio $\frac{current\ in\ S}{current\ in\ T}$

A $\frac{1}{4}$ B $\frac{1}{2}$ C 2 D 4

7.

The diagram shows a network of resistors. Each resistor has resistance R.

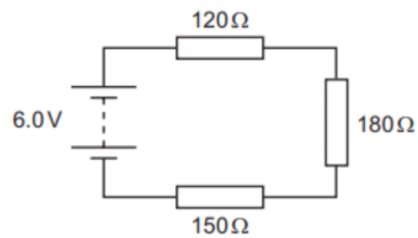


What is the total resistance of the network between points X and Y?

A $\frac{R}{6}$ **B** $\frac{6R}{11}$ **C** $\frac{11R}{6}$ **D** $6R$

8.

Three resistors are connected in series with a battery, as shown. The battery has negligible internal resistance.



What is the potential difference across the 180 Ω resistor?

A 1.6V **B** 2.4V **C** 3.6V **D** 4.0V

10.

A cylindrical metal wire X has resistance R. The same volume of the same metal is made into a cylindrical wire Y of double the length.

What is the resistance of wire Y?

A R B 2R C 4R D 8R