Cambridge

IGCSE

Physics

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Chapter 06

Momentum

The idea of momentum

Snooker players perform impressive moves without Newton's laws of motion, but physics laws can help understand collisions and bounces on the table.

Figure 6.2 If you play pool often enough, you will be able to predict how the balls will move on the table. Alternatively, you can use the laws of physics to predict their motion.

Modelling collisions

<u>Springy collisions</u>

Figure 6.3a illustrates a surprising collision between a snooker ball and a stationary ball, where the moving ball stops dead and the stationary ball moves off with the same velocity.

To achieve this, a snooker player must observe two conditions:

■ The collision must be head-on. (If one ball strikes a glancing blow on the side of the other, they will both move off at different angles.)

■ The moving ball must not be given any spin. (Spin is an added complication which we will ignore in our present study, although it plays a vital part in the games of pool and snooker.)

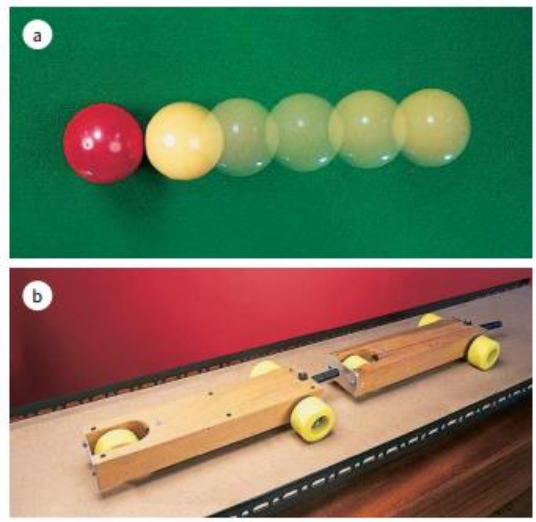


Figure 6.3 **a** The red snooker ball, coming from the left, has hit the yellow ball head-on. **b** You can do the same thing with two trolleys in the laboratory.

Sticky collisions

Figure 6.4 illustrates sticky collisions where trolleys have adhesive pads, causing them to stick together. When a single trolley collides with an identical stationary one, the combined speed is half that of the original trolley. When a trolley's mass increases, its velocity decreases, and vice versa. This demonstrates the importance of adhesion in collisions.

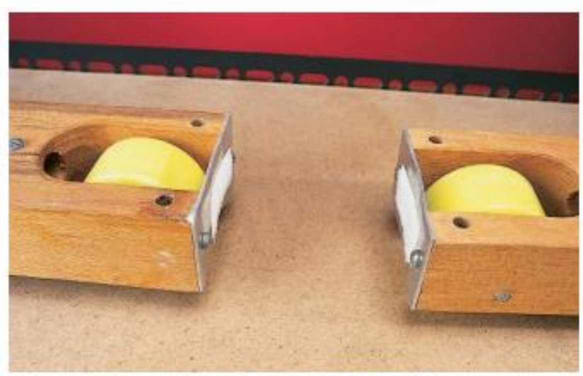


Figure 6.4 If a moving trolley sticks to a stationary trolley, they both move off together.

Defining linear momentum

From the examples discussed above, we can see that two quantities are important in understanding collisions:

■The mass m of the object

■The velocity v of the object.

These are combined to give a single quantity, called the **linear momentum** (or simply momentum) p of an object.

momentum = mass × velocity

$$p = mv$$

The unit of momentum is $\mathrm{kg\,m\,s^{-1}}$. There is no special name for this unit in the SI system.

We have to consider objects which form a **closed system** – that is, no external force acts on them. The principle of **conservation of momentum** states that:

Within a closed system, the total momentum in any direction is constant.

The principle of conservation of momentum can also be expressed as follows:

For a closed system, in any direction:
total momentum of objects before collision
= total momentum of objects after collision

Figure 6.7 The front of each car has crumpled in, as a result of a head-on collision.

Understanding collisions

The car in Figure 6.7 suffered significant damage in a collision due to its crumple zone, designed to absorb the impact. Manufacturers use test labs to study how their cars respond to impacts, combining soft materials with rigid structures. Old-fashioned cars had rigid structures, which increased the likelihood of bouncing back and fatal forces.

Two types of collisions

Collisions between two objects can be either inelastic, where kinetic energy disappears completely, or perfectly elastic, where kinetic energy is converted into heat or sound, resulting in a collision that is either springy or sticky.

We should now use the correct scientific terms, **perfectly elastic** and **inelastic.**

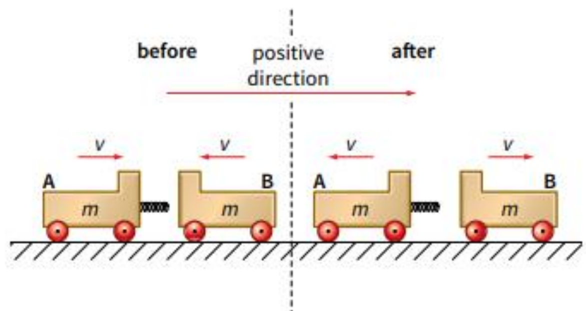


Figure 6.8 Two objects may collide in different ways: this is an elastic collision. An inelastic collision of the same two objects is shown in Figure 6.9.

A perfectly elastic collision

A perfectly elastic collision Two identical objects A and B, moving at the same speed but in opposite directions, have a head-on collision, as shown in Figure 6.8. Each object bounces back with its velocity reversed. This is a perfectly elastic collision.

Before the collision

object A: mass $= m$ velocity $= v$ momentum $= mv$

object B: mass $= m$ velocity $= -v$ momentum $= -mv$

Object B has negative velocity and momentum because it is travelling in the opposite direction to object A. Therefore we have:

$$\text{total momentum before collision} = \text{momentum of A} + \text{momentum of B} = mv + (-mv) = 0$$

$$\text{total kinetic energy before collision} = \text{k.e. of A} + \text{k.e. of B} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 = mv^2$$

After the collision

Both objects have their velocities reversed, and we have:

$$\text{total momentum after collision} = (-mv) + mv = 0$$

$$\text{total kinetic energy after collision} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 = mv^2$$

So the total momentum and the total kinetic energy are unchanged. They are both conserved in a perfectly elastic collision such as this.

In this collision, the objects have a **relative speed** of $2v$ before the collision. After their collision, their velocities are reversed so their relative speed is $2v$ again. This is a feature of perfectly elastic collisions.

An inelastic collision

In Figure 6.9, the same two objects collide, but this time they stick together after the collision and come to a halt. Clearly, the total momentum and the total kinetic energy are both zero after the collision, since neither mass is moving. We have:

	Before collision	After collision
momentum	0	0
kinetic energy	$\frac{1}{2}mv^2$	0

Momentum is conserved in collisions, unlike kinetic energy which can be converted into other forms like sound or internal energy. The principle of conservation of energy ensures that the total amount of energy remains constant.

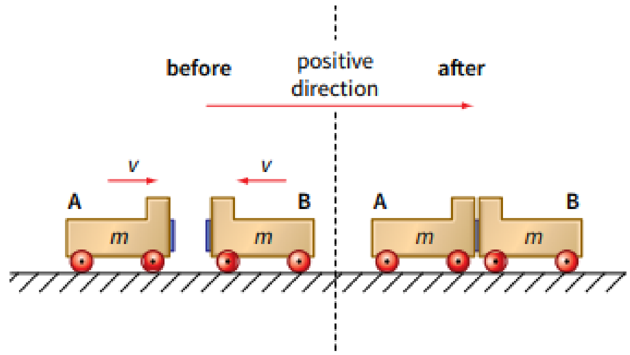


Figure 6.9 An inelastic collision between two identical objects. The trolleys are stationary after the collision.

Explosions and crash-landings

The principle of conservation of momentum suggests that momentum is created or disappears without trace. In situations like rocket explosions, momentum is created by the spreading out of burning materials equally in all directions. However, since momentum is a vector quantity, the total amount of momentum created is zero.

More fireworks

The Roman candle fires a jet of burning material upwards, creating momentum. However, the chemical molecules have momentum upwards, while the Roman candle pushes downwards on Earth, causing an equal amount of downward momentum. This tiny change in Earth's velocity is not noticeable.

Down to Earth

The momentum of a large rock falling over a cliff is due to the Earth's gravity, which causes the rock to gain kinetic energy and momentum downwards. As the rock falls, the Earth's momentum also decreases, causing the Earth to move upwards. When the rock hits the ground, its momentum becomes zero, and the Earth's momentum also cancels out. This process conserves momentum throughout the fall and crash-landing.

total momentum of Earth and rock = 0

Hence:

$$(60 \times 20) + (6.0 \times 10^{24} \times v) = 0$$

$$v = -2.0 \times 10^{-22}\,\mathrm{m\,s^{-1}}$$

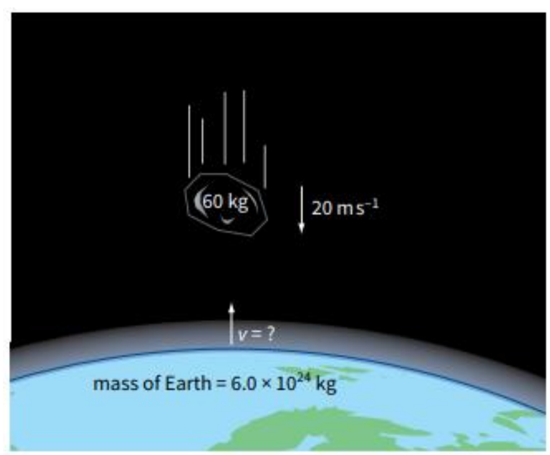


Figure 6.13 The rock and Earth gain momentum in opposite directions.

Collisions in two dimensions

Figure 6.14 shows a two-dimensional collision between two snooker balls. From the multiple images, we can see how the velocities of the two balls change:

■ At first, the white ball is moving straight forwards. When it hits the red ball, it moves off to the right. Its speed decreases: we can see this because the images get closer together.

■ The red ball moves off to the left. It moves off at a bigger angle than the white ball, but more slowly – the images are even closer together.

The momentum of each ball in a collision is conserved, as each has a sideways component of velocity. The red ball moves at a greater angle but has less velocity than the white ball, resulting in the same component of velocity at right angles to the original track.

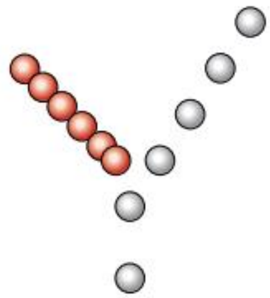


Figure 6.14 The white ball strikes the red ball a glancing blow. The two balls move off in different directions.

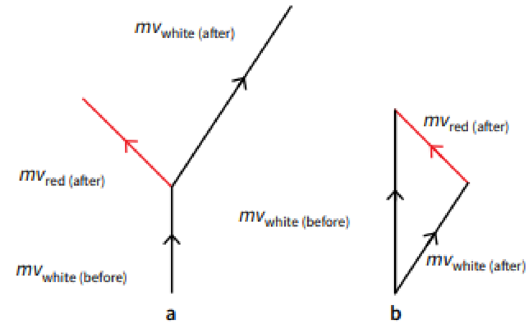


Figure 6.15 **a** These vectors represent the momenta of the colliding balls shown in Figure 6.14. **b** The closed vector triangle shows that momentum is conserved in the collision.

Momentum and Newton's laws

The big ideas of physics are often simple and applicable in various situations. Concepts like force, energy, and voltage require imagination and refinement over decades. Isaac Newton's work on motion, published in the Principia, is an example of this process.

Understanding motion

In Chapter 3, we looked at Newton's laws of motion. We can get further insight into these laws by thinking about them in terms of momentum.

Newton's first law of motion

Momentum refers to something's free will tendency to keep moving, as seen in an oil tanker's ability to stop at sea. This concept is related to Newton's first law of motion.

An object will remain at rest or keep travelling at constant velocity unless it is acted on by a resultant force.

Newton's second law of motion

Newton's second law of motion links the idea of the resultant force acting on an object and its momentum. A statement of Newton's second law is:

The resultant force acting on an object is directly proportional to the rate of change of the linear momentum of that object. The resultant force and the change in momentum are in the same direction.

The resultant force acting on an object is equal to the rate of change of its momentum. The resultant force and the change in momentum are in the same direction.

Hence:

resultant force ∝ rate of change of momentum

This can be written as:

$$F \propto \frac{\Delta p}{\Delta t}$$

force = rate of change of momentum

$$F = \frac{\Delta p}{\Delta t}$$

A special case of Newton's second law of motion

Imagine an object of constant mass m acted upon by a resultant force F. Th e force will change the momentum of the object. According to Newton's second law of motion, we have:

$$F = \frac{\Delta p}{\Delta t} = \frac{mv - mu}{t}$$

where u is the initial velocity of the object, v is the fi nal velocity of the object and t is the time taken for the change in velocity. Th e mass m of the object is a constant; hence the above equation can be rewritten as:

$$F = \frac{m(v-u)}{t} = m\left(\frac{v-u}{t}\right)$$

Th e term in brackets on the right-hand side is the acceleration a of the object. Th erefore a special case of Newton's second law is:

F = ma

Newton's third law of motion

Newton's third law of motion is about interacting objects. Th ese could be two magnets attracting or repelling each other, two electrons repelling each other, etc. Newton's third law states:

When two bodies interact, the forces they exert on each other are equal and opposite.

The concept of momentum can be illustrated by comparing two magnets, one in each hand, attracting each other due to the same force. Even if one magnet is stronger, they still attract each other equally. When released, the magnets gain momentum, with one gaining momentum to the left and the other equal to the right.

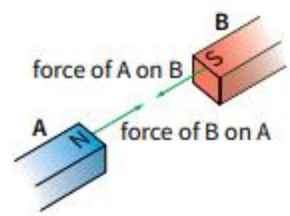


Figure 6.21 Newton's third law states that the forces these two magnets exert one each other must be equal and opposite.

Revision questions

1)A golf club undergoes an inelastic collision with a stationary golf ball and gives it an initial velocity of 60 m s^{-1}. The ball is in contact with the club for 15 ms and the mass of the ball is 4.5×10^{-2} kg.

(a) Explain what is meant by an inelastic collision.
(b) Calculate
(i) the change in momentum of the ball
(ii) the average force the club exerts on the ball.

2) In a football match, a player kicks a stationary football of mass 0.44 kg and gives it a speed of 32 m s^{-1}

(a) (i) Calculate the change of momentum of the football.

(ii) The contact time between the football and the footballer's boot was 9.2 m s. Calculate the average force of impact on the football

3) A video recording showed that the toe of the boot was moving on a circular arc of radius 0.62 m centred on the knee joint when the football was struck. The force of the impact slowed the boot down from a speed of 24 m s^{-1} to a speed of 15 m s^{-1}

Figure 1

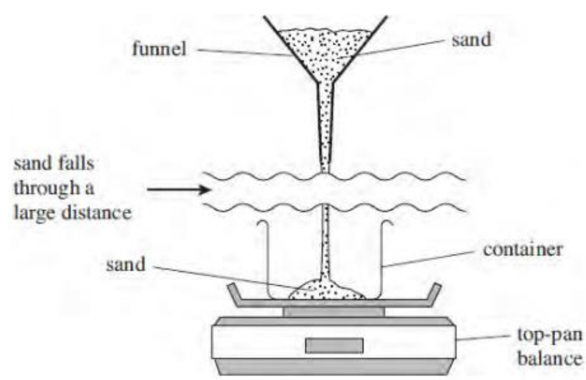
(i) Calculate the deceleration of the boot along the line of the impact force when it struck the football.

(ii) Calculate the centripetal acceleration of the boot just before impact.

iii) Discuss briefly the radial force on the knee joint before impact and during the impact.

4) a) State, in words, the relationship between the force acting on a body and the momentum of the body.

B)A container rests on a top-pan balance, which measures mass in kg. A funnel above the container holds some sand. The sand falls at a constant rate of 0.300 kg s–1 into the container, having fallen through an average vertical height of 1.60 m. This arrangement is shown in the figure below.

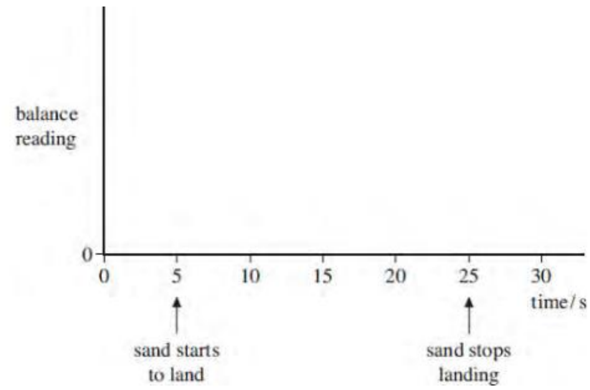


(b) (i) Show that the velocity of the sand as it lands in the container is 5.6 ms^{-1}

ii) Calculate the magnitude of the momentum of the sand that lands in the container in each second.

(iii) The mass of the container is 0.650 kg. Show that the reading of the balance, 10.0 s after the sand starts landing continuously in the container, will be 3.82 kg. You may assume that the sand comes to rest without rebounding when it lands in the container.

(c) It takes 20.0 s for all of the sand to fall into the container. On the axes below, sketch a graph to show how the reading of the balance will change over a 30.0 s period, where t = 5.0 s is the time at which the sand starts to land in the container. No further calculations are required and values need not be shown on the vertical axis of the graph.



5)(a) State two quantities that are conserved in an elastic collision.

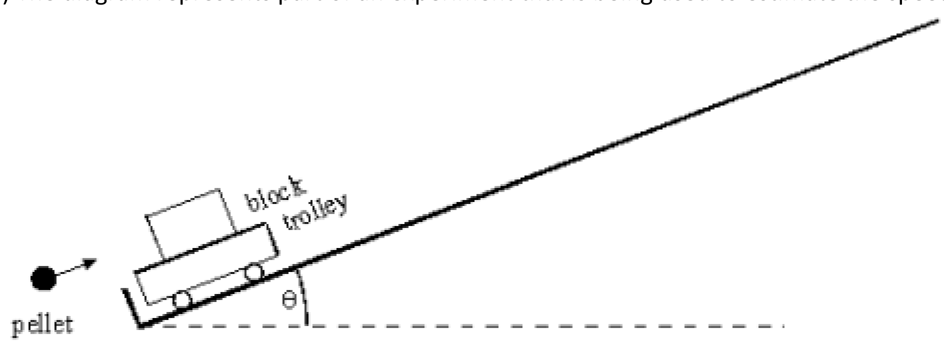
(b) A gas molecule makes an elastic collision with the walls of a gas cylinder. The molecule is travelling at 450 m s^{-1} at right angles towards the wall before the collision.

(i) What is the magnitude and direction of its velocity after the collision?

(ii) Calculate the change in momentum of the molecule during the collision if it has a mass of 8.0×10^{-26} kg.

(c) Use Newton's laws of motion to explain how the molecules of a gas exert a force on the wall of a container. You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

7) The diagram represents part of an experiment that is being used to estimate the speed of an air gun pellet.



The pellet which is moving parallel to the track, strikes the block, embedding itself. The trolley and the block then move along the track, rising a vertical height, h.

(a) Using energy considerations explain how the speed of the trolley and block immediately after it has been struck by the pellet, may be determined from measurements of h. Assume frictional forces are negligible.

(b) The following data is collected from the experiment

mass of trolley and block	0.50 kg
mass of pellet	0.0020 kg
speed of trolley and block immediately after impact	0.40 m s^{-1}

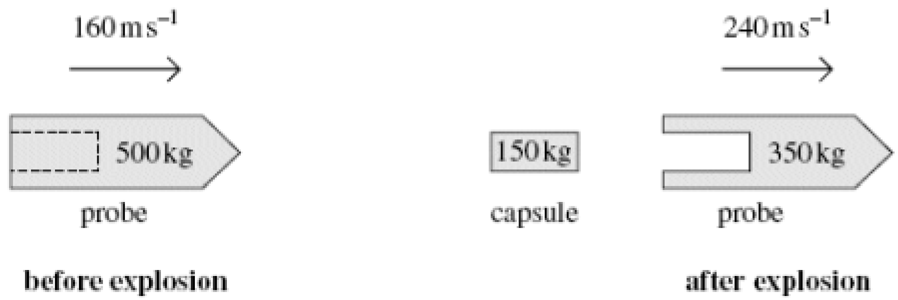
Calculate (i) the momentum of the trolley and block immediately after impact,

(ii) the speed of the pellet just before impact.

(c) (i) State what is meant by an inelastic collision.

(ii) Use the data from part (b) to show that the collision between the pellet and block is inelastic.

8) Deep space probes often carry modules which may be ejected from them by an explosion. A space probe of total mass 500 kg is travelling in a straight line through free space at 160 m s^{-1} when it ejects a capsule of mass 150 kg explosively, releasing energy. Immediately after the explosion the probe, now of mass 350 kg, continues to travel in the original straight line but travels at 240 m s^{-1} as shown in the figure below.



(a) Discuss how the principles of conservation of momentum and conservation of energy apply in this instance. The quality of your written communication will be assessed in this question.

(b) (i) Calculate the magnitude of the velocity of the capsule immediately after the explosion and state its direction of movement.

ii) Determine the total amount of energy given to the probe and capsule by the explosion.